

Obdelava digitalnih slik

Metode obdelave digitalnih slik

- Quantization
 - Uniform Quantization
 - Random dither
 - Ordered dither
 - Floyd-Steinberg dither
- Pixel operations
 - Add random noise
 - Add luminance
 - Add contrast
 - Add saturation
- Filtering
 - Blur
 - Detect edges
- Warping
 - Scale
 - Rotate
 - Warp
- Combining
 - Composite
 - Morph

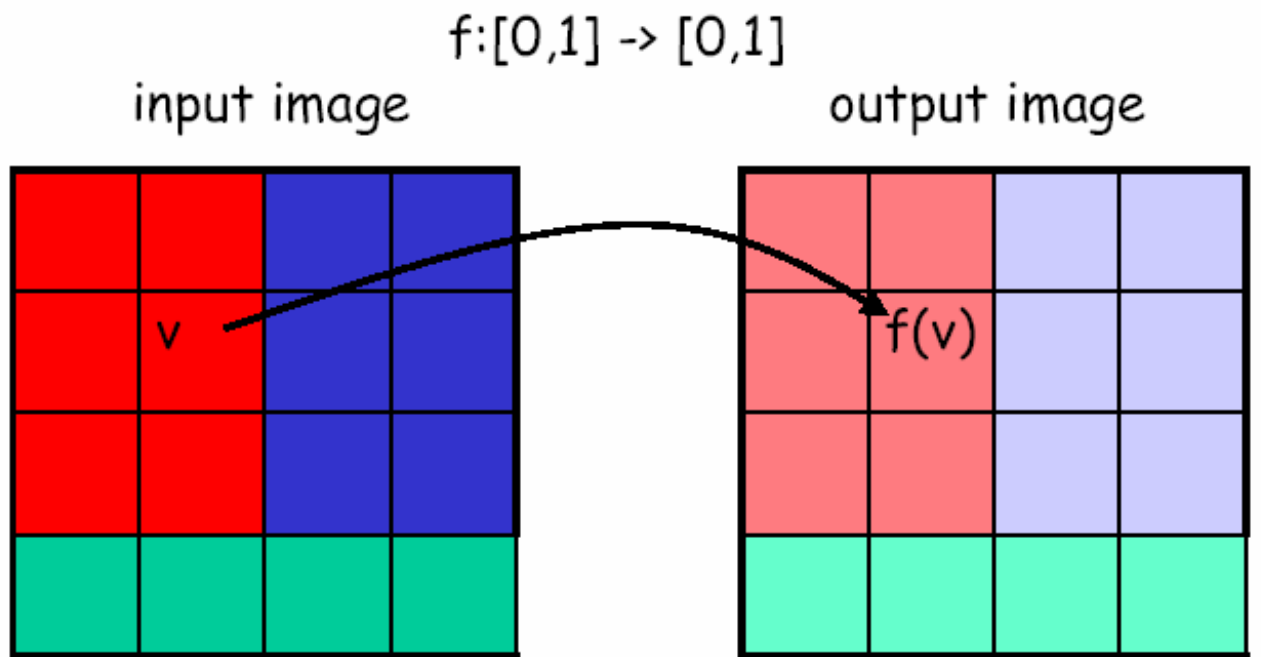
types of techniques

- simple pixel modification
 - interpolation/extrapolation
 - compositing
 - convolution
 - dithering
 - warping
 - morphing
 - misc. effects
-

types of techniques

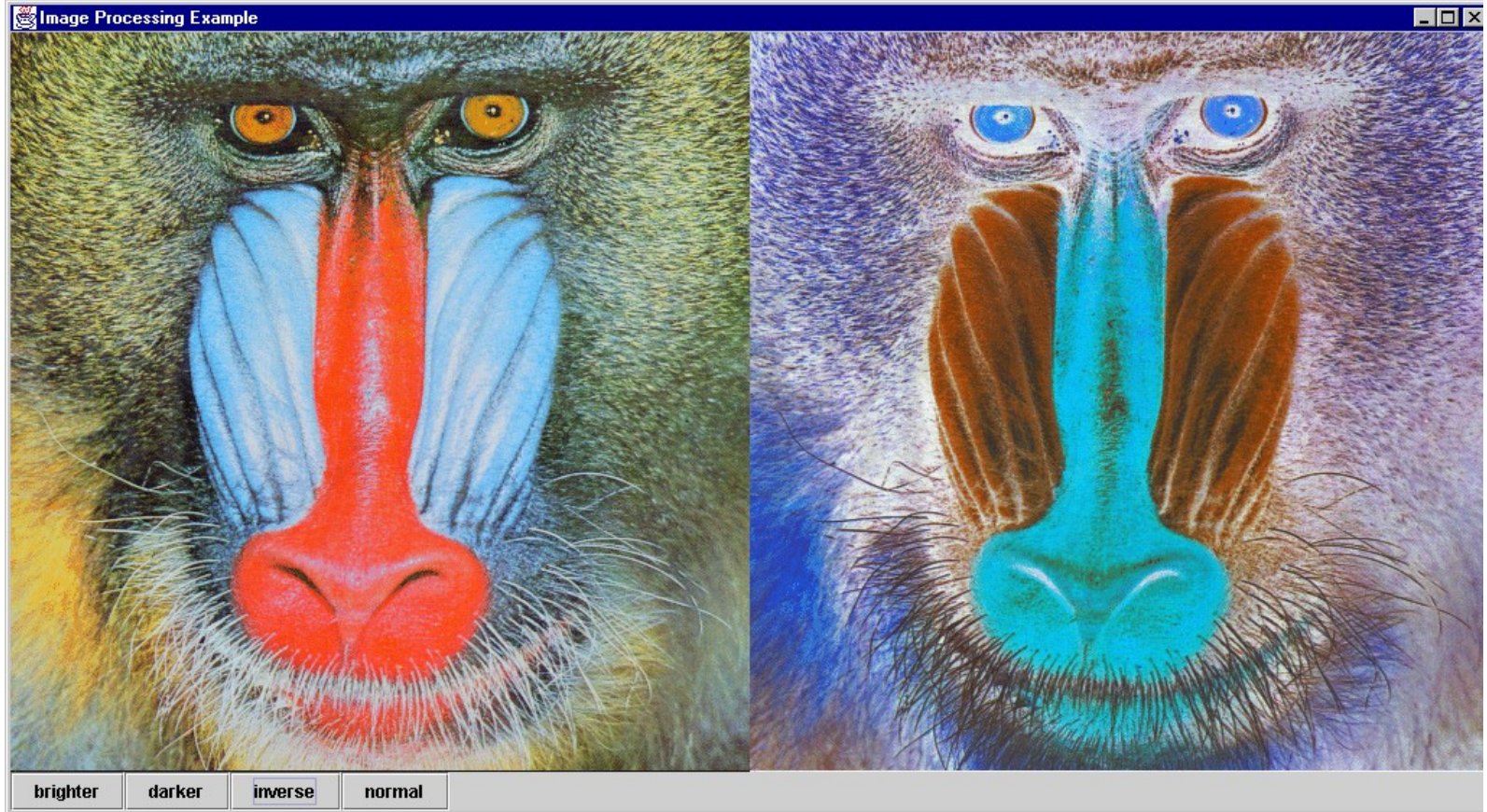
- **simple pixel modification**
- interpolation/extrapolation
- compositing
- convolution
- dithering
- warping
- morphing
- misc. effects

Preprosto spreminjanje pikslov



apply function f to each channel of each pixel of input image

Primer



Preprosto spreminjanje pikslov

Homogeneous Point Processing--

An image processing that maps an input image to an output image without performing geometric changes.

Theory--

If an image is represented by an array of pixels, each with a color value, one can manipulate the color values to change the image.

Examples--

- washing out an image,
- brightening an image,
- adjusting an image's contrast

Negativi slik

Princip: $f(v) = 1-v$

A negative image may be created by taking a color, and finding its opposite. If 8 bits represent a given red, blue, or green value, the inverse would be 255 minus that color value.

Formula: $f(v_{ij}) = 255 - v_{ij}$



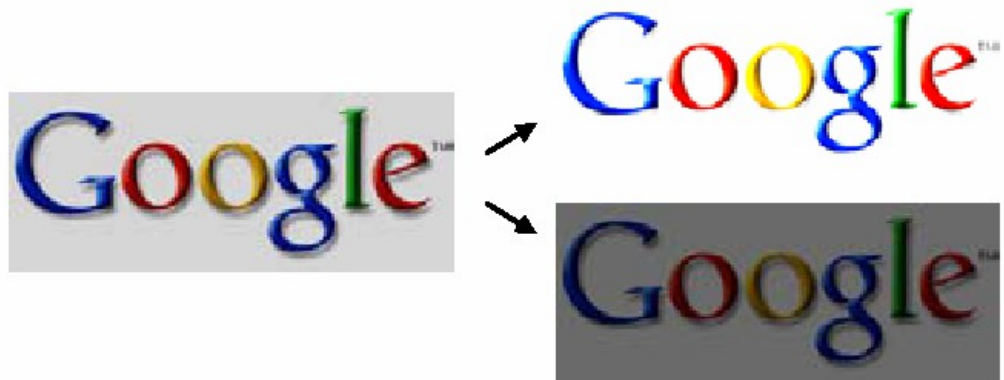
Popravljanje svetlosti (Brightness)

Princip: $f(v) = \alpha v$ for $\alpha \geq 0$ clamp to $[0,1]$

The brightness of each pixel may be adjusted by a similar use of the byte range:

$$f(v_{ij}) = 255 (v_{ij}/255)^{pow}$$

With $pow > 1$, the image darkens



Popravljanje kontrasta

Likewise, we can build a linear transform to alter contrast

$$v_{ij} = cv_{ij} + b,$$

where

$$c = \text{delta } D / \text{delta } V,$$

$$b = (D_{\min} * V_{\max} - D_{\max} * V_{\min}) / \text{delta } V$$

given

V = value in image

D = value that can be displayed

(This is also called linear gray level scaling)

Koda v Javi: naložimo sliko (in njene piksele)

```
public void loadImage(String strFile) {  
    ImageIcon i = new ImageIcon(strFile);  
    image = i.getImage();  
    width = image.getWidth(this);  
    height = image.getHeight(this);  
}
```

```
public void getData() {  
    pixels = new int[width * height];  
    PixelGrabber pg = new PixelGrabber  
        (image, 0, 0, width, height, pixels, 0, width);  
    try {  
        pg.grabPixels();  
    }  
    catch (InterruptedException e){  
        System.err.println("interrupted waiting for pixels!");  
        return;  
    }  
    if ((pg.getStatus() & ImageObserver.ABORT) != 0){  
        System.err.println("image fetch aborted or errored");  
        return;  
    }  
}
```

