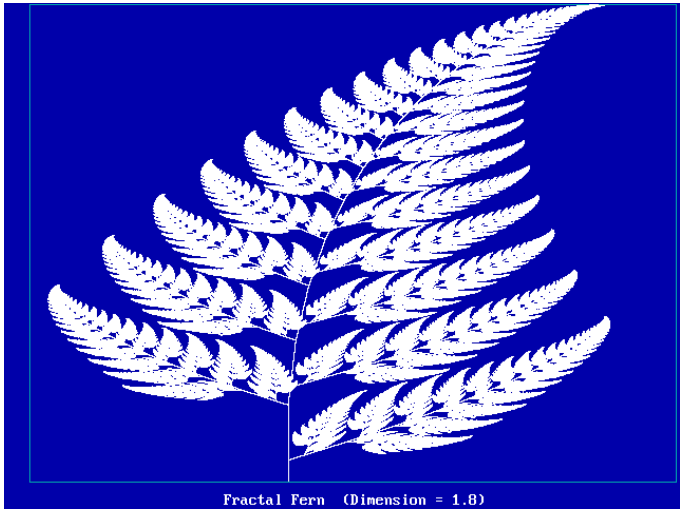
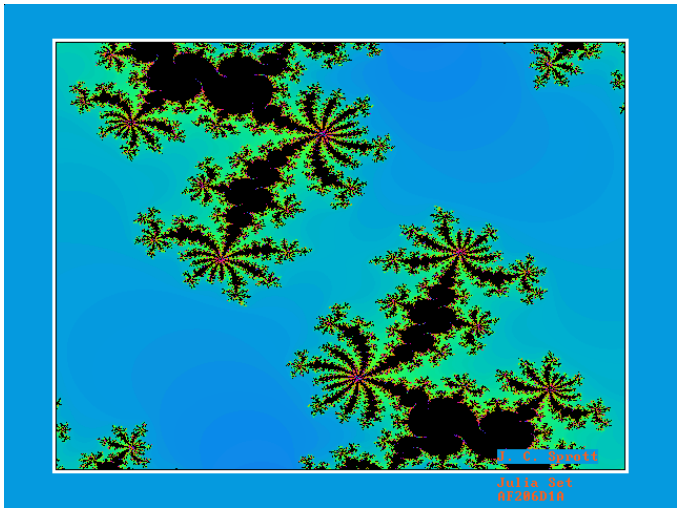
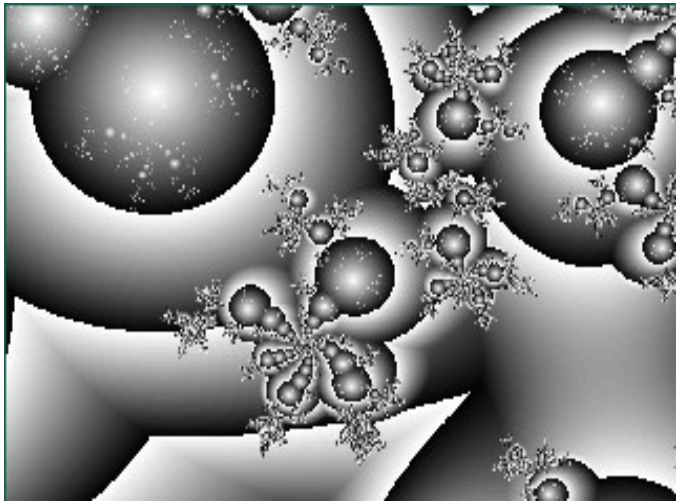
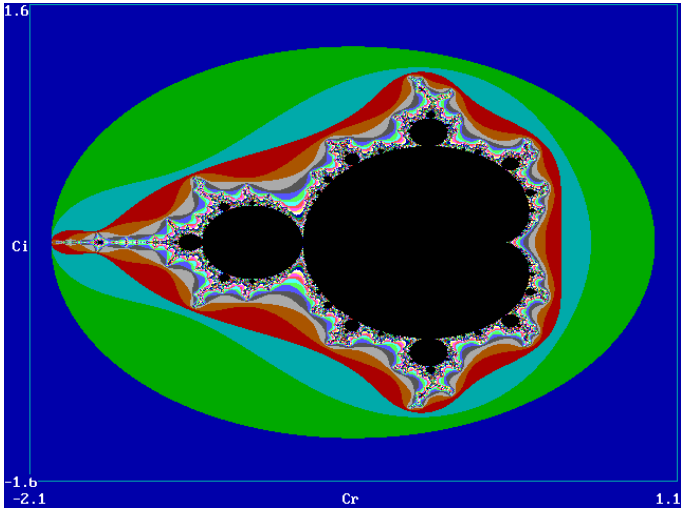
Fraktali

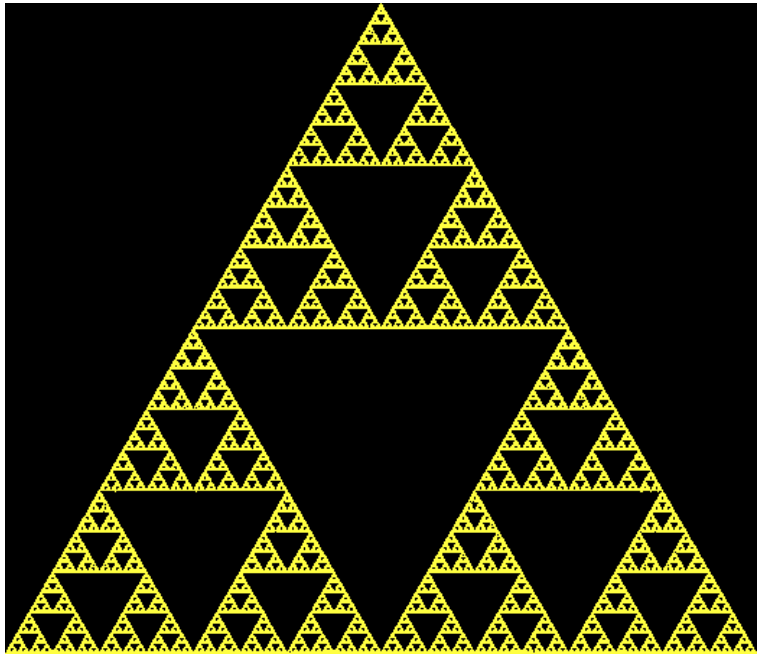

Fractal Fern (Dimension = 1.8)


