2.4 Naloge

Reši naslednje diferencialne enačbe z ločljivima spremenljivkama:

1. $\left(xy^2 + x\right)dx + \left(y - x^2 y\right)dy = 0\,;$

2. $\left(1 + y^2\right)x\,dx + \left(1 + x^2\right)dy = 0\,;$

3. $\left(1 + 2y\right)x\,dx + \left(1 + x^2\right)dy = 0\,;$

4. $xy\left(1 + x^2\right)y' = 1 + y^2\ ;$

5. $e^x\left(\frac{dy}{dx} + 1\right) = 1\,;$

6. $y\frac{dy}{dx} + x = 1\,;$

7. $xy' + y = y^2\,;$

8. $\left(1 + y^2\right)dx - xy\,dy = 0\ ,\ \ y(2) = 1\,;$

9. $\sin x\ y' = y\ln y\ \ ,\ y\left(\frac{\pi}{2}\right) = 1\,;$

10. $\left(2x + 1\right)dy + y^2\,dx = 0\,,\quad y(4) = 1\,;$

11. $2\sqrt{y}\,dx = dy\ ,\ \ y(0) = 1\,;$

Reši naslednje homogene enačbe:

12. $y^2\,dx + x(x-y)dy = 0$;

13. $\left(x^3 + y^3\right)dx - 3xy^2dy = 0$;

14. $xy\,dx + \left(x^2 + y^2\right)dy = 0$;

15. $(x+y)dx + (y-x)dy = 0$;

16. $xy' - y = y\ln\frac{y}{x}$;

17. $x^2 - y^2 + 2xyy' = 0$;

18. $\left(y + \sqrt{x^2 - y^2}\right)dx - x\,dy = 0$;

19. $x(\ln x - \ln y)dy - y\,dx = 0$;

20. $y' = \frac{y}{x} + e^{\frac{y}{x}}$;

Reši naslednje linearne diferencialne enačbe:

21. $y' + y\cos x = \cos x \sin x$;

22. $y' - y = e^x$;

23. $\frac{dy}{dx} = x + y$;

24. $y' + ay = ce^{bx}$;

25. $y' + x^2 y = x^2$;

26. $y' - \frac{n}{x+1} y = e^x (x+1)^n$;

27. $x(x-1)y' + (1-2x)y + x^2 = 0$;

28. $xy' + y = \ln x$;

29. $y' - \frac{2}{x} y = x^4$;

30. $(1+x^2)y' + xy = \frac{1}{1+x^2}$;

31. $y' + y\cos x = e^{-\sin x}$;

32. $y' + y\tan x = \sin 2x$;

33. $x\frac{dy}{dx} = (x-1)e^x + y$;

34. $y' = a\sin x + by$;

35. $\frac{dy}{dx} + \frac{4xy}{x^2+1} = \frac{1}{x^2+1}$;

Reši naslednje Bernullijeve enačbe:

36. $y' + 2ax^3 y^3 + 2xy = 0$;

37. $y' + y = xy^3$;

38. $xy' + xy^2 - y = 0$;

39. $y' + y = x\sqrt{y}$;

40. $\left(x^2 - 1\right)y' - y\left(y - x\right) = 0$;

41. $2xyy' - y^2 + ax = 0$;

42. $y^2 + \left(1 - xy\right)y' = 0$;

43. $y' + \frac{2y}{x} = \frac{2\sqrt{y}}{\cos^2 x}$;

44. $x\frac{dy}{dx} + 2y = x^5 y^2$.

2.5 Rešitve

1. $y^2 = C\left(x^2 - 1\right) - 1$;

2. $\arctan y + \frac{1}{2}\ln\left(1 + x^2\right) = C$;

3. $y = \frac{2C - x^2}{2\left(1 + x^2\right)}$;

4. $\frac{x}{\sqrt{1 + y^2}\sqrt{1 + x^2}} = C$;

5. $y = -x - e^x + C$;

6. $y^2 + x^2 - 2x = C$;

7. $y = 1 + Ce^x, \quad y = 0$;

8. $y = \pm\sqrt{\frac{x^2}{2} - 1}$;

9. $\ln y = \frac{1 - \cos x}{\sin x}$;

10. $y = \frac{1}{1 + \frac{1}{2}\ln\frac{1+2x}{9}}$;

11. $y = (1+x)^2$;

12. $x = \frac{y}{\ln(Cy)}$;

13. $x^3 - 2y^3 = Cx$;

14. $x^2y^2 + \frac{y^4}{2} = C$;

15. $\sqrt{x^2 + y^2} = C\arctan\left(\frac{y}{x}\right)$;

16. $x = C\ln\frac{y}{x}$;

17. $y^2 + x^2 = Cx$;

18. $x = Ce^{\arcsin\frac{y}{x}}$;

19. $x = ye^{Cy+1}$;

20. $x = Ce^{-e^{-\frac{y}{x}}}$;

21. $y = \sin x - 1 + Ce^{-\sin x}$;

22. $y = \frac{e^x}{2} + Ce^{-x}$;

23. $y = -x - 1 + Ce^x$;

24. $y = \frac{ce^{bx}}{a+b} + Ce^{-ax}$;

25. $y = 1 + Ce^{-\frac{x^3}{3}}$;

26. $y = (1+x)^n\left(e^x + C\right)$;

27. $y = x + Cx(x-1)$;

28. $y = \ln x - 1 + \frac{C}{x}$;

29. $y = \frac{x^5}{3} + Cx^2$;

30. $y = \frac{x + C\sqrt{1+x^2}}{1+x^2}$;

31. $y = e^{-\sin x}(x + C)$;

32. $y = (C - 2\cos x)\cos x$;

33. $y = e^x + Cx$;

34. $y = -\frac{a(\cos x + b\sin x)}{1+b^2} + Ce^{bx}$;

35. $y = \frac{1}{3}\frac{x^3 + 3x + 3C}{\left(x^2+1\right)^2}$;

36. $\frac{1}{y^2} = -a\left(x^2 + \frac{1}{2}\right) - Ce^{2x^2}$;

37. $\frac{1}{y^2} = x + \frac{1}{2} + Ce^{2x}$;

38. $\frac{1}{y} = \frac{x^2 + C}{2x}$;

39. $\sqrt{y} = x - 2 + Ce^{-\frac{x}{2}}$;

40. $\frac{1}{y} = x + C\sqrt{x^2 - 1}$;

41. $y^2 = -ax\ln x + Cx$;

42. $\frac{x}{y} - \frac{1}{2y^2} = C$;

43. $\sqrt{y} = \tan x + \frac{1}{x}\ln\cos x + \frac{C}{x}$;

44. $\frac{1}{y} = -\frac{x^5}{3} + Cx^2$.