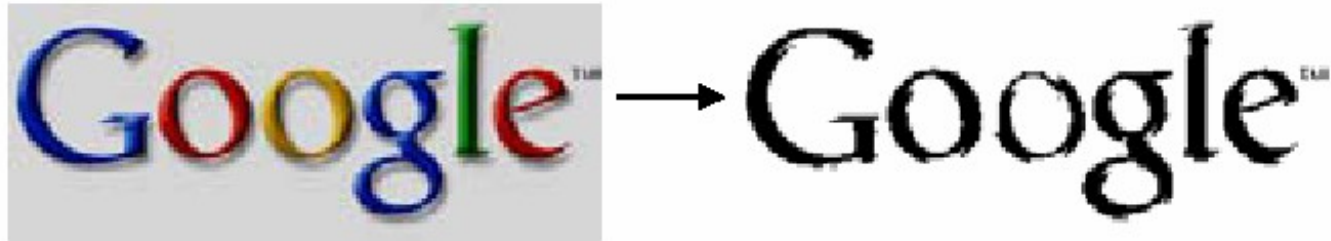
*How does
PixelGrabber
work? Check
your API*

Obdelava pikslov

```
public int inversePixel(int x, int y, int pixel) {
    int alpha = (pixel >> 24) & 0xff;
    int red    = (pixel >> 16) & 0xff;
    int green  = (pixel >>  8) & 0xff;
    int blue   = (pixel          ) & 0xff;
    red = Math.abs(255-red);
    green = Math.abs(255-green);
    blue  = Math.abs(255-blue);
    return (alpha << 24) | (red << 16) | (green << 8) | blue;
}
```

```
public int brightenPixel(int x, int y, int pixel, double p){
    int alpha = (pixel >> 24) & 0xff;
    int red    = (pixel >> 16) & 0xff;
    int green  = (pixel >>  8) & 0xff;
    int blue   = (pixel          ) & 0xff;
    red    = (int)(255 * Math.pow(red/255.0, p));
    green  = (int)(255 * Math.pow(green/255.0, p));
    blue   = (int)(255 * Math.pow(blue/255.0, p));
    return (alpha << 24) | (red << 16) | (green << 8) | blue;
}
```

Prag (treshold)



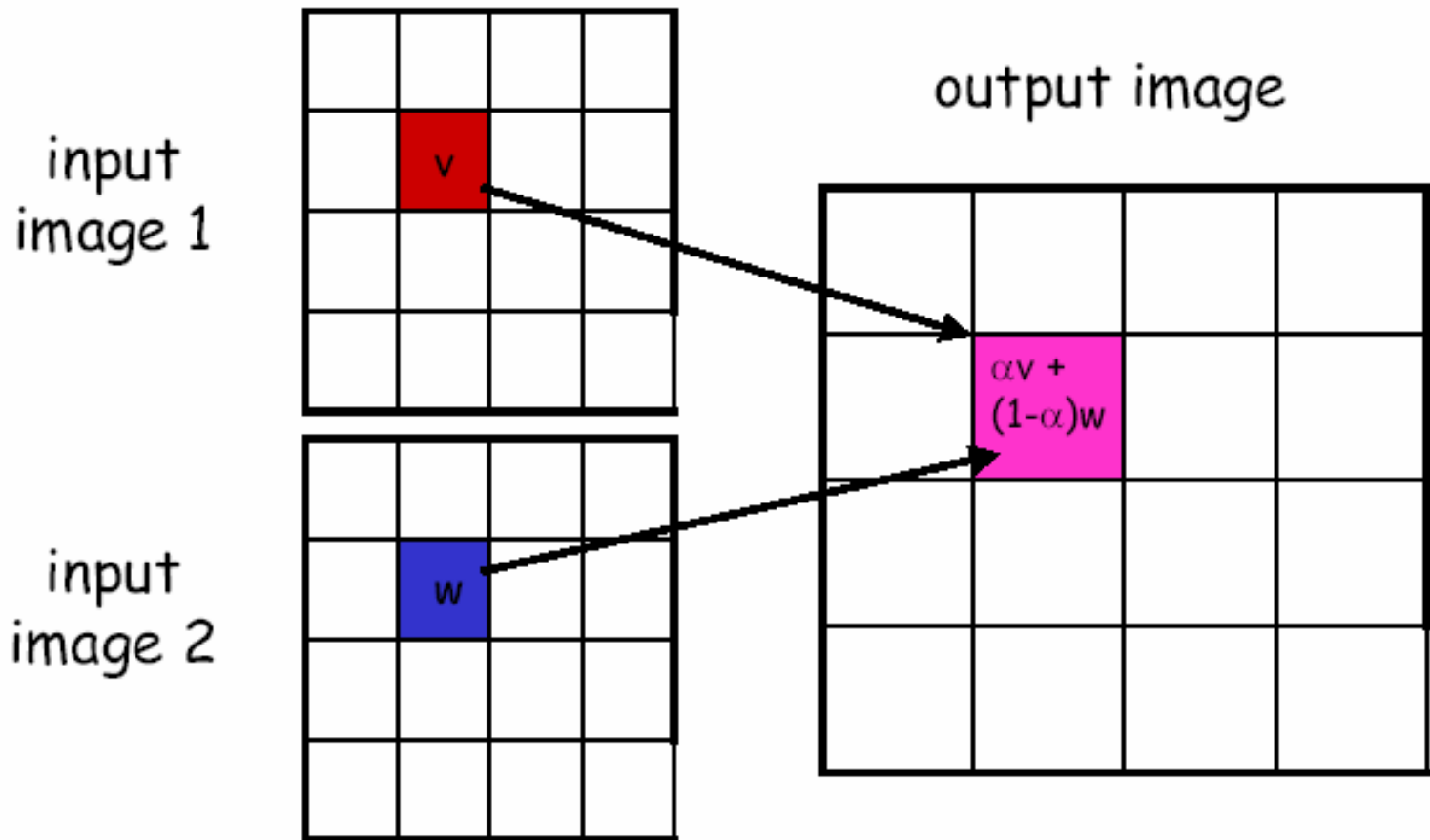
if $v > t$ then $f(v) = 1$

else $f(v) = 0$

types of techniques

- simple pixel modification
- **interpolation/extrapolation**
- compositing
- convolution
- dithering
- warping
- morphing
- non-photo-realistic effects

Interpolacija - ekstrapolacija

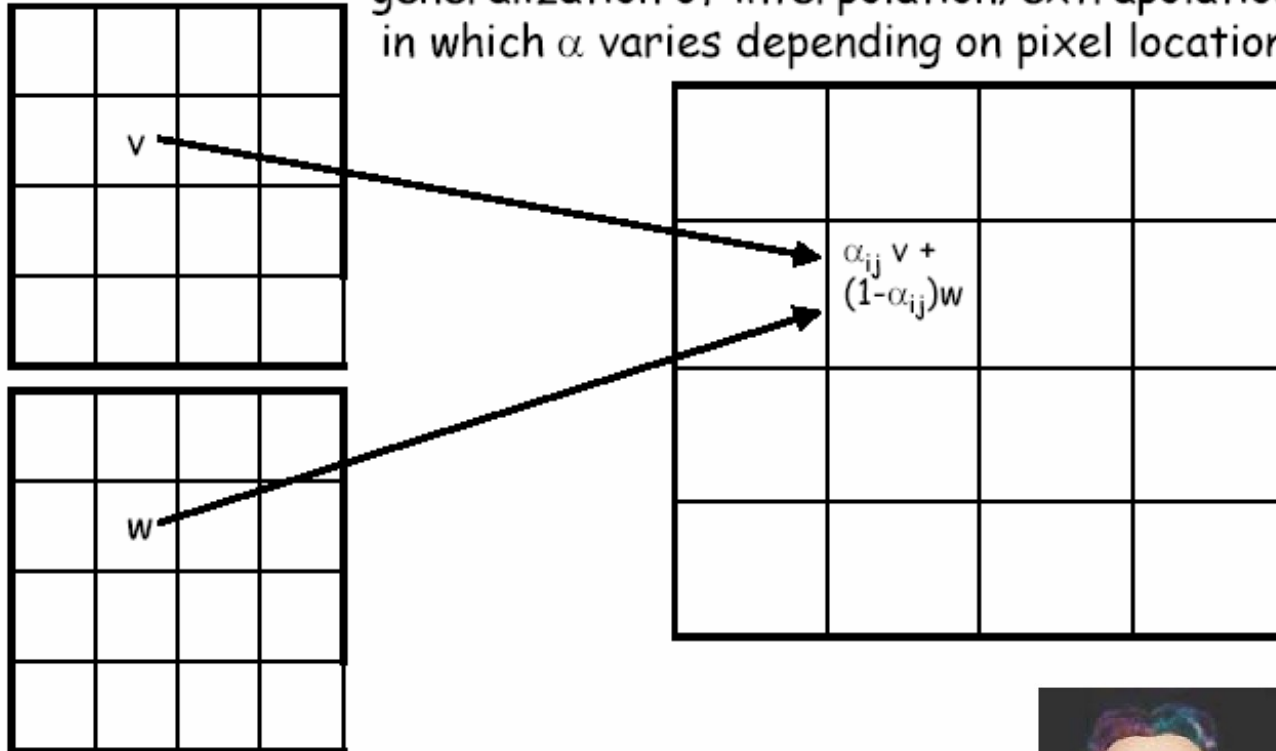


type of techniques

- simple pixel modification
- interpolation/extrapolation
- **compositing**
- convolution
- dithering
- warping
- morphing
- misc. effects

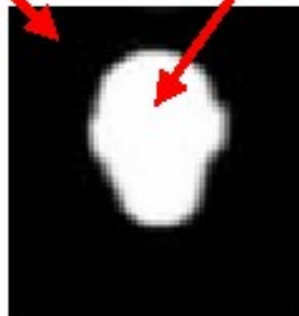
Kompozicija

generalization of interpolation/extrapolation
in which α varies depending on pixel location



Kompozicija

typically $\alpha \in [0,1]$ so the array of α values can be represented by a single channel image called a *mask*



type of techniques

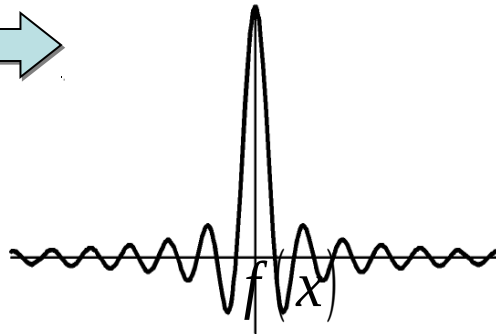
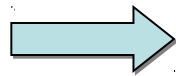
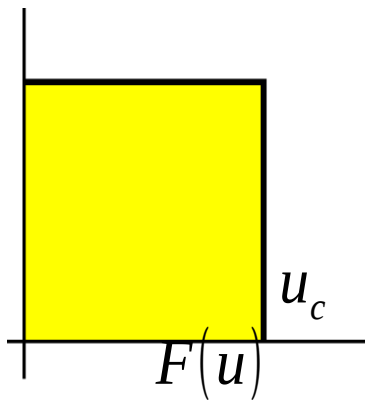
- simple pixel modification
- interpolation/extrapolation
- compositing
- **convolution**
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Filtri

- Filters are characterised by:
 - *frequency response* (e.g. high-pass, low-pass, band-pass, notch...)
 - *support* = region over which filter is non-zero. If non-zero over a finite domain, filter is said to have finite support.
 - *order*: most filters are *linear* (i.e. do not depend on the signal being filtered)
 - *variance*: a filter which does not alter with position is known as *shift-invariant* otherwise *spatially variant*.
- The frequency representation of a filter determines its frequency response.
- The spatial representation is used to characterise its support.