Julia Set
8U286D1R

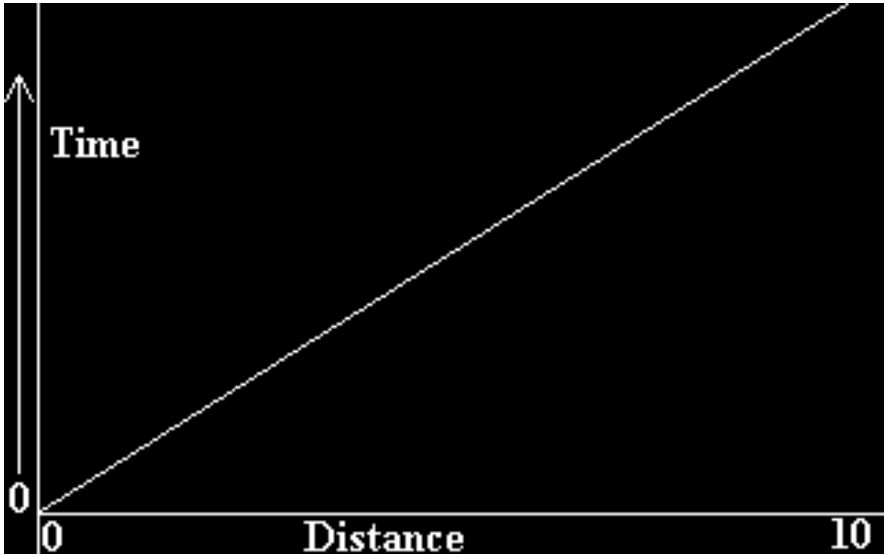


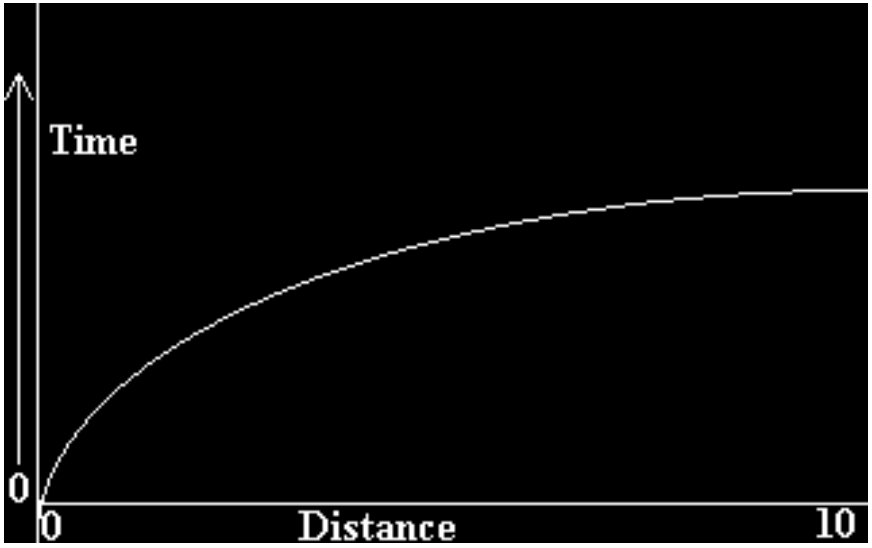

1.6
Ci
-1.6
-2.1
Cr
1.1

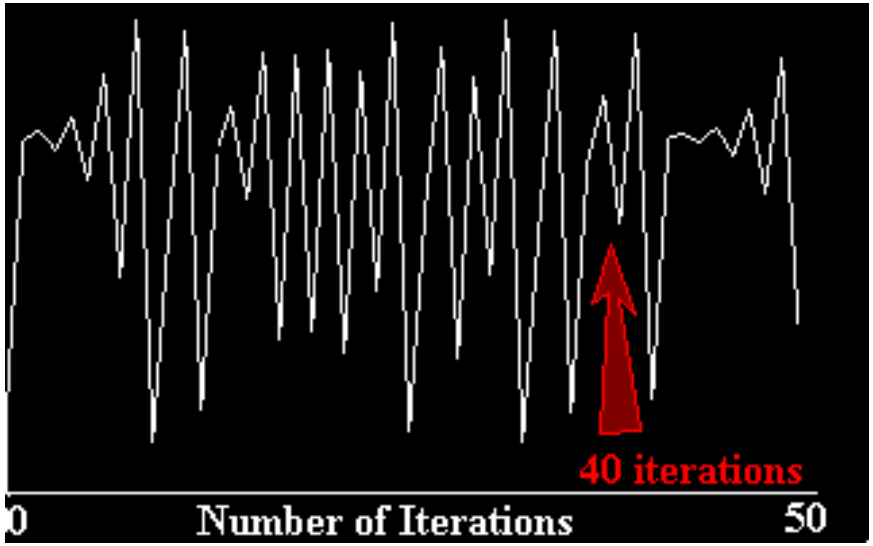
Kaj je fraktal

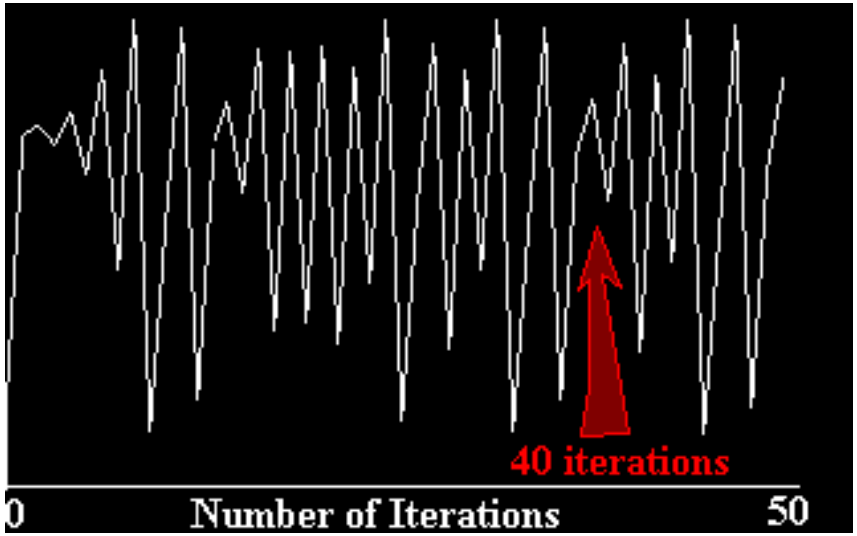


Fraktal je geometrični konstrukt, za katerega velja pri različnih nivojih podrobnosti samopodobnost.


Time
0
0
Distance
10

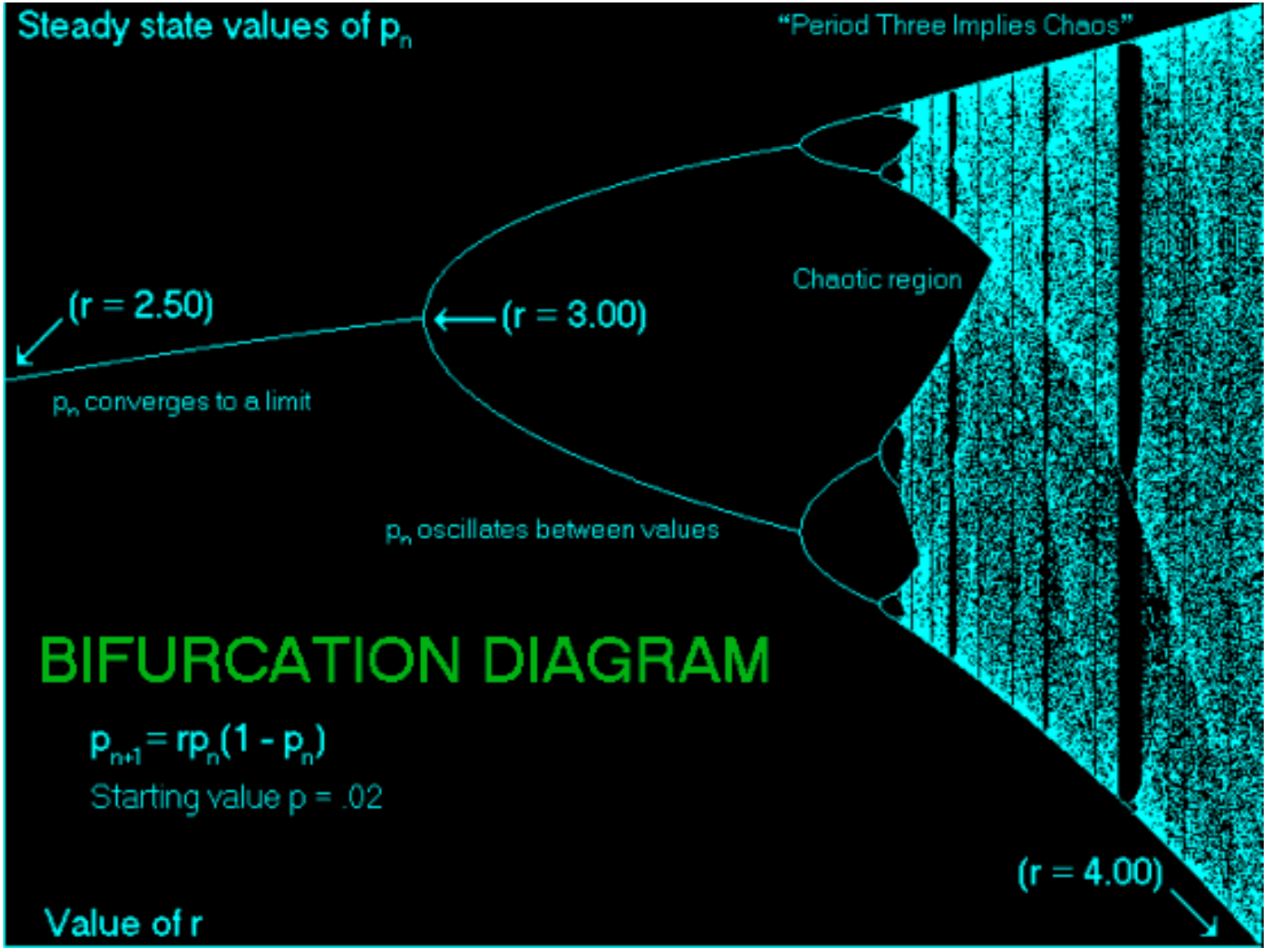

Time
0
0
Distance
10





V ilustracijo poglejmo spodnja dva grafa. Že po 40 iteracijah se zelo razlikujeta, čeprav je bil njun začetni potek navidezno enak.

Bifurkacija in periodičnost



Demo

Teorija kaosa

Teorija kaosa temelji na tem, da lahko majhne spremembe povzroče velika nihanja. Res je sicer, da ne moremo napovedovati natančnega stanja nekega sistema, lahko pa modeliramo splošno obnašanje takega sistema.

Lorenz je študiral sisteme enačb.Tako je za področje fluidne dinamike uporabil naslednji poenostavljeni sistem enačb:
dx/dt = delta * (y - x) dy/dt = r * x - y - x * z
dz/dt = x * y - b * z
Pri tem je Lorenz oporabil naslednje parametre:
delta = 10
r = 28
b = 8/3

Lorenz, efekt metulja, Lorenzov atraktor Demo

Kaos in fraktali - nihalo Demo

Lastnosti fraktalov

Ena od lastnosti je **samopodobnost** oziroma ponavljanje vzorcev pri vseh povečavah. Druga tipična lastnost je **neskončna kompleksnost** podrobnosti. Formalna definicija fraktala uvaja še pojem **fraktalske dimenzije**.

Fraktalska dimenzija (Anglija) Demo

Fraktalska dimenzija (Kochova snežinka) Demo

Samopodobnost fraktalov

- Parts are scaled down versions of the entire object
  - use same scaling on subparts
  - use different scaling factors for subparts
- Statistically self-similar
  - Apply random variation to subparts
    - Trees, shrubs, other vegetation

Samopodobnost (Mandelbrot) Video

Samopodobnost (Lorenz) Video

Samopodobnost (Mandelbrot) Video

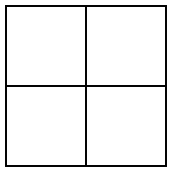
Fraktalska dimenzija

- D=fractal dimension
  - Amount of variation in the structure
  - Measure of roughness or fragmentation of the object
    - Small d-less jagged
    - Large d-more jagged
    - s=scaling factor
    - n number of subparts in subdivision
    - d=log(n)/log(1/s)

Fraktalska dimenzija

Točka: D = 0, N=1, s=1/2

Črta: D = 1, N=2, s=1/2

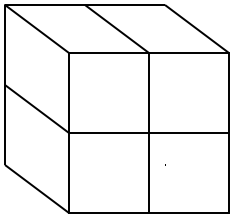


Kvadrat: D = 2, N=4, s=1/2

$N = (1/s)^D$

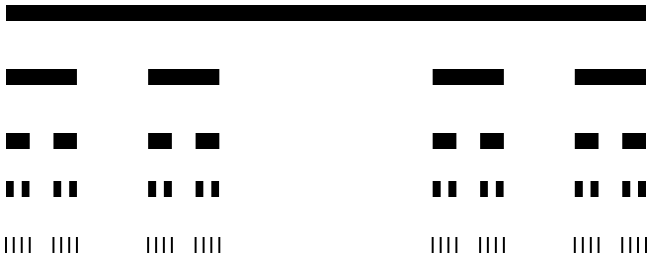
$\log N = D \log (1/s)$

$$D = \log(N)/\log(1/s)$$

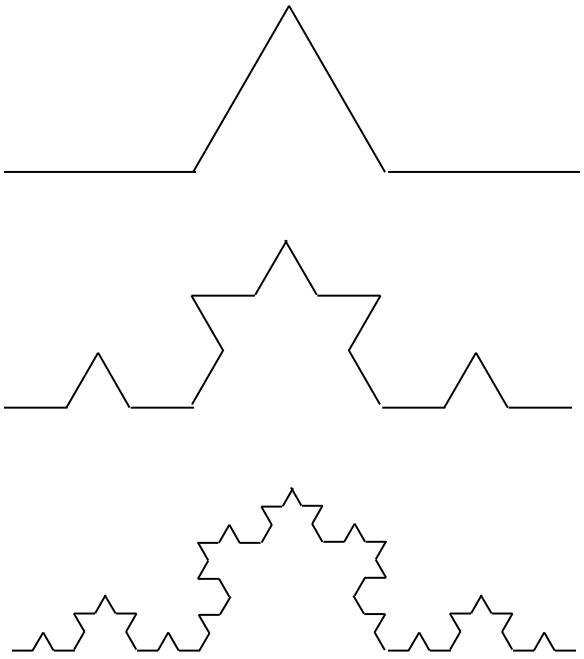


Kocka: D = 3, N=8, s=1/2

Dimenzija fraktala: primeri



N=2
s=1/3
D = log 2/log 3
D = .6...

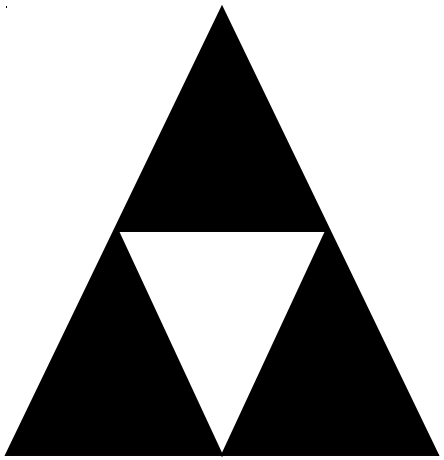


N=4
s=1/3
D = log 4/log 3
D = 1.3...

Kaj je torej dimenzija fraktala

- Dimension is a ratio of the (new size)/(old size)
  - Divide line into n identical segments
    - n=s
  - Divide lines on square into small squares by dividing each line into n identical segments
    - $n=s^2$ small squares
  - Divide cube
    - Get $n=s^3$ small cubes
- Koch's snowflake
  - After division have 4 segments
    - n=4 (new segments)
    - s=3 (old segments)
    - Fractal Dimension
      - D=log4/log3 = 1.262
  - For your reference: Book method
    - n=4
      - Number of new segments
    - s=1/3
      - segments reduced by 1/3
    - d=log4/log(1/(1/3))

Dimenzija fraktala Sierpinski



- Divide each side by 2
  - Makes 4 triangles
  - We keep 3
  - Therefore n=3
    - Get 3 new triangles from 1 old triangle
  - s=2 (2 new segments from one old segment)
- Fractal dimension
  - D=ln(3)/ln(2) = 1.585

Dimenzija fraktala kocka (cube)

- Apply fractal algorithm
  - Divide each side by 3
  - Now push out the middle face of each cube
  - Now push out the center of the cube
- What is the fractal dimension?
  - Well we have 20 cubes, where we used to have 1
    - n=20
  - We have divided each side by 3
    - s=3
  - Fractal dimension ln(20)/ln(3) = **2.727**

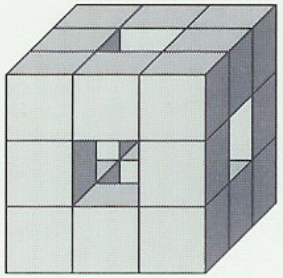
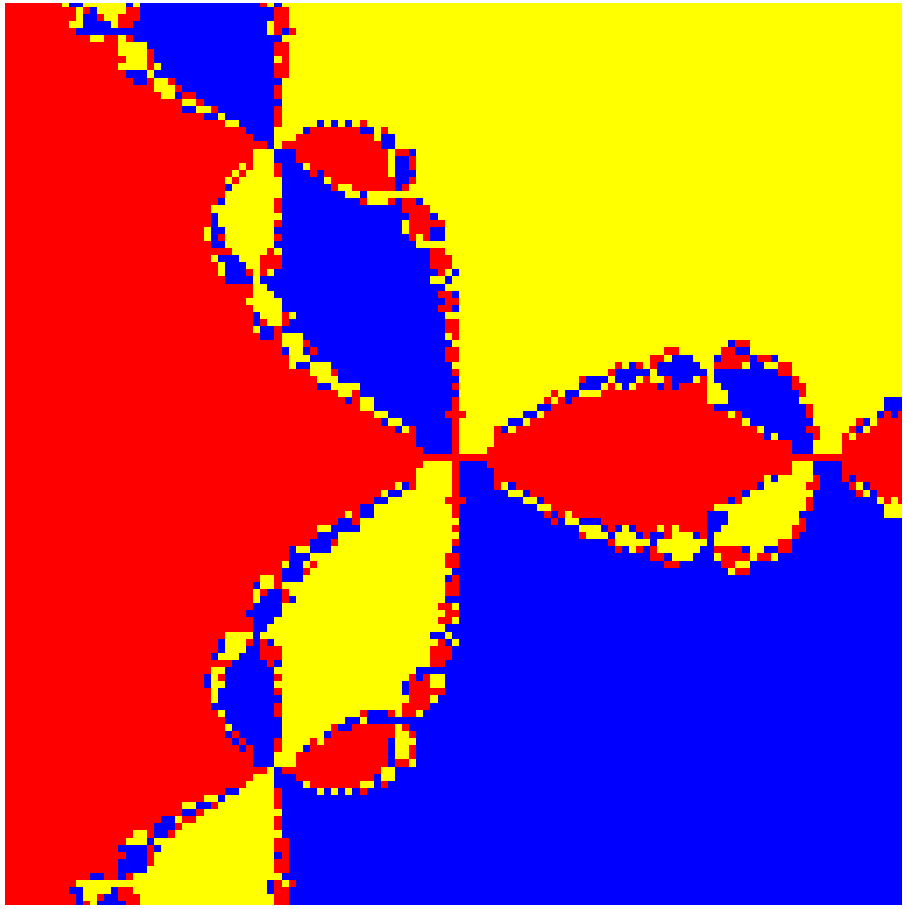


Figure 11.23 Subdivision of a cube.

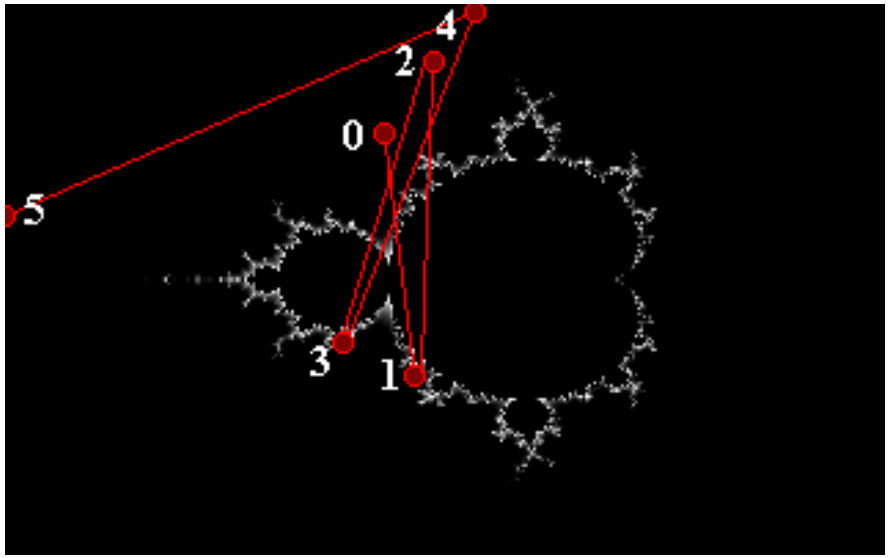
Newtonov fraktal

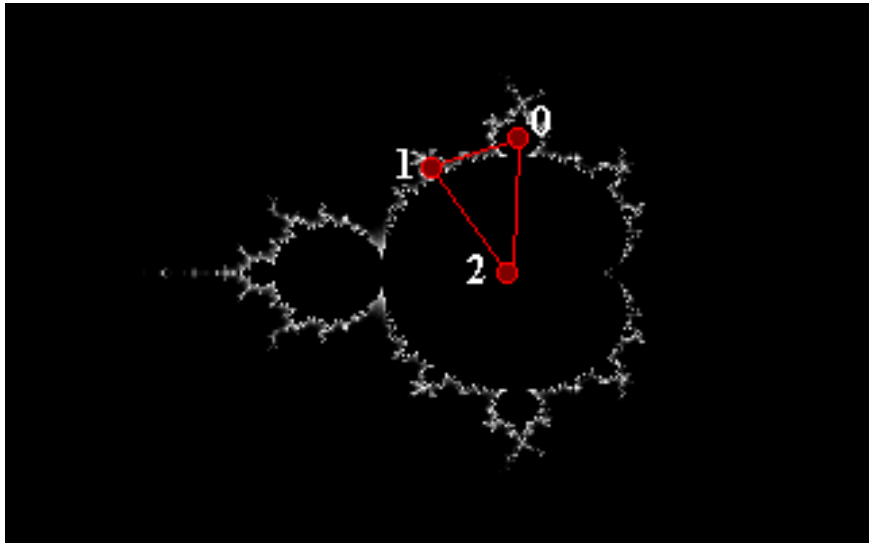


Video

Demo

Mandelbrotov fraktal





Mandelbrot set is the graph of all the complex numbers c, that do not go to infinity when iterated in