Jedra filtrov (Filter Kernels)

- The ideal filter is a box function in the frequency domain.
- In the spatial domain, this becomes the *sinc* filter kernel which has non-finite support.



$$F(u) = \begin{cases} 1 & |u| \leq u_c \\ 0 & |u| > u_c \end{cases}$$

$$f(x) = \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

Filtri

- Kernels with larger support can have better frequency response properties (more effective filters).
 - ⇒ *more memory* required per filter = $(2s+1)^2$ for discrete 2D filter
 - ⇒ convolution is *more expensive* = $n(2s+1)^2$
- Filter *gain* is usually assumed to be 1, i.e. *unity gain* ⇒ filter is normalised:
$$g = \sum_{t=-s}^s g[t]$$
- If gain > 1, the filtered signal will be scaled by the gain
 - images will get brighter or darker
- We can compensate by dividing by the gain (= *scale*):

$$h(x) = f * g = \sum_{t=-s}^s \frac{g[t]}{\text{gain}} f[x-t] = \frac{1}{\text{gain}} \sum_{t=-s}^s g[t] f[x-t]$$

Konvolucija

- Multiplying the frequency representation of an image by a filter may be expressed in the spatial domain as *convolution*.

- If the image is $f(x)$ and the filter (in the spatial domain) is $g(x)$ then convolution is defined by:

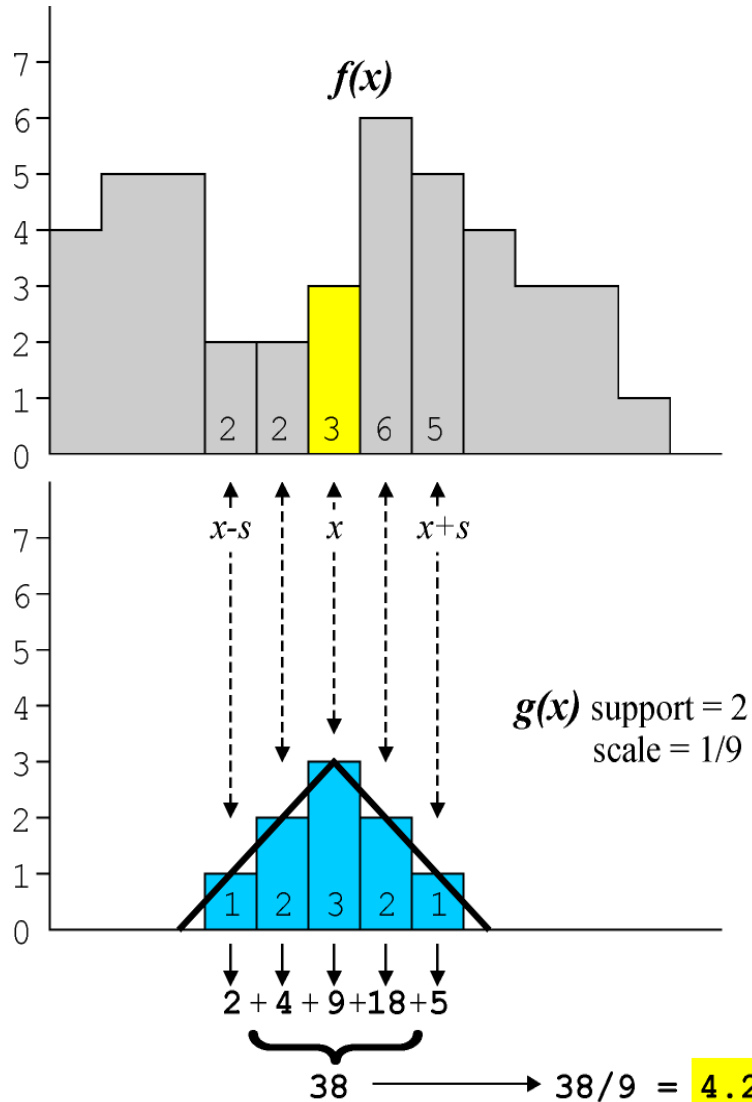
$$f(x) * g(x) = \mathbf{F}(f(x))\mathbf{F}(g(x)) = \int_{-\infty}^{\infty} g(t) f(x-t) dt$$

- Normally we are concerned with *discrete convolution*:

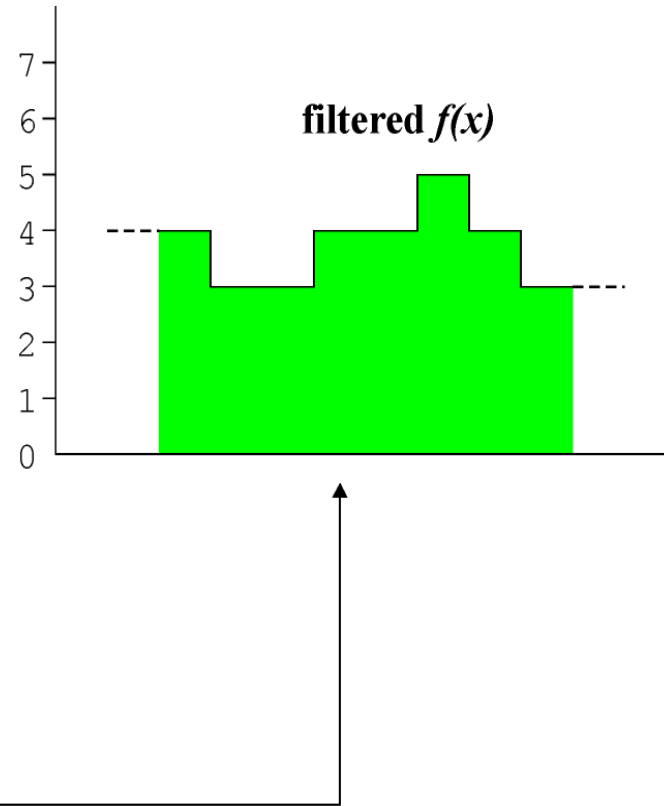
$$f * g = \sum_{t=-s} g[t] f[x-t]$$

- In this case, the filter $g(x)$ is assumed to have finite discrete support $[-s,s]$