$$z = z^2 + c,$$

with a starting value of

$z = 0 + 0i.$

Juliajeve množice

What is the difference between Julia sets and Mandelbrot set?
Julia sets are strictly connected with Mandelbrot set. The iterative function used to produce both Mandelbrot and Julia sets is:

z(n) = z(n - 1) ^ 2 - c

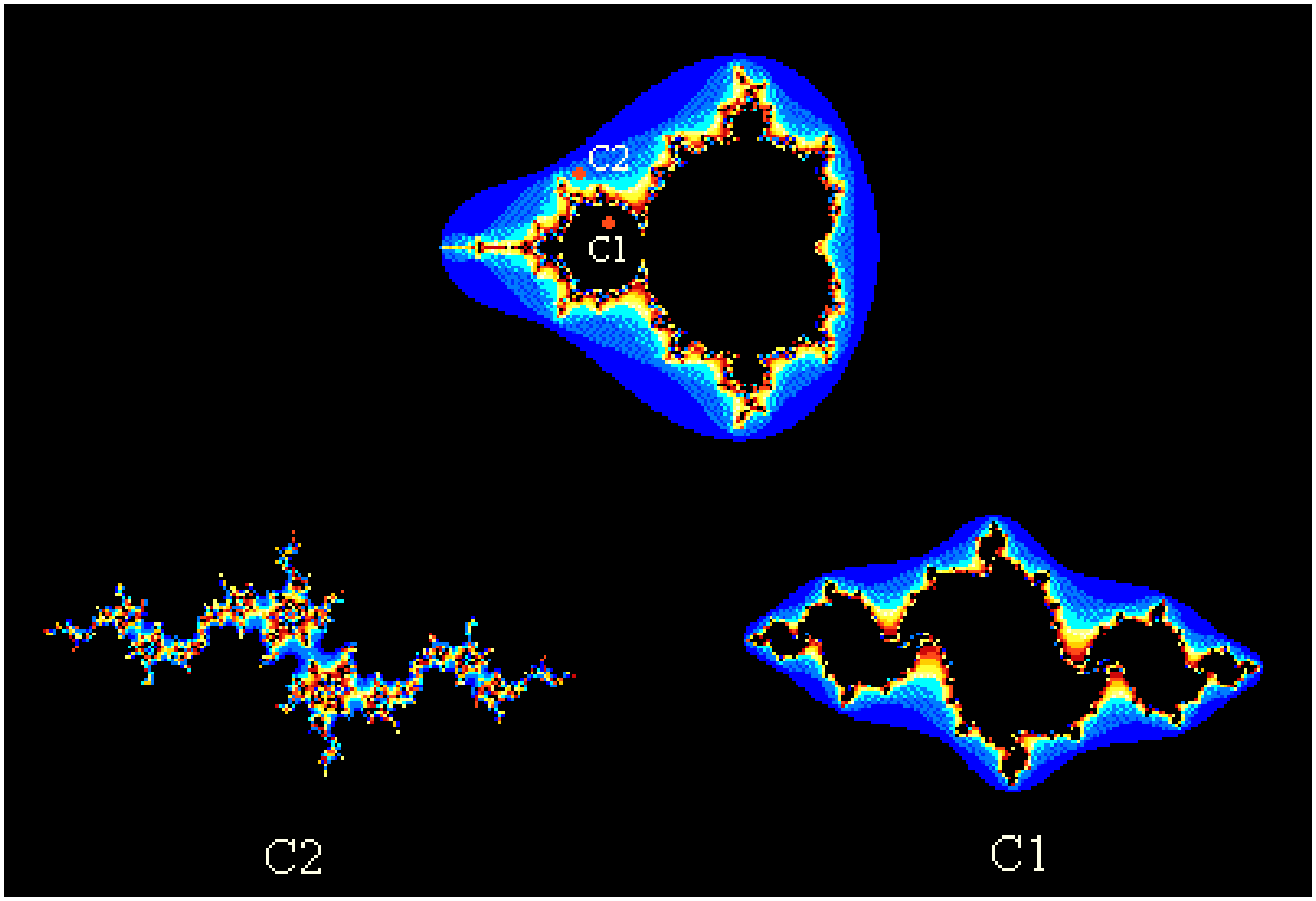
where values of z and c are complex numbers of the form a + ib and i is the square root of -1.

What is different is the way this formula is used. Each point of the complex plane is associated with a Julia set, so you can think to the Mandelbrot set as an "index" for Julia sets: each point of the Mandelbrot set is associated with a particular Julia set.

Julia set is the graph of all the complex numbers z, that do not go to infinity when iterated in **f(z) -- > z 2 + c**, where c is constant.

Video

Razmerje med Mandelbrotom in Juliajem



Demo

IFS – iterirani funkcijski sistemi

IFS stands for *Iterated Function System*. Fractals of this type are created by applying one of a number of functions, chosen randomly from the rules set up for the IFS, repeatedly to an intitial point, and graphing each new point.

With IFS fractals, it can be seen that the starting point does not effect the shape of the fractal too much. This means that a particular fractal can be defined by the rules used to find the next point, and the probabilities that an individual function will be chosen. This is the model used by my IFS fractal generator.

Hopalong orbit Demo

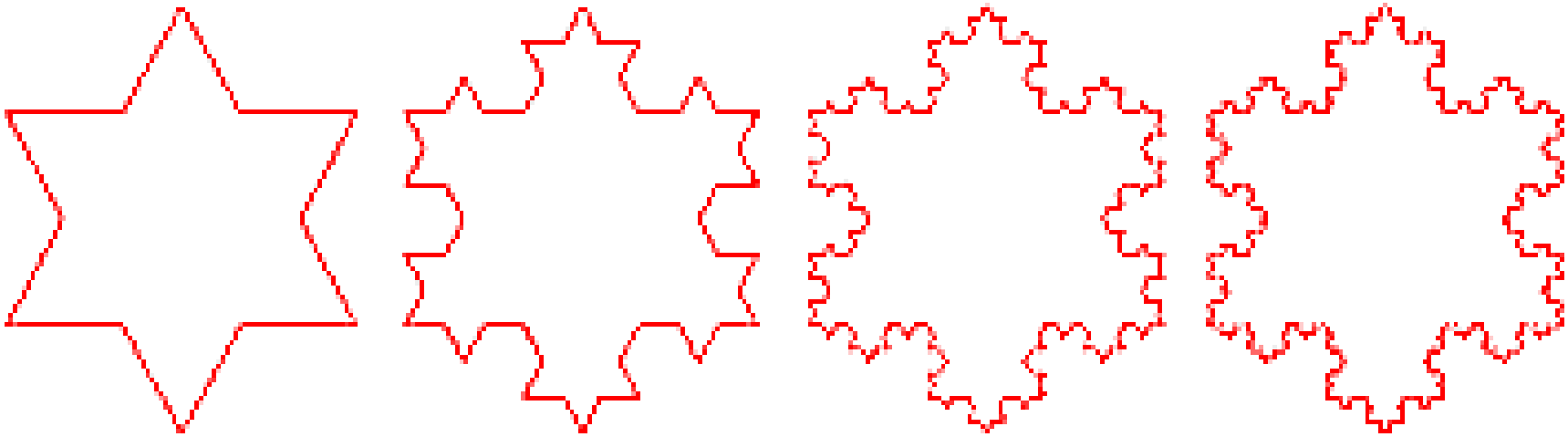
Ply orbit Demo

Dragon Demo

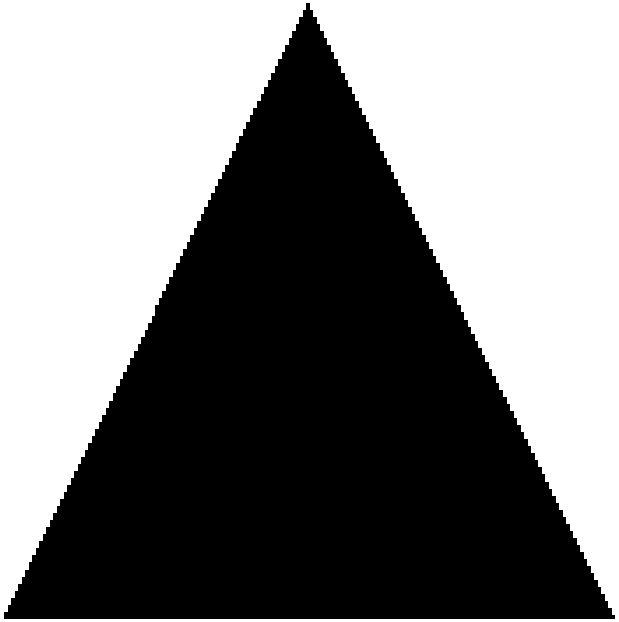
Gingerbreadman Demo

Popcorn Demo

Iterativni proces

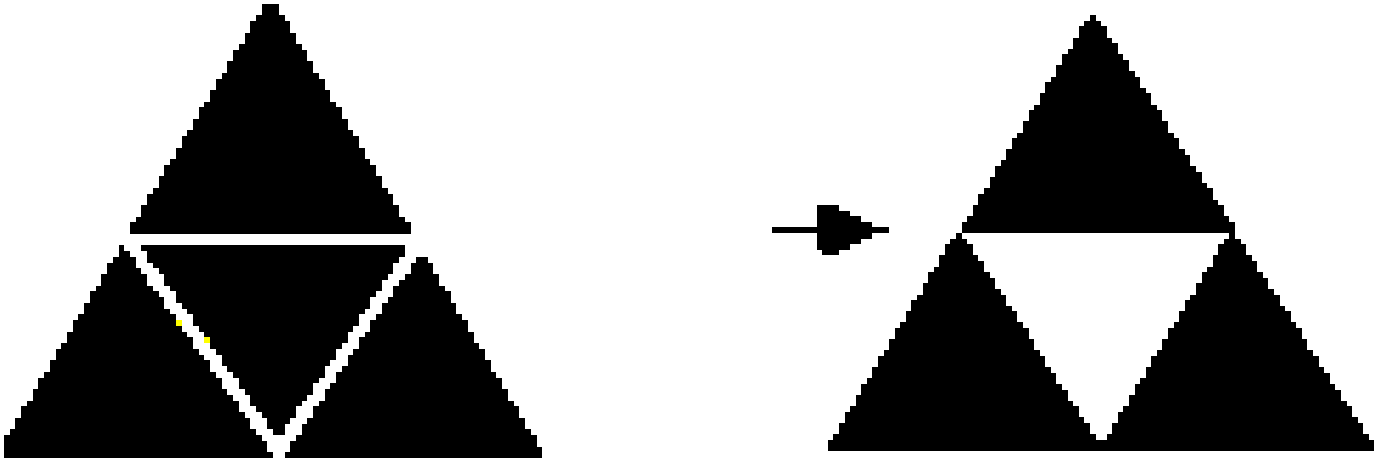


Trikotnik Sierpinskega



Kako tvorimo trikotnik Sierpinskega

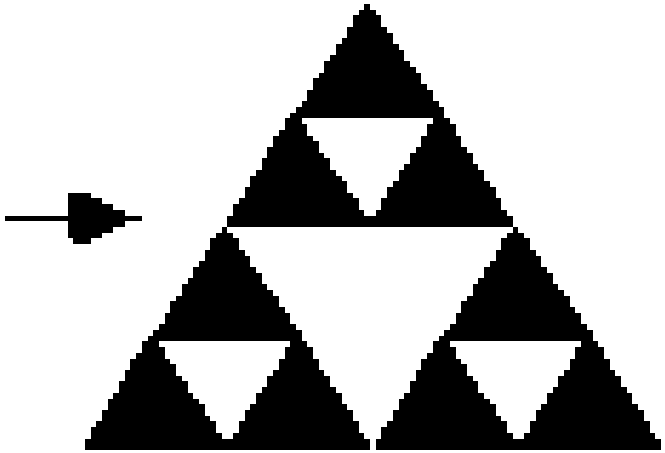
Step One:
Draw an equilateral triangle with sides of 2 triangle lengths each. Connect the midpoints of each side. How many equilateral triangles do you now have? Shade out the triangle in the center. Think of this as cutting a hole in the triangle.



Kako tvorimo trikotnik Sierpinskega

Step Two:

Draw another equilateral triangle with sides of 4 triangle lengths each. Connect the midpoints of the sides and shade the triangle in the center as before. Notice the three small triangles that also need to be shaded out in each of the three triangles on each corner - three more holes.

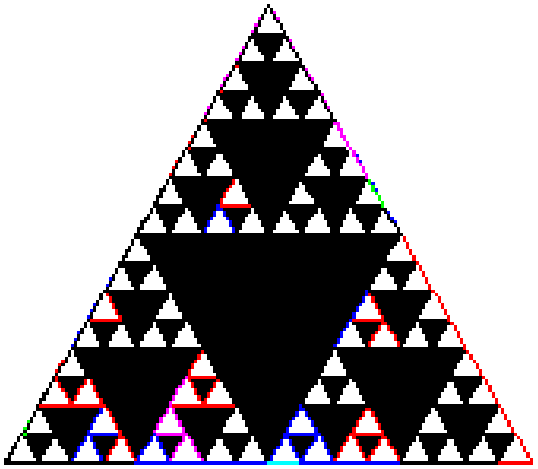


Kako tvorimo trikotnik Sierpinskega

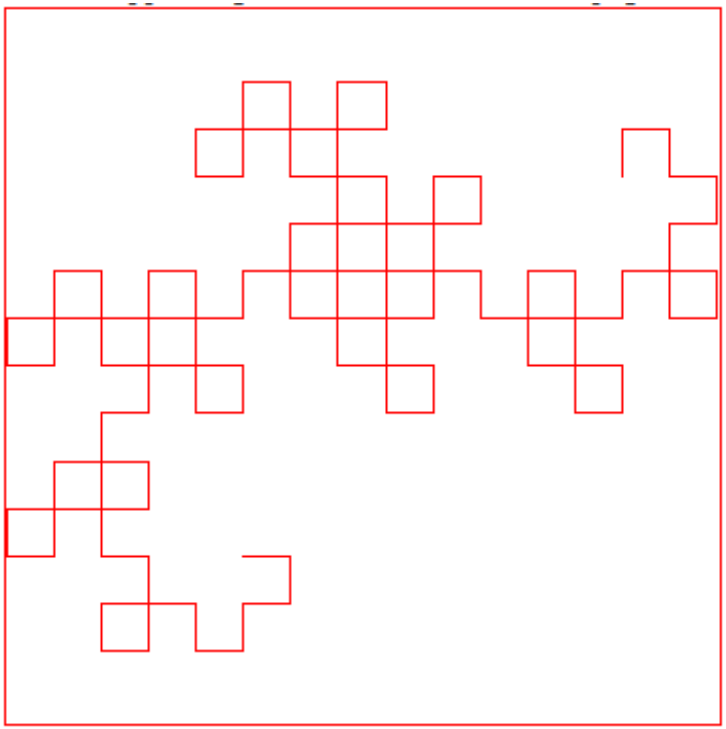
OR:

1. Draw a large equilateral triangle.
2. Draw the midpoint of each side.
3. Connect Midpoints.
4. Shade the middle triangle.
5. Repeat the process with the un-shaded triangles….

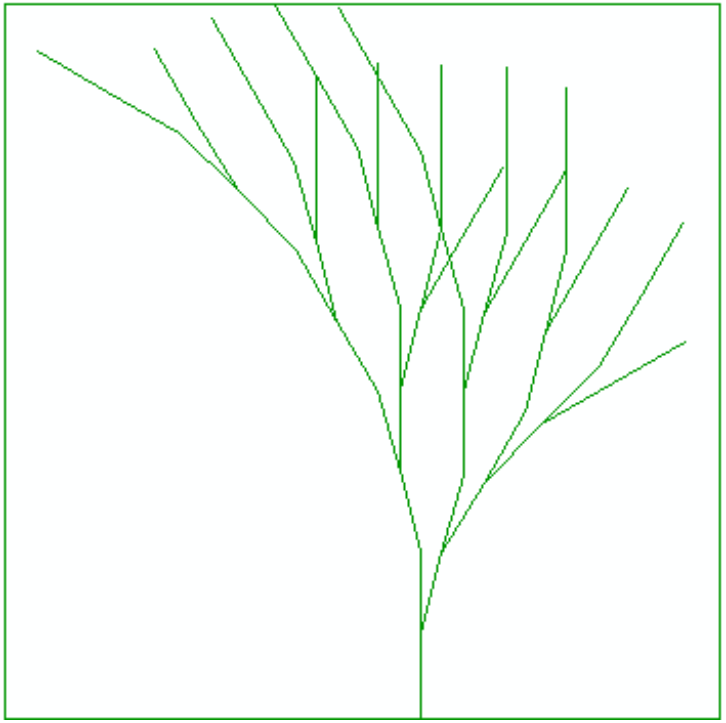
6. How far can you get…



L – sistem fraktali (language based fractals)



Demo



Drevesa

Demo

L – sistem fraktali

The fractals are constructed from line segments using a set of rules.

The rules include commands such as draw forward, move forward, turn by a specified angle, etc.

The commands start with an initial drawing string, called the axiom.

The axiom can reference additional command strings, which themselves can be recursive. The axiom is then executed recursively.