Diskretna konvolucija

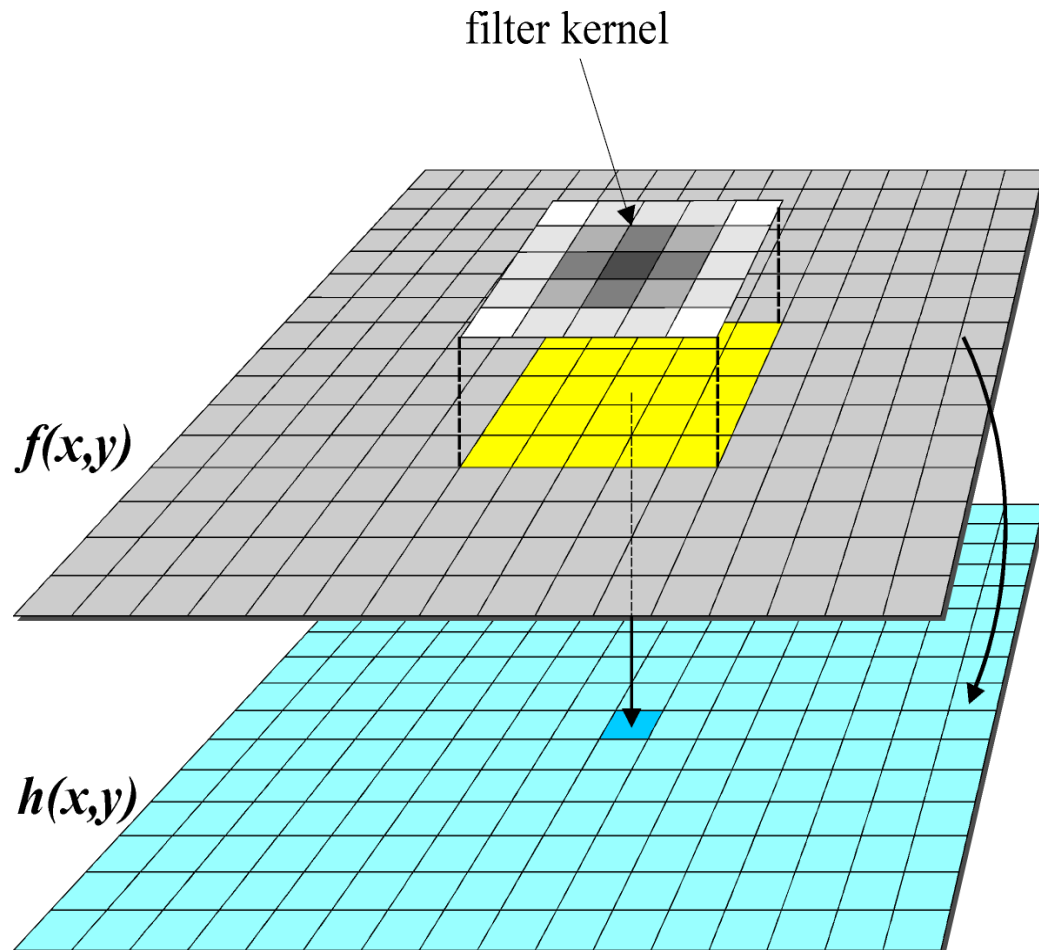


$$f * g = \frac{1}{\text{gain}} \sum_{t=-s}^s g[t] f[x-t]$$

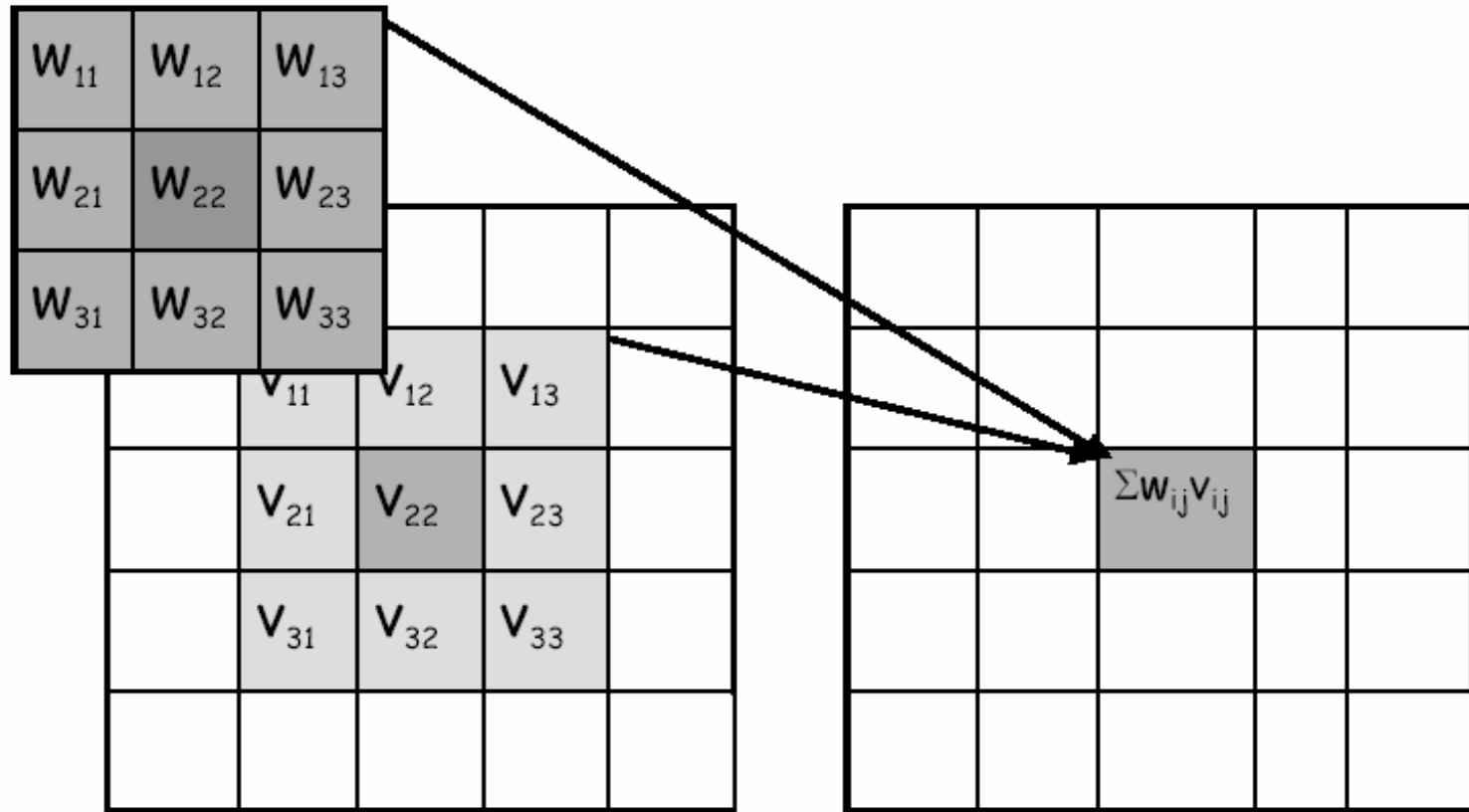


Diskretna 2D konvolucija

$$f * g = \frac{1}{\text{gain}} \sum_{v=-s}^s \sum_{u=-s}^s g[u,v] f[x-u, y-v]$$



Konvolucija

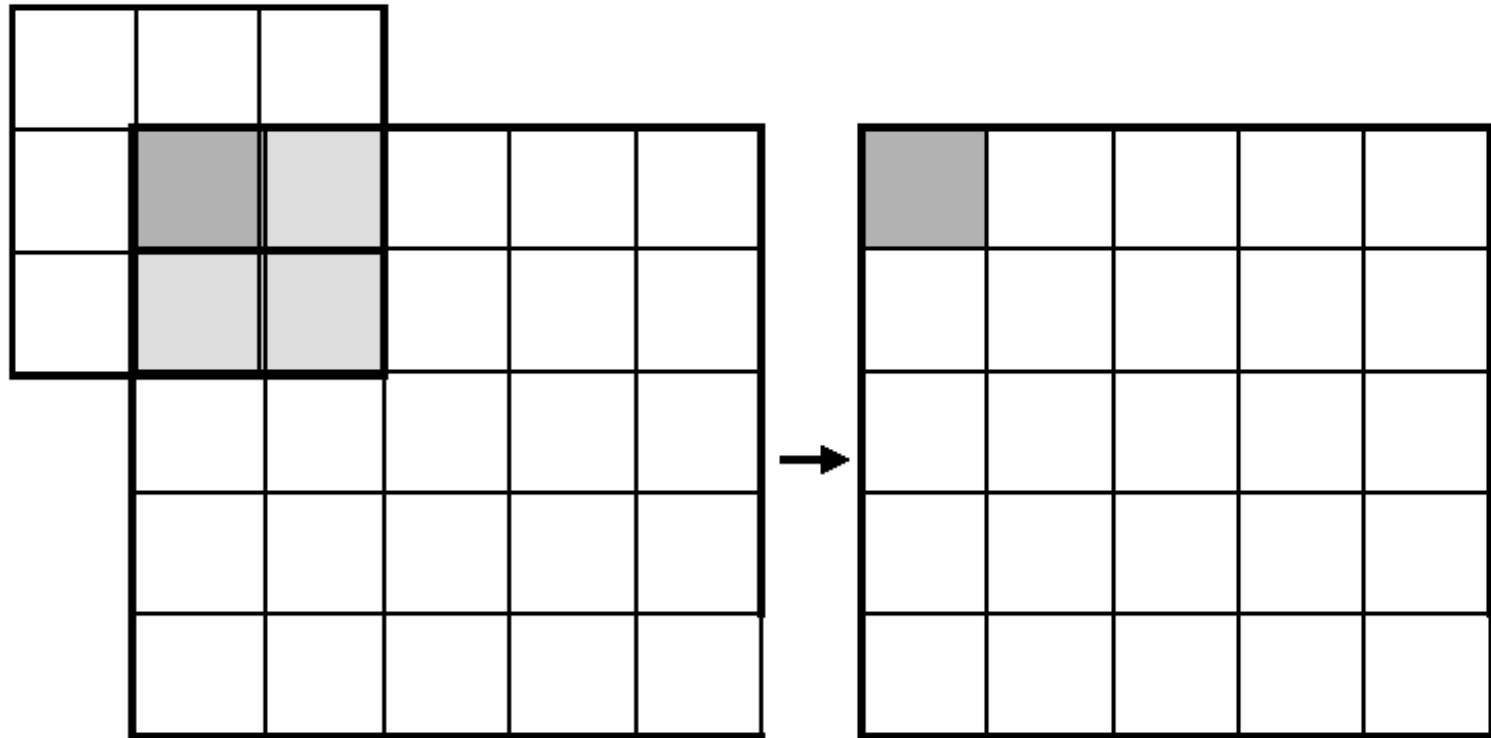


Splošna oblika jedra

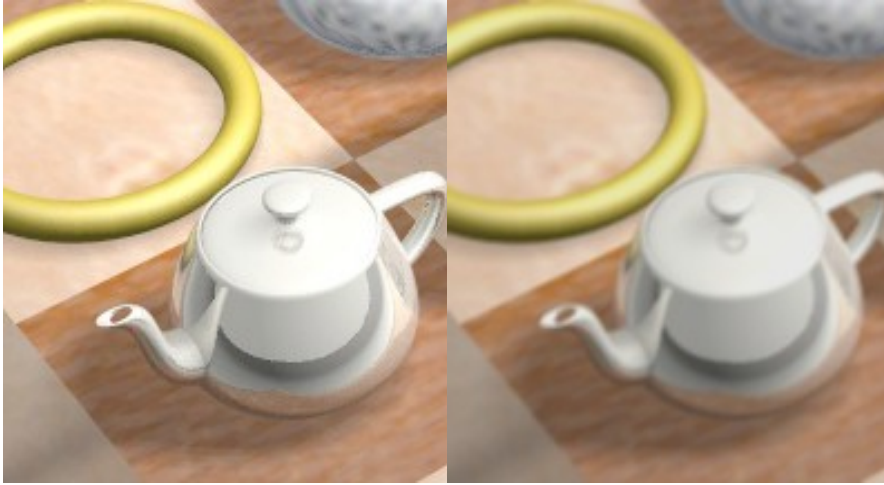
w_{11}	...	w_{1n}
\vdots	■	\vdots
w_{n1}	...	w_{nn}

n.... liho število

Kaj narediti na robovih?