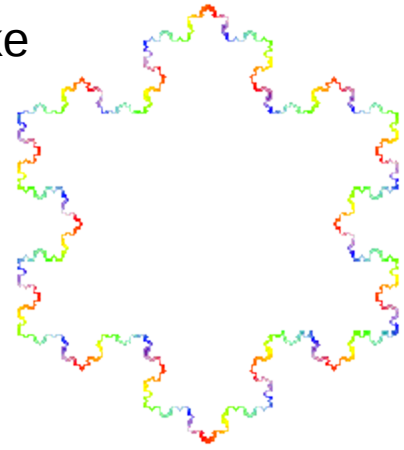
With each iteration, every line segment is replaced with all the commands in the axiom.

L system fraktali

Lets define some commands, create an axiom, and we what types of structures are produced.

Angle n	default rotation by 360/n degrees
+	rotate counterclockwise
-	rotate clockwise
F	Draw forward
G	Move forward
Cnn	Select color nn
<nn	Increment color by nn
>nn	Decrement color by nn

The following set of rules will produce a colored Koch snowflake



Angle 6

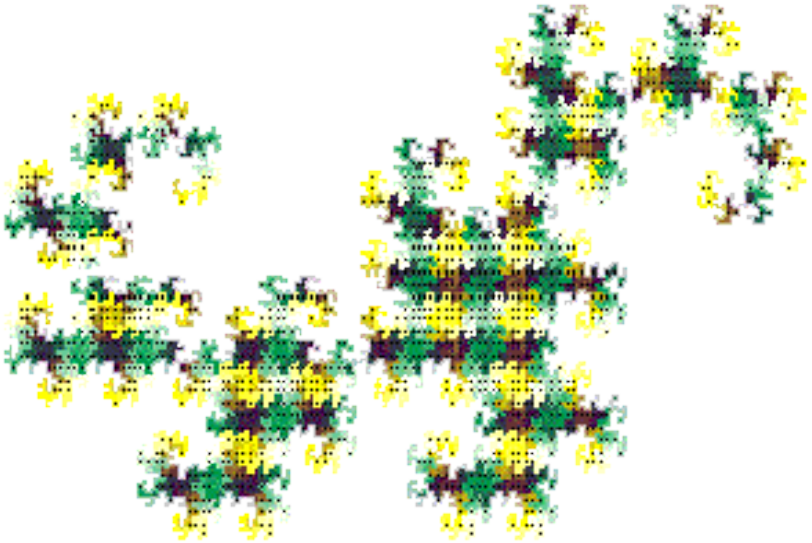
Axiom F--F--F

F=F>1+F--F+F

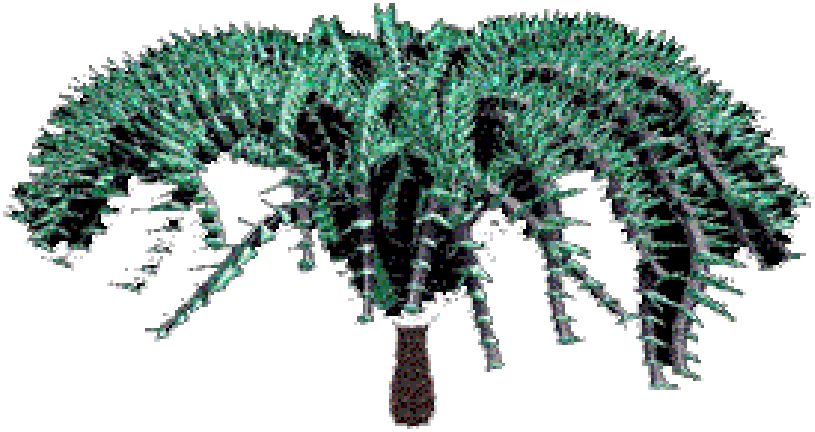
L system fraktali

The following rules produce a dragon curve.

Angle 8
Axiom FX
F=>1
Y=+FX--FY+
X=-FX++FY-



L system fraktali



Drevesa 2

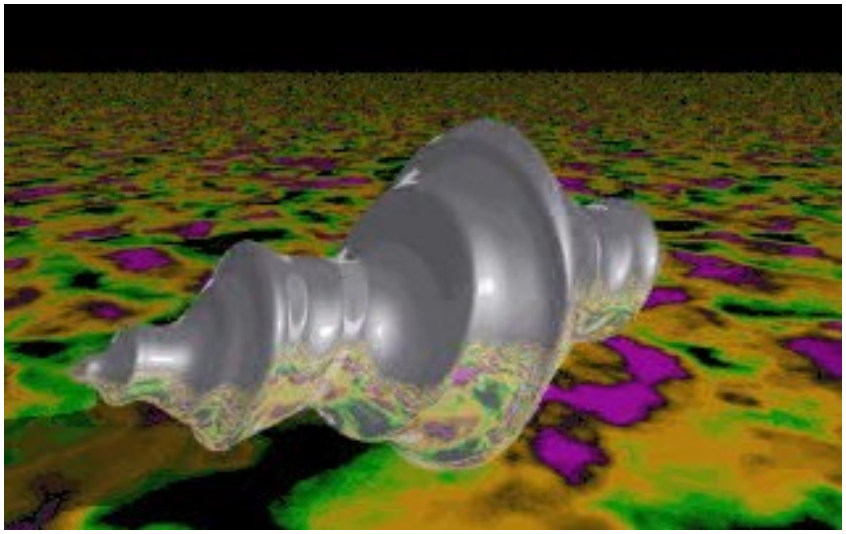
Demo

Kvaternionski fraktali

Mandelbrotova množica in vrsta drugih fraktalov temelji na uporabi kompleksnih števil, ki imajo dve komponenti in jih lahko prikažemo v dveh dimenzijah.

Kvaternioni so spremenljivke, ki imajo 4 komponente. Prikažemo jih lahko v 4 dimenzijah.

Seveda ne moremo prikazati 4 dimenzionalnega fraktala, lahko pa prikažemo njegovo 3 dimenzionalno "rezino"



Video

Video

Kvaternionski fraktali

Goblin Park The fractal sculpture was created from two Fractal Zplot quaternions, stone texture from a Dofo-Zon Elite fractal, and the trees behind the river from l-system fractals by L-System and Fractal Zplot. The rest of the scene and rendering by Vue d'Esprit.

Kvaternionski fraktali

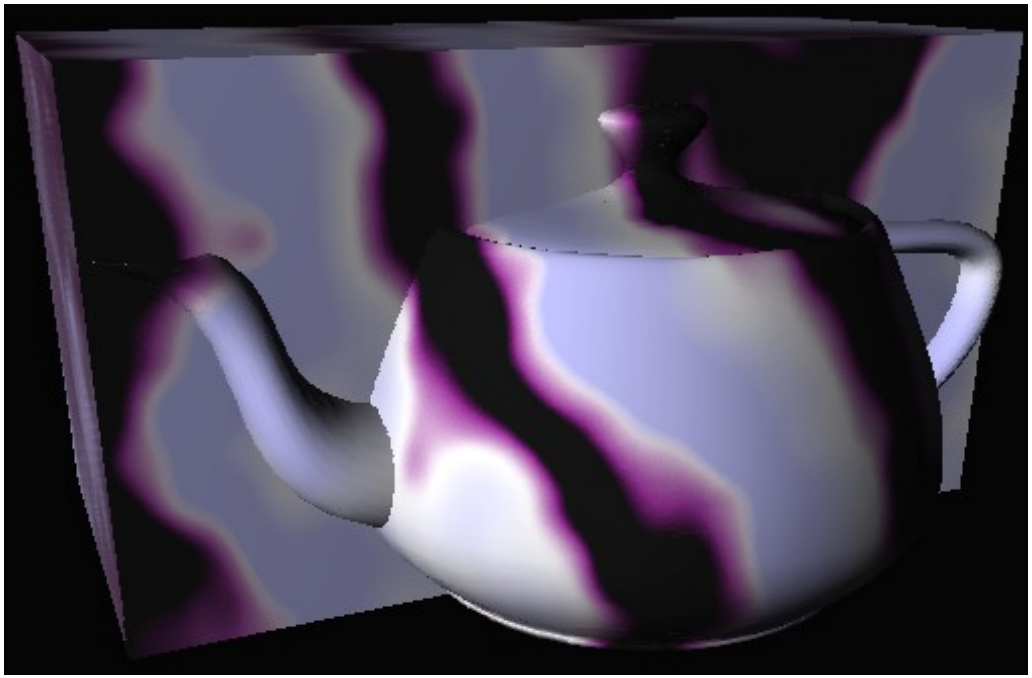
Dancers A fractal sculpture using two versions of a complex quaternion fractal by QuarSZ; scenery and rendering by Vue d'Esprit.