Nizkopasovni filtri

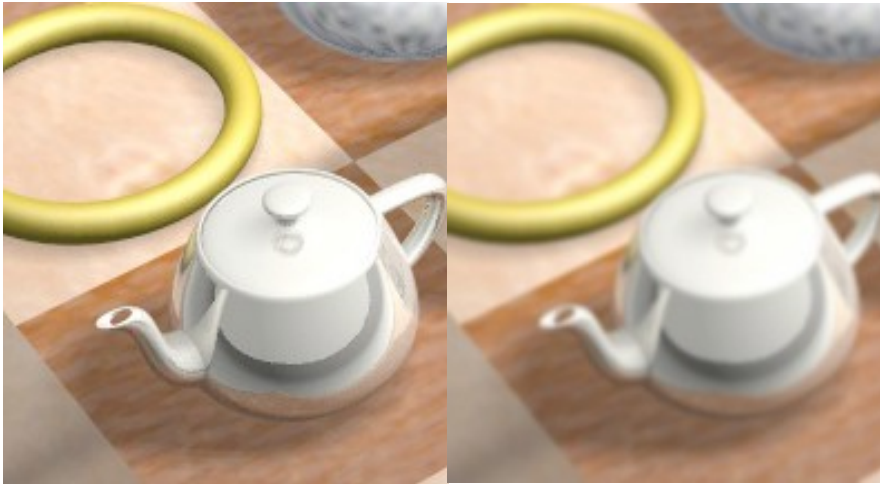


$$\text{Box (order 3)} = \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

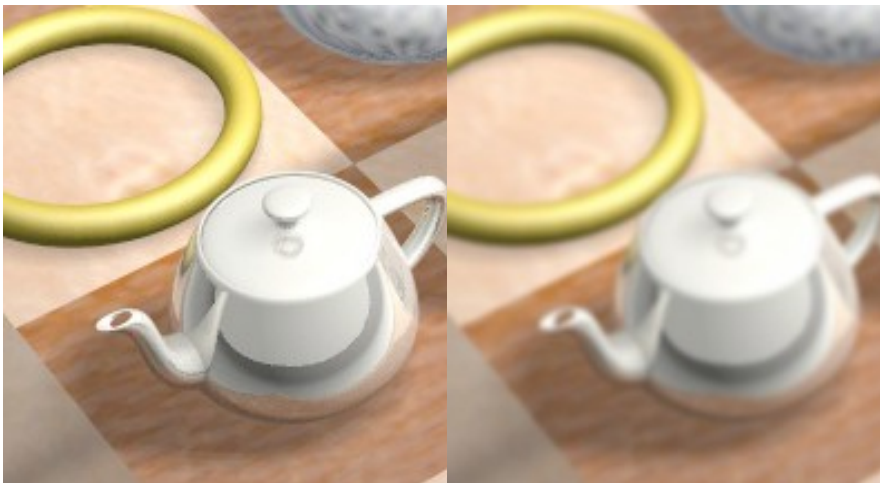


$$\text{Box (order 5)} = \frac{1}{25} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Nizkopasovni filtri

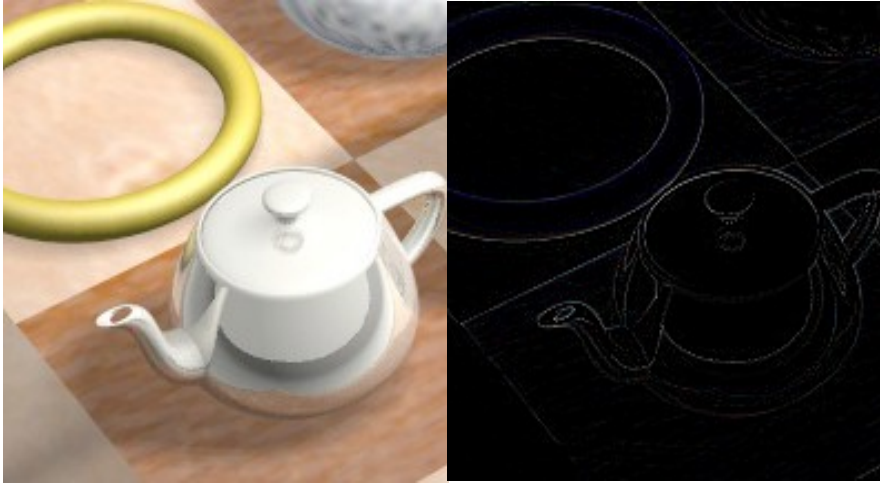


$$\text{Gaussian} = \frac{1}{256} \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$



$$\text{Triangle (Bartlett)} = \frac{1}{81} \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 2 & 4 & 6 & 4 & 2 \\ 3 & 6 & 9 & 6 & 3 \\ 2 & 4 & 6 & 4 & 2 \\ 1 & 2 & 3 & 4 & 1 \end{bmatrix}$$

Visokopasovni filtri

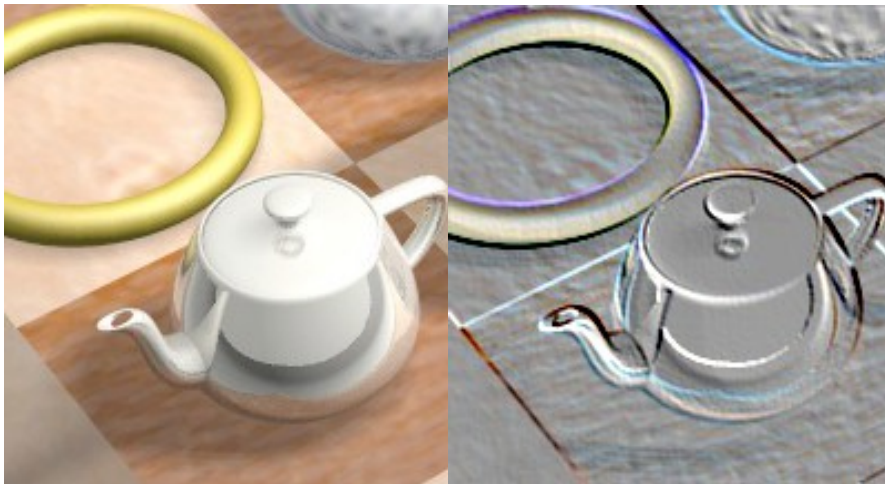


$$\begin{aligned}\text{Highpass} &= \frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & -12 & 2 \\ 1 & 2 & 1 \end{bmatrix} \\ &= \frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\end{aligned}$$

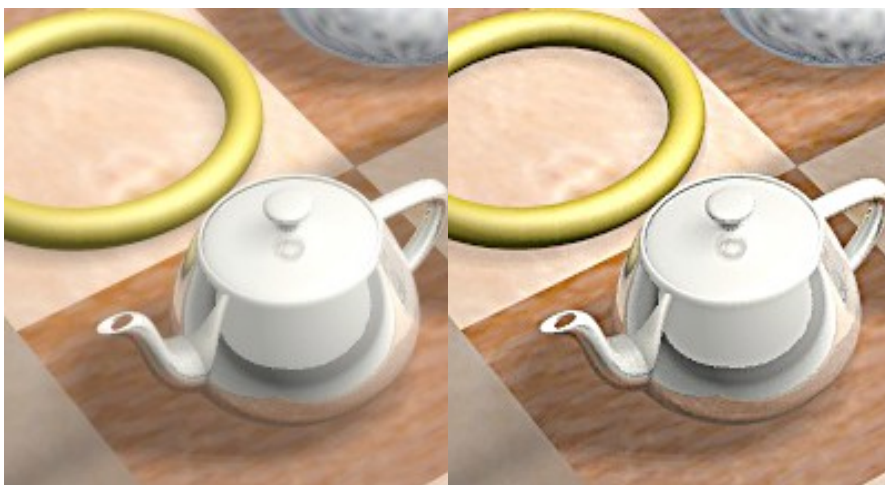


$$\text{Laplacian} = \frac{1}{1} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Drugi filtri



$$\text{Emboss} = \frac{1}{2} + \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$



$$\text{unsharp} = f + (f - f * g_{LP})$$

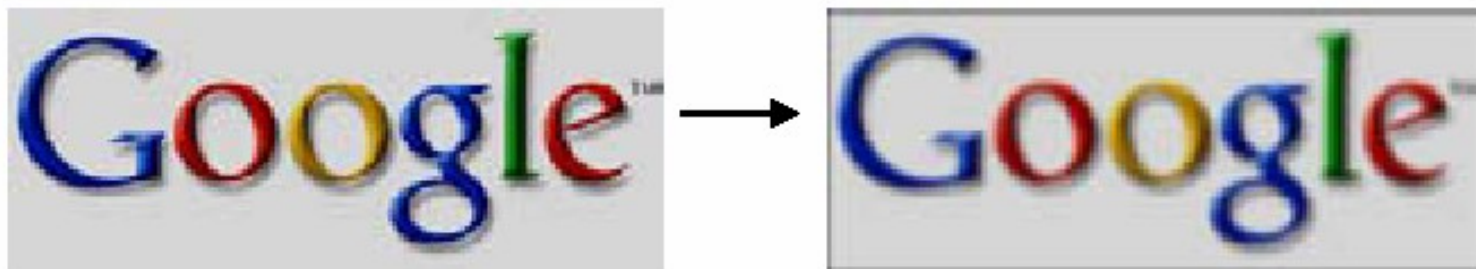
Odkrivanje robov



Jedro za odkrivanje robov

$-1/8$	$-1/8$	$-1/8$
$-1/8$	1	$-1/8$
$-1/8$	$-1/8$	$-1/8$

Zameglitev (blur)



3x3 box blur

$1/9$	$1/9$	$1/9$
$1/9$	$1/9$	$1/9$
$1/9$	$1/9$	$1/9$

$n \times n$ box blur

w	...	w
\vdots	■	\vdots
w	...	w

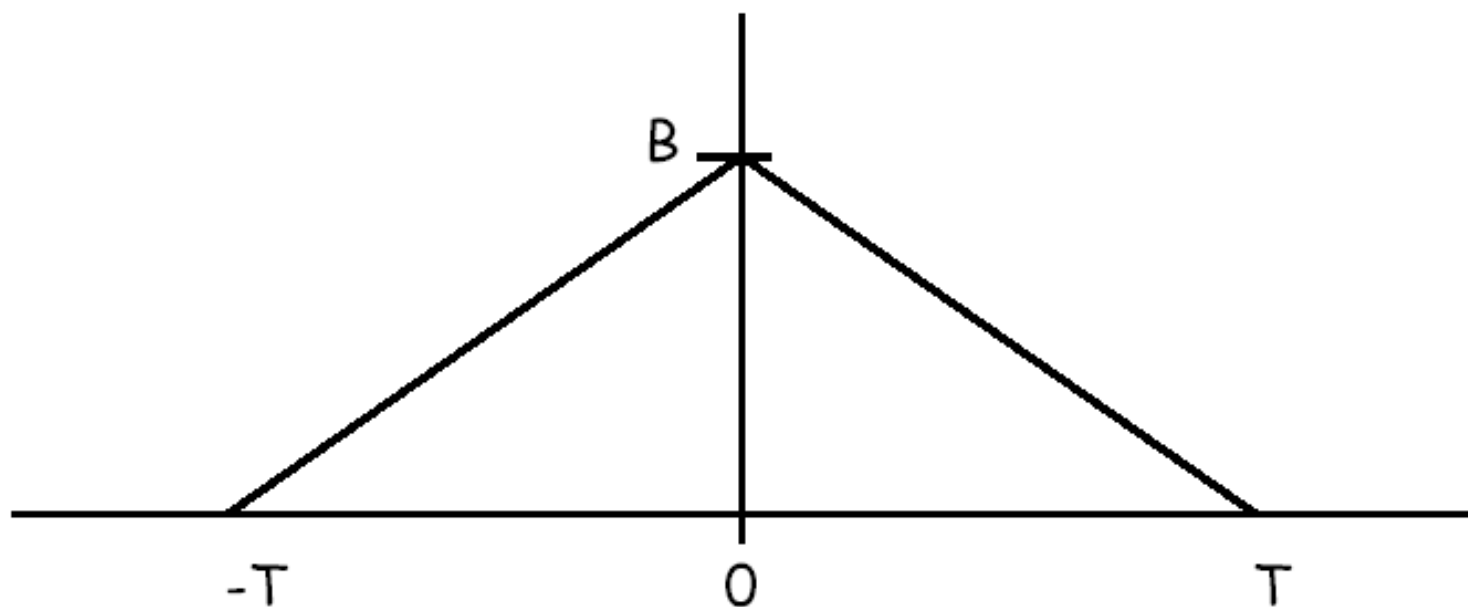
$$w = 1/n^2$$

why is it important that the sum of the weights is 1?

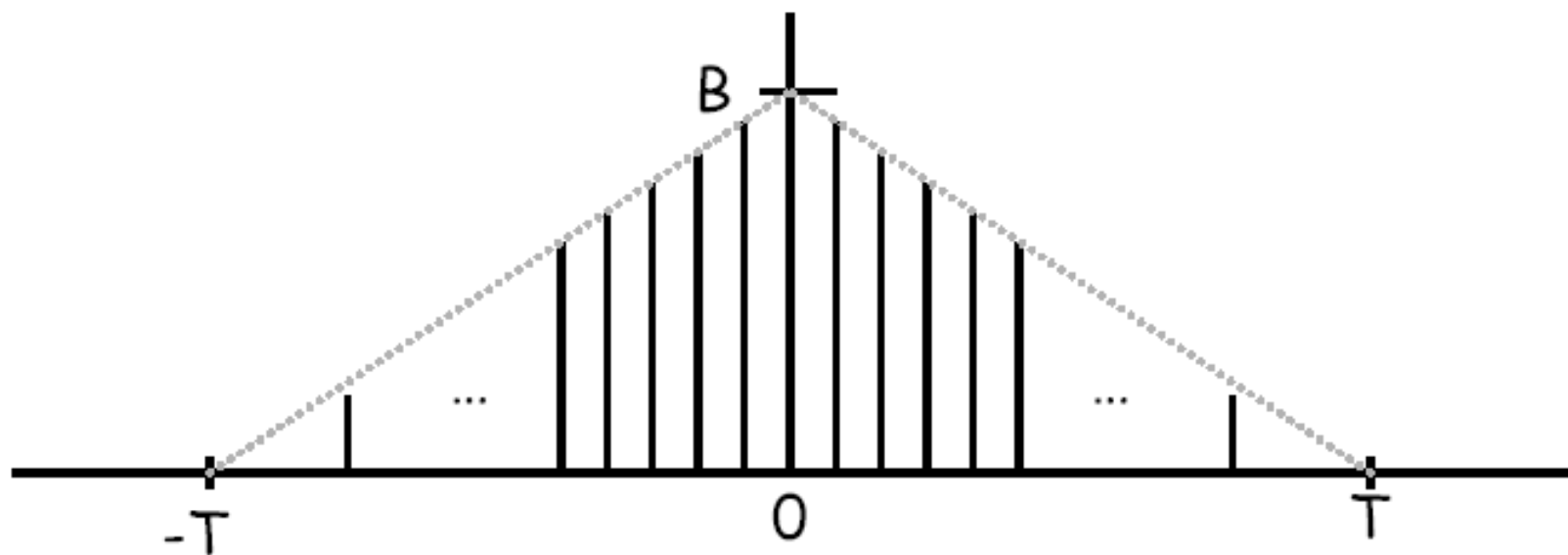
3x3 triangle blur

$1/16$	$1/8$	$1/16$
$1/8$	$1/4$	$1/8$
$1/16$	$1/8$	$1/16$

triangle function

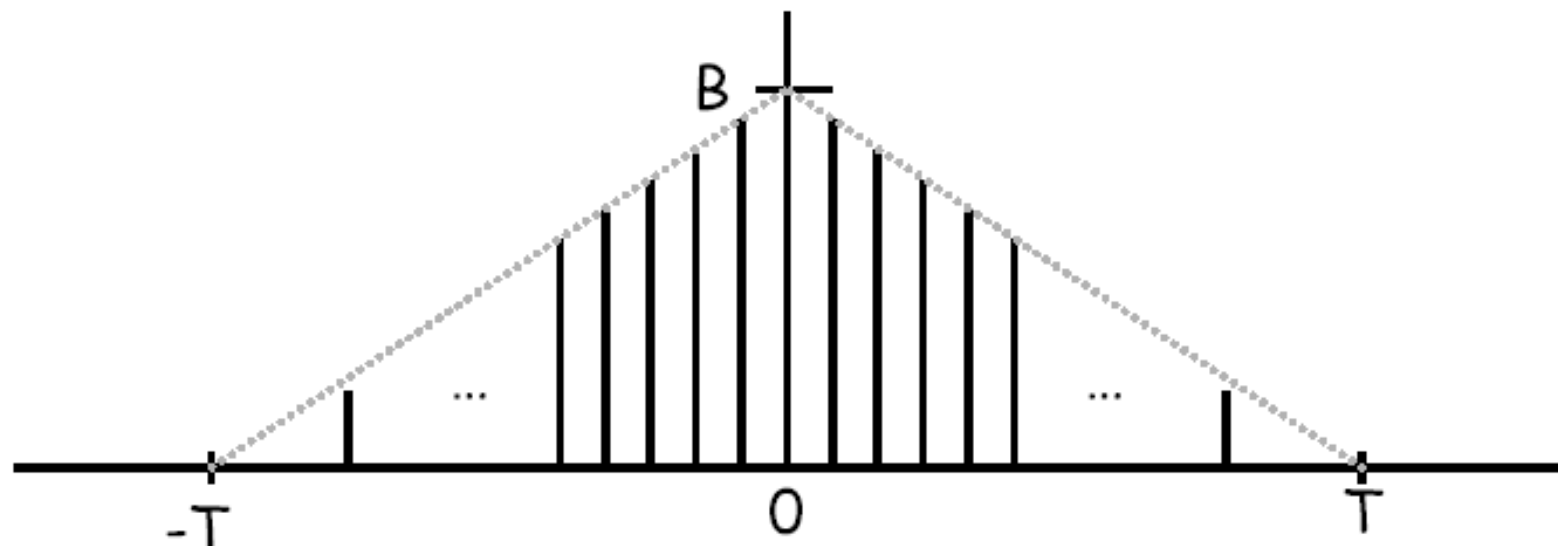


discrete triangle

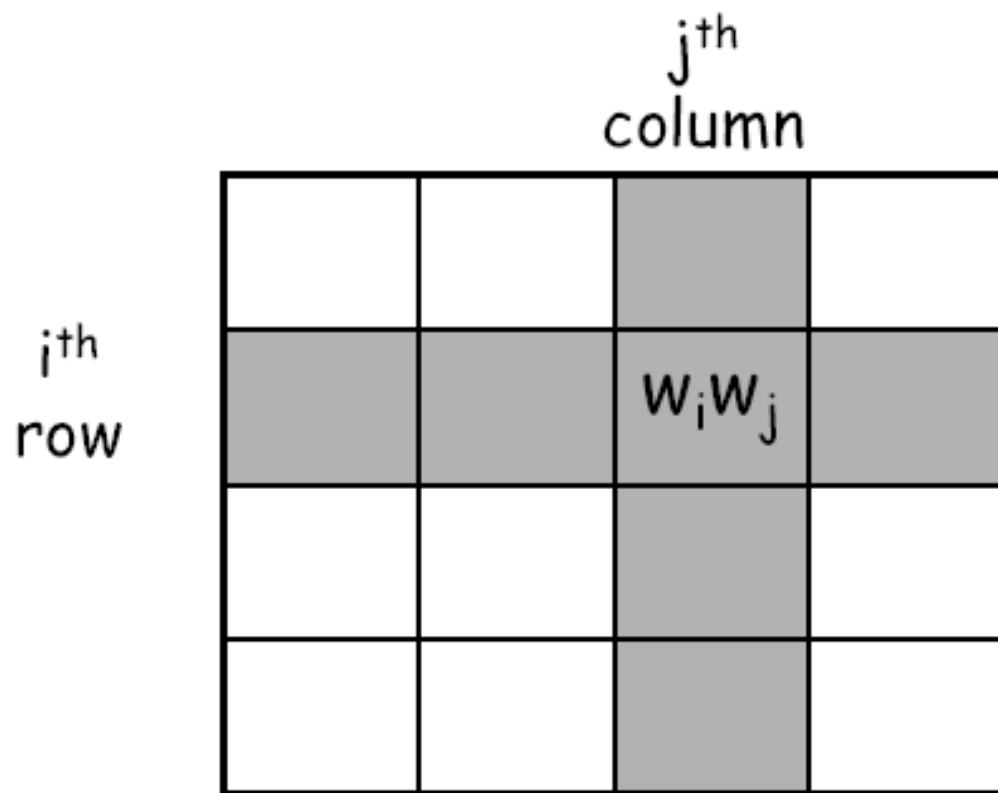


normalized, discrete triangle

1. $T = (n+1)/2$ gives n non-zero samples
2. $\sum_{j=-T}^T f(j) = 1$ provided $B = 2/(n+1)$