Uporaba fraktalov

- Teksture
- Pokrajine
- Posebni efekti
- Fraktalska glasba

Marmor

$$f(s,t,r) = r + \sum_{i=1}^{4} 2^{-i} n(2^i s, 2^i t, 2^i r))$$

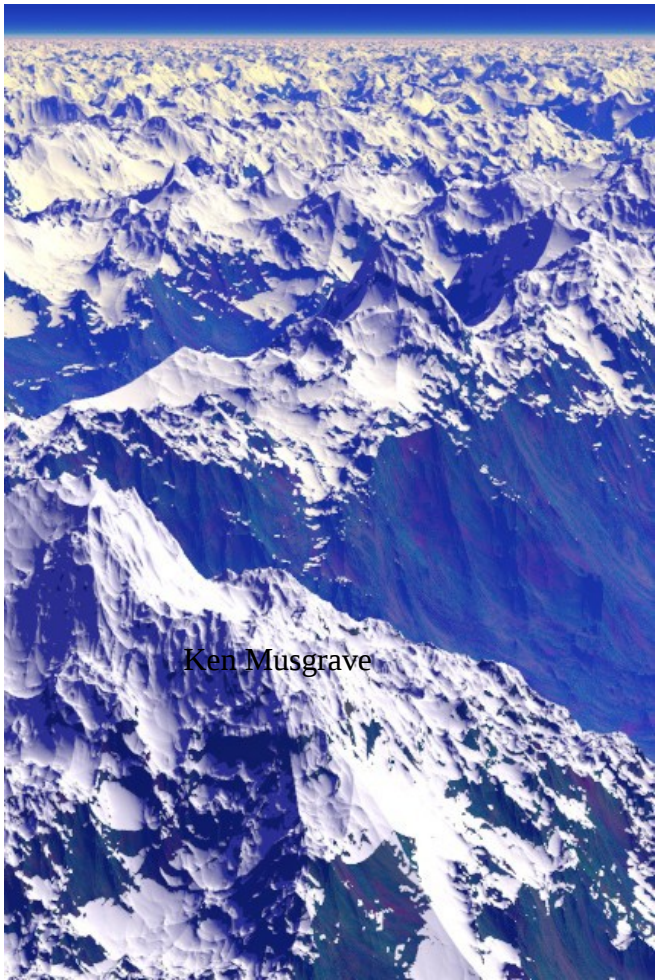


Ken Perlin, 1985

Fraktalske gore

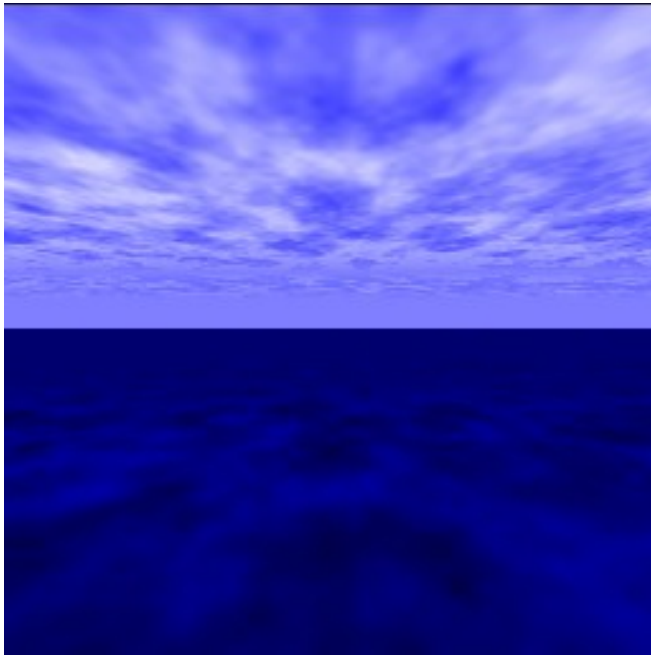
- Displacement map of meshed plane
- Can also be formed using midpoint displacement

Gunther Berkus via Mojoworld



Oblaki - voda

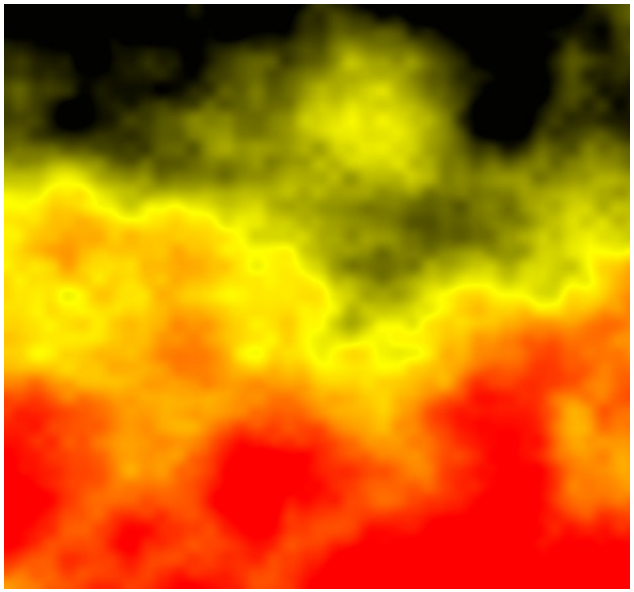
$$f(\mathbf{s}) = \sum_{i=1}^{4} 2^{-i} n(2^i \mathbf{s})$$

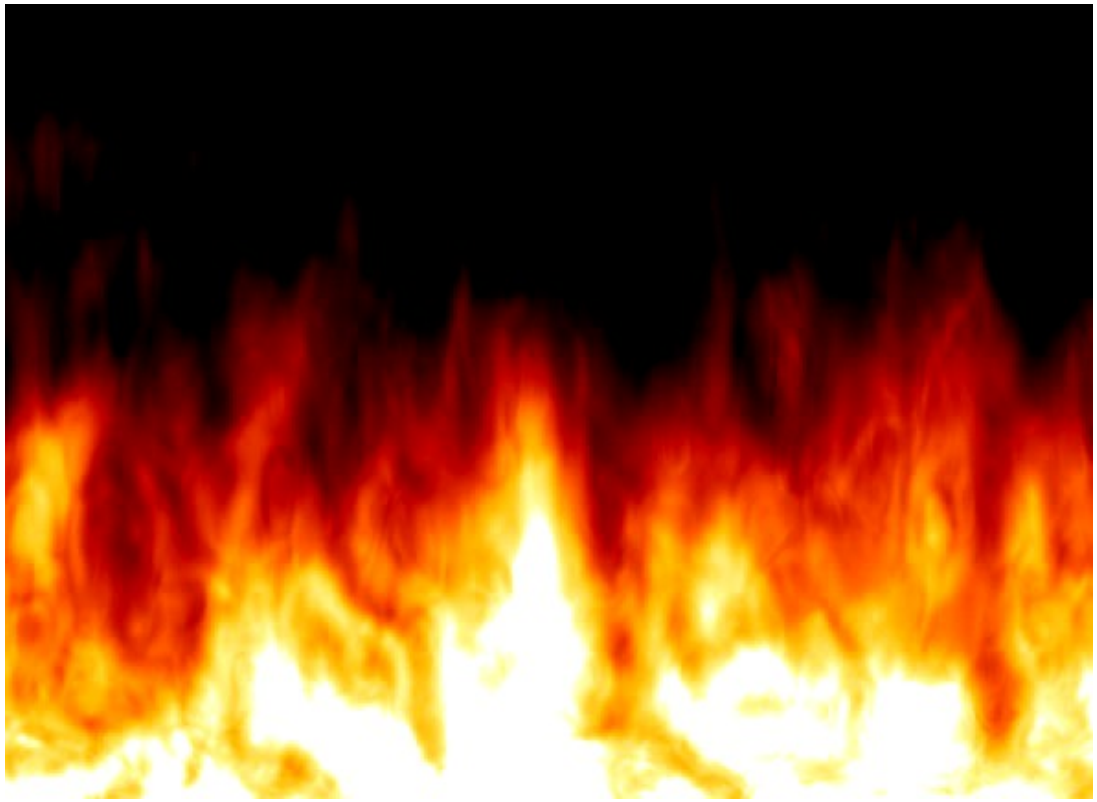


Gunther Berkus via Mojoworld

Ogenj

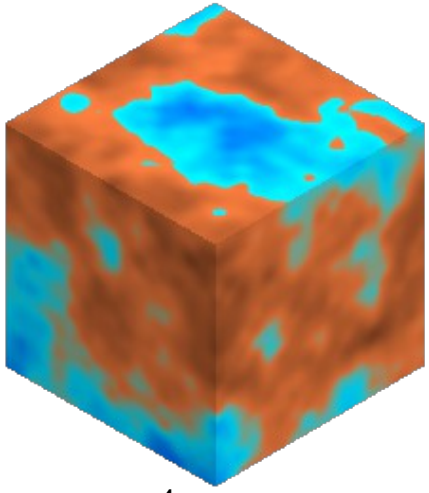
$$f(s,t,r) = r + \sum_{i=1}^{4} 2^{-i} n(2^i s, 0, 2^i r + \phi)).$$

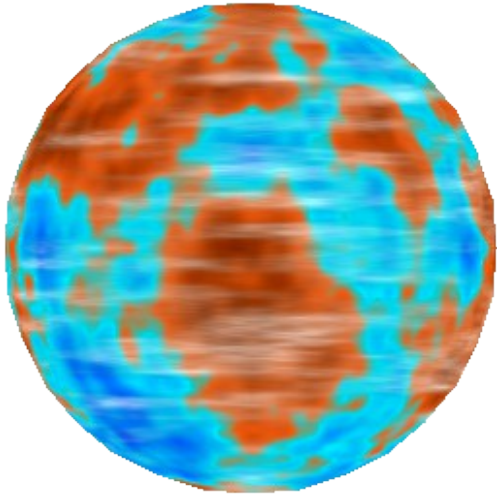


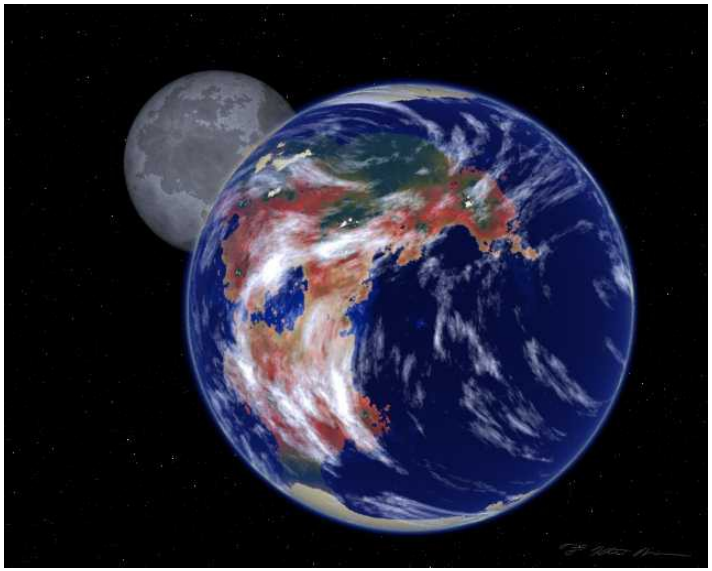


Ken Musgrave

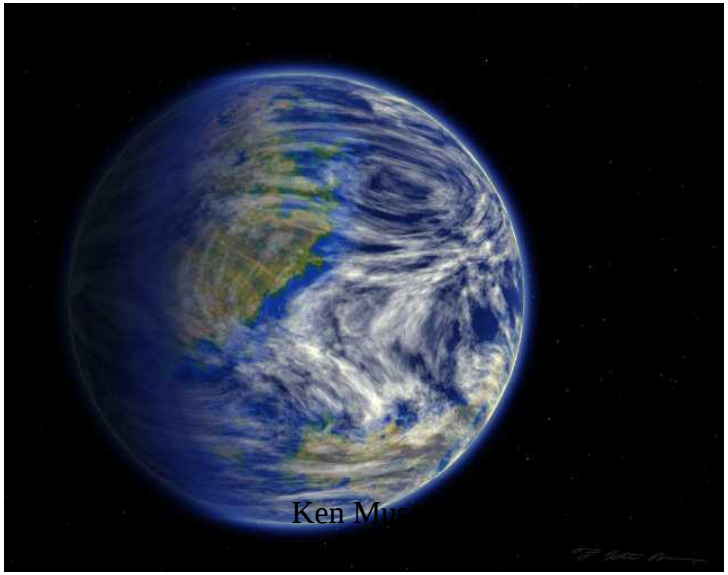
Planeti



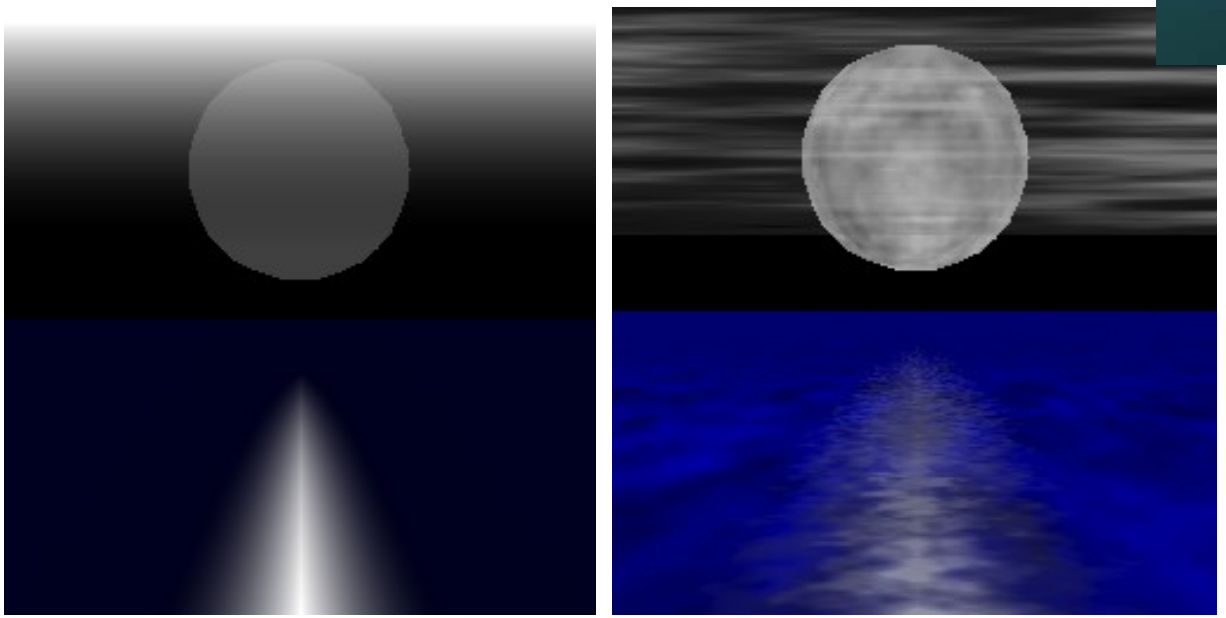




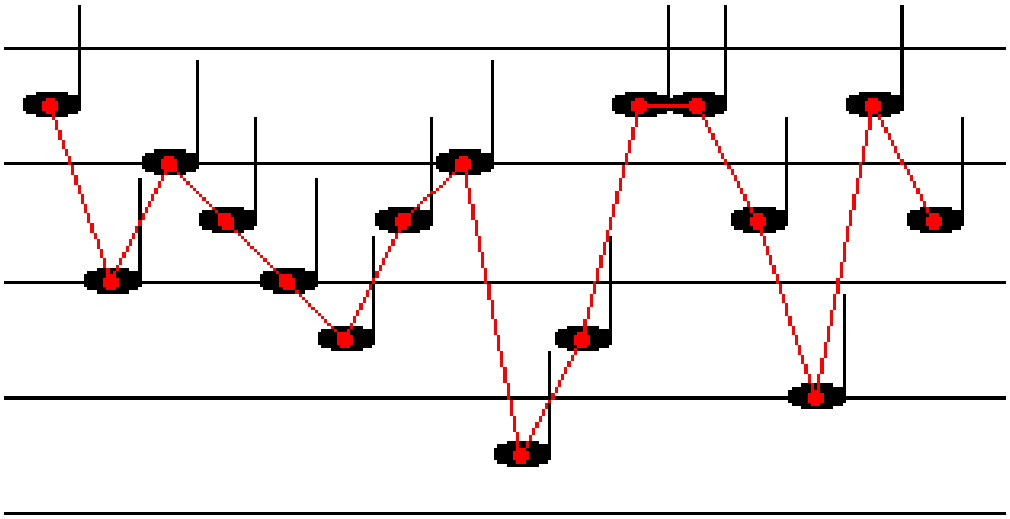
$$f(\mathbf{s}) = \sum_{i=1}^{4} 2^{-i} n(2^i \mathbf{s})$$



Mesečev vzhod



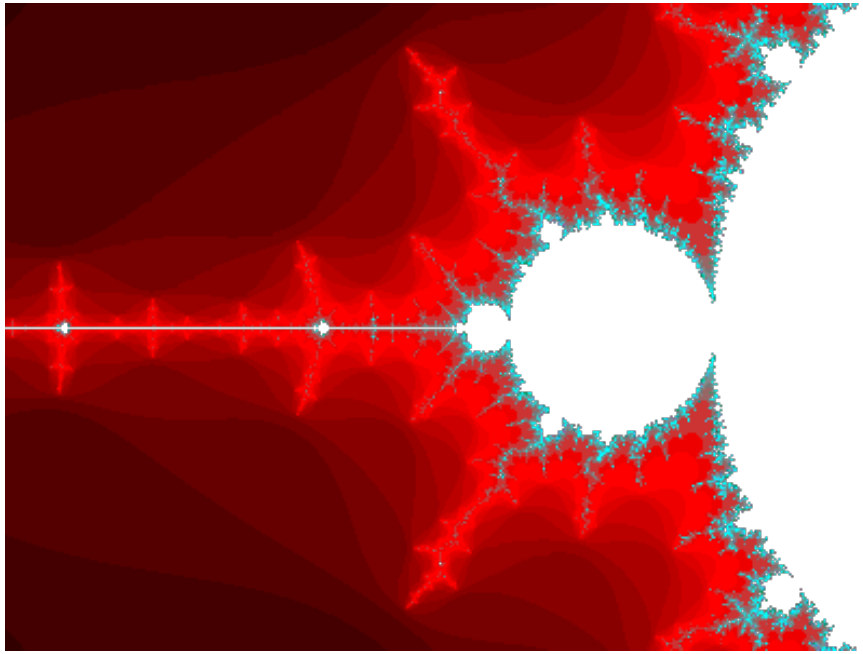
Fraktalska glasba



Fraktalsko glasbo lahko dobimo tako, da s šumom, ki ga predstavlja, primerno popravimo višino tona nekega glasbila, lahko pa z nekim drugim šumom tudi vplivamo na dolžino posameznih not.

Audio Audio Audio Audio Audio

Primer Mandelbrotove uglasbitve



Najprej izračunamo del Mandelbrotove množice. Matematične podatke interpretiramo kot podatke midi. Večje, kot je število iteracij, višji je na primer ton.

Audio

Fraktalska kompresija

Fraktalska kompresija

- Fractal compression is a very complex (lossy) compression technique.
- It is based on the transformation of a bitmap image to a vector-like mathematical representation using *iterated function systems* (e.g.fractals).
- Fractal compression is asymmetrical as the compression step is very much slower than decompression (decompression is, in fact, just a rendering algorithm) but there is a lot of work going on to overcome this problem.
- The advantages of fractal compression are the good compression ratio that can be achieved with little degradation of the image quality and the ability (just like with vector formats) to scale the image without losing information and adding noise.
- The drawback is that not everyone agrees on the advantages.