triangle blur filter

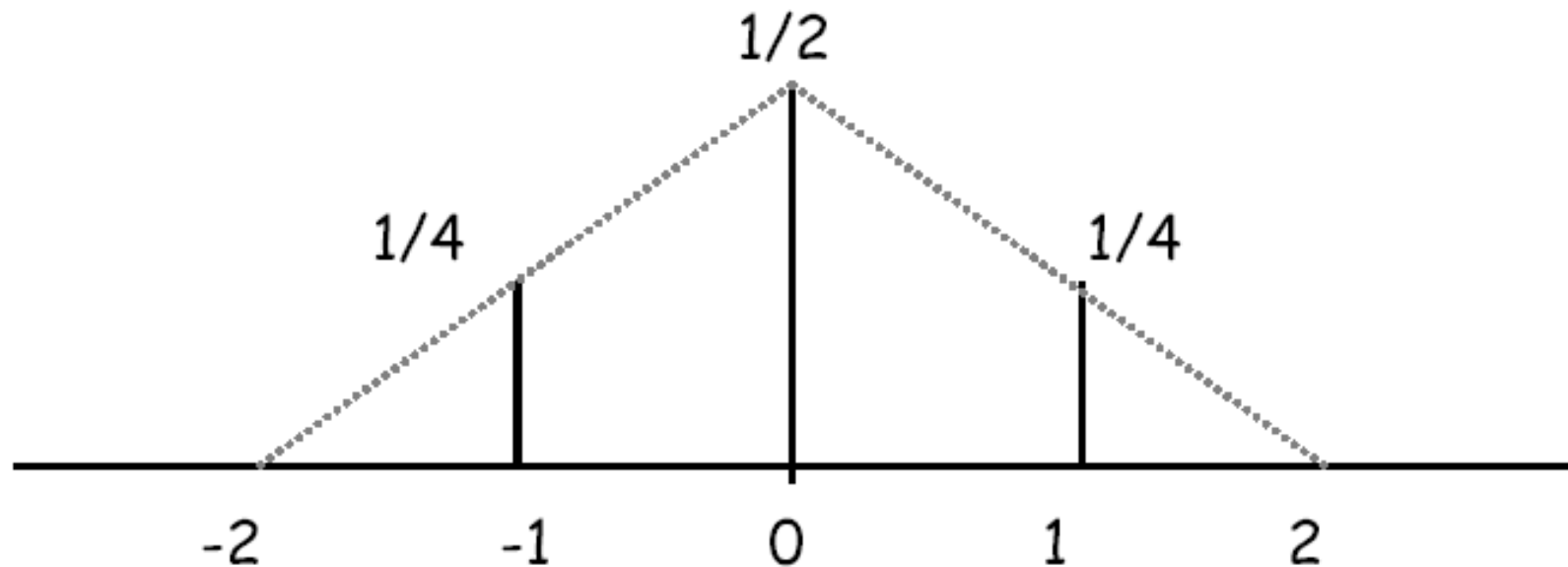


triangle samples: w_1, w_2, \dots, w_n

example: $n=3$

$$T = (n+1)/2 = 2$$

$$B = 2/(n+1) = 1/2$$

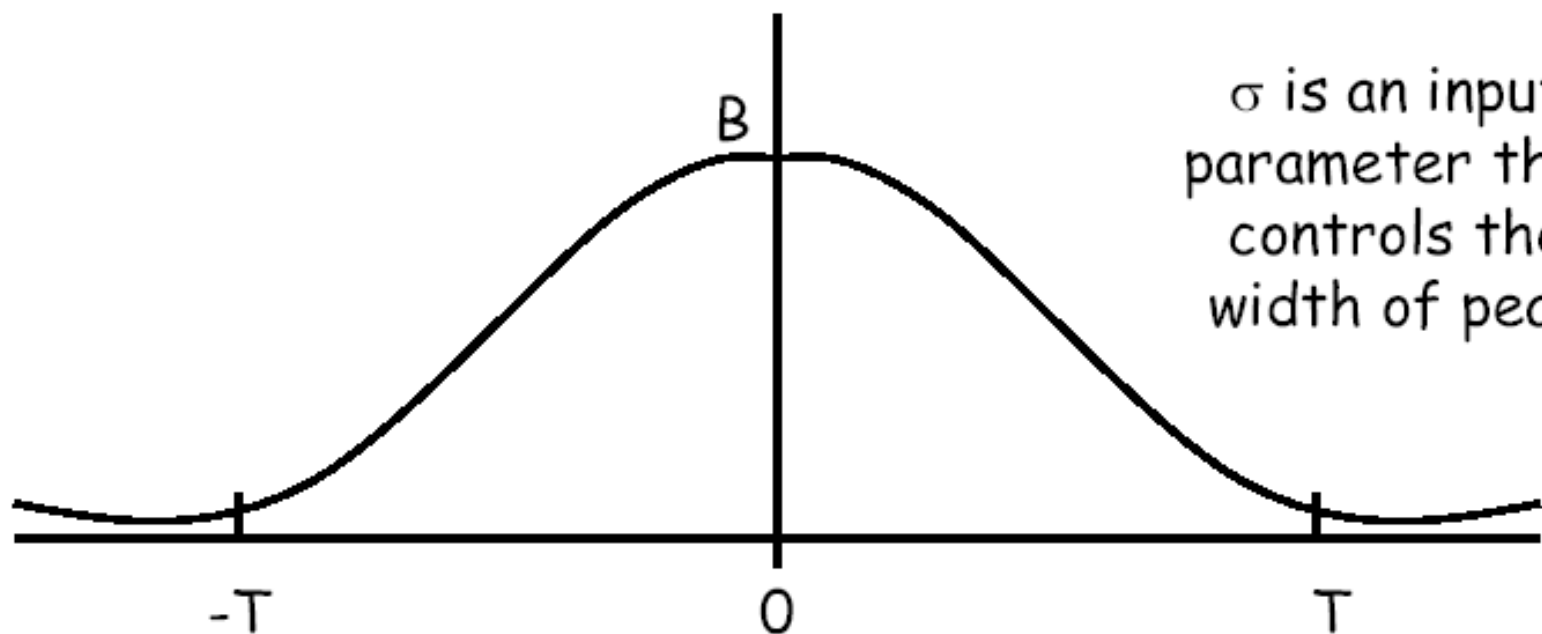


3x3 triangle blur filter

$1/4$	$1/16$	$1/8$	$1/16$
$1/2$	$1/8$	$1/4$	$1/8$
$1/4$	$1/16$	$1/8$	$1/16$
	$1/4$	$1/2$	$1/4$

gaussian function

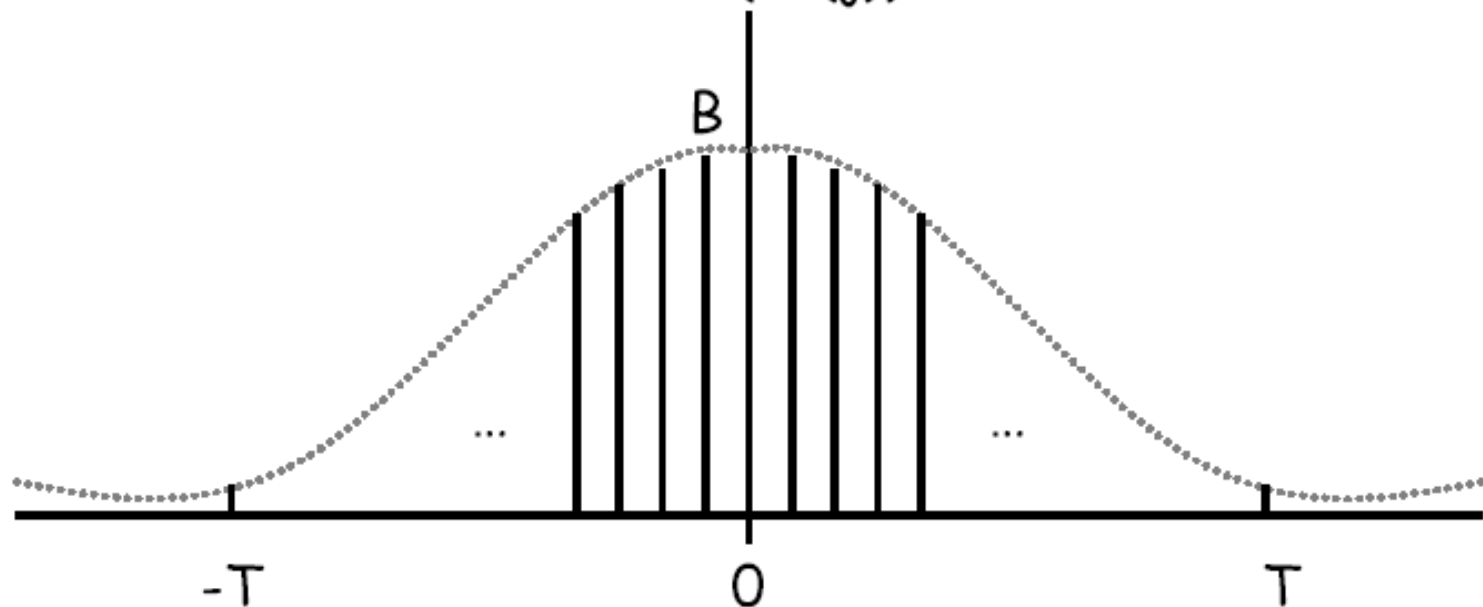
$$f(x) = Be^{-x^2/\sigma^2}$$



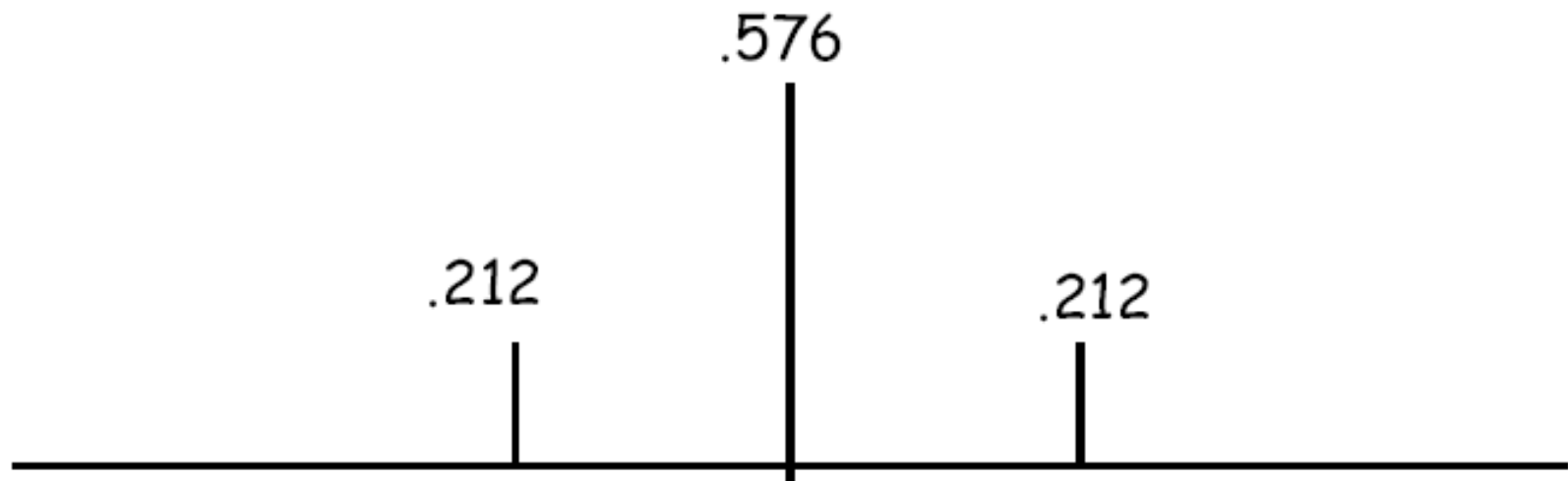
normalized, discrete version

$T = (n+1)/2$ gives n samples

$$B = 1/(\sum f(j))$$



example: $n=3$, $\sigma=1$



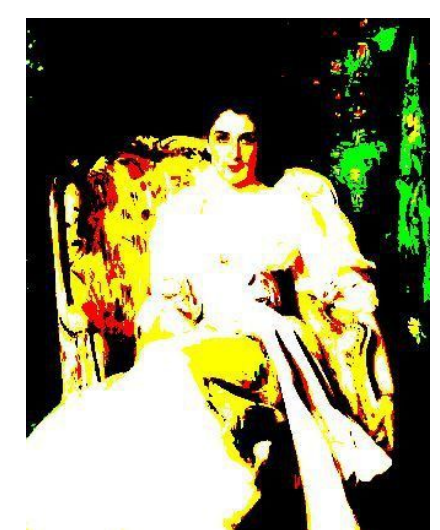
3x3 gaussian blur, $\sigma = 1$

.212	.045	.122	.045
.576	.122	.332	.122
.212	.045	.122	.045
	.212	.576	.212

Box, triangle, Gaussian blurr



Obdelava slik in java2D



type of techniques

- simple pixel modification
- interpolation/extrapolation
- compositing
- convolution
- **dithering** Razprševanje
- warping
- morphing
- misc. effects

Razprševanje in poltoni (dithering, halftoning)

Trade spatial for intensity resolution

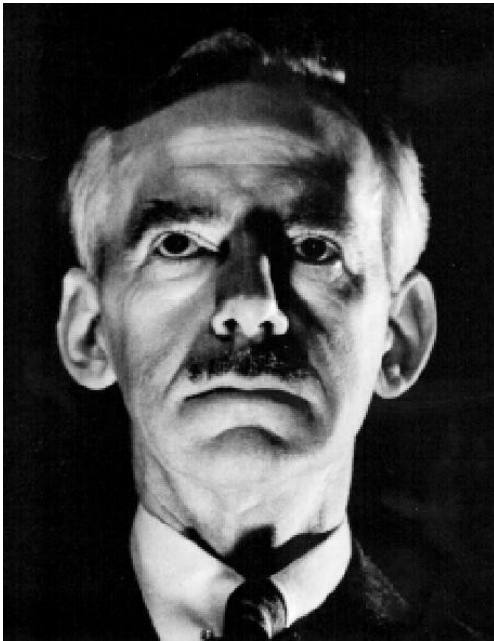
- Thresholding.
- Random dither; Robert's algorithm
- Ordered dither
- Error diffusion

Your eye will average over an area

- Spatial Integration

Prag (Thresholding)

Original image.



$n = 0.5$

Simple threshold.



$n = 0.7$



$$v(x, y) = \text{trunc}(\lfloor v(x, y) + n \rfloor)$$

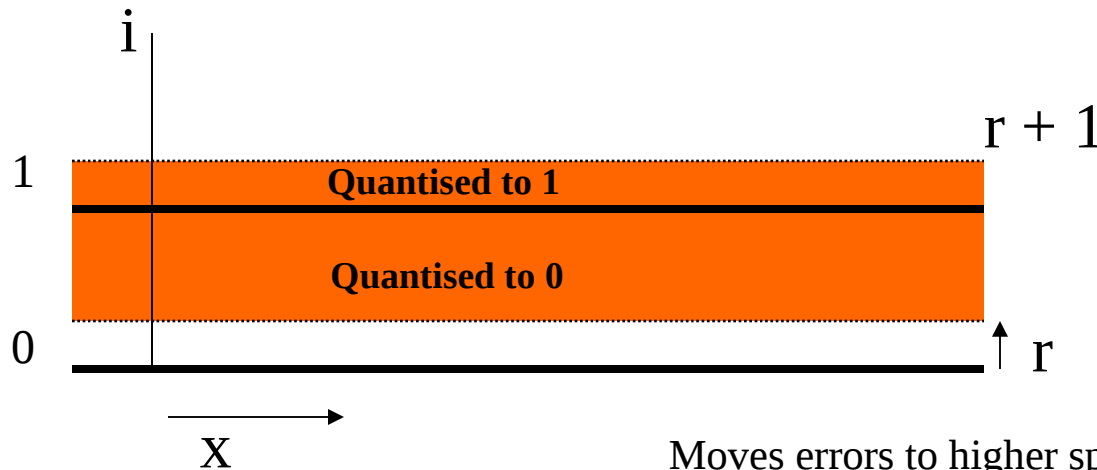
Errors are low spatial frequencies.

Robertov algoritem

- First add noise
- Then quantize

$$\hat{v}(x, y) = \text{trunc}(K \times v(x, y) + \text{noise}(x, y))$$

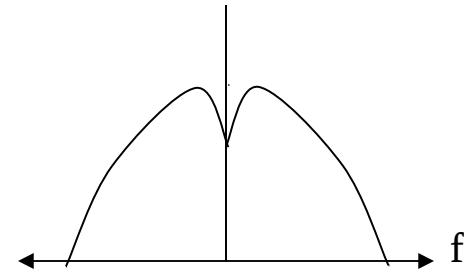
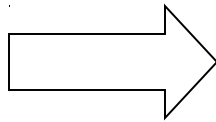
$$0 \leq \text{noise} < 1$$



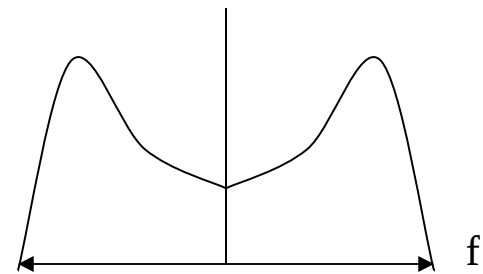
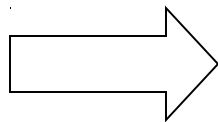
Moves errors to higher spatial frequencies.
-> eye averages over an area.

Spektri šumov

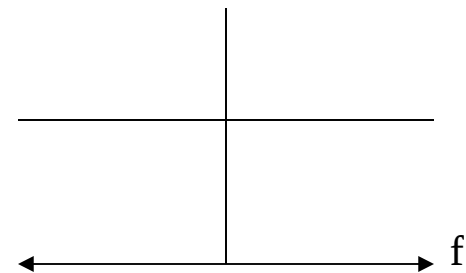
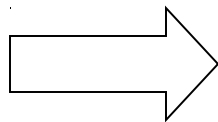
Pink Noise



Blue Noise



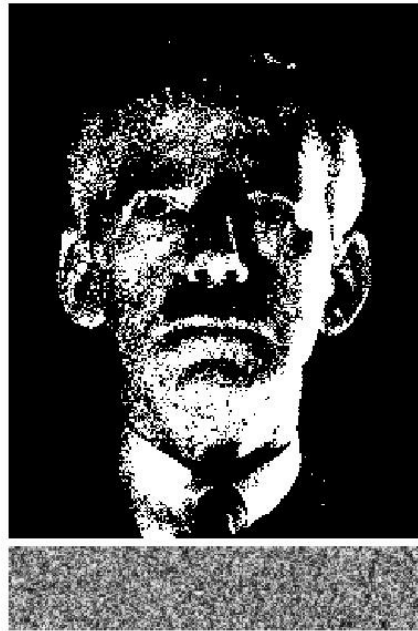
White Noise



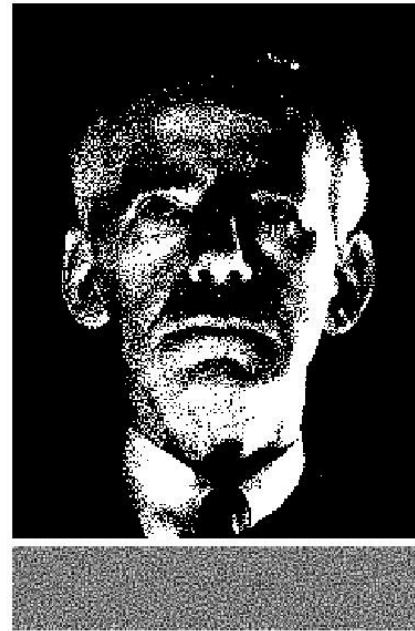
Robertov algoritem

Moves low frequency (average error) to high frequency
Pink(low), **Blue (high)**, White(all) frequency noise

Pink



Blue

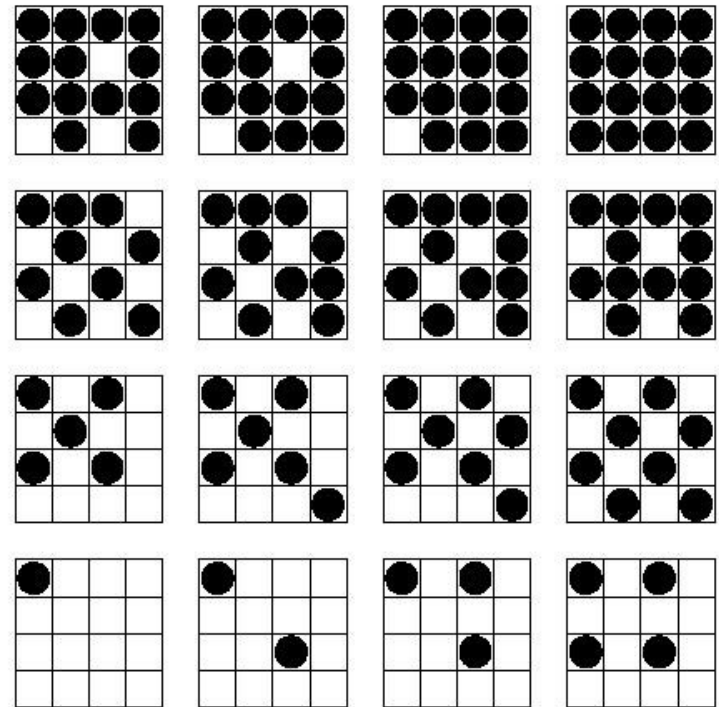


Problemi s šumom

- Difficult to compute quickly.
- Not reproducible.
- Pre-compute pseudo-random function and store in table.
- Small tiled patterns sufficient

Urejeno razprševanje (Ordered Dithering)

- Trade off spatial resolution for intensity resolution.
- Use dither patterns.
- Can be represented as a matrix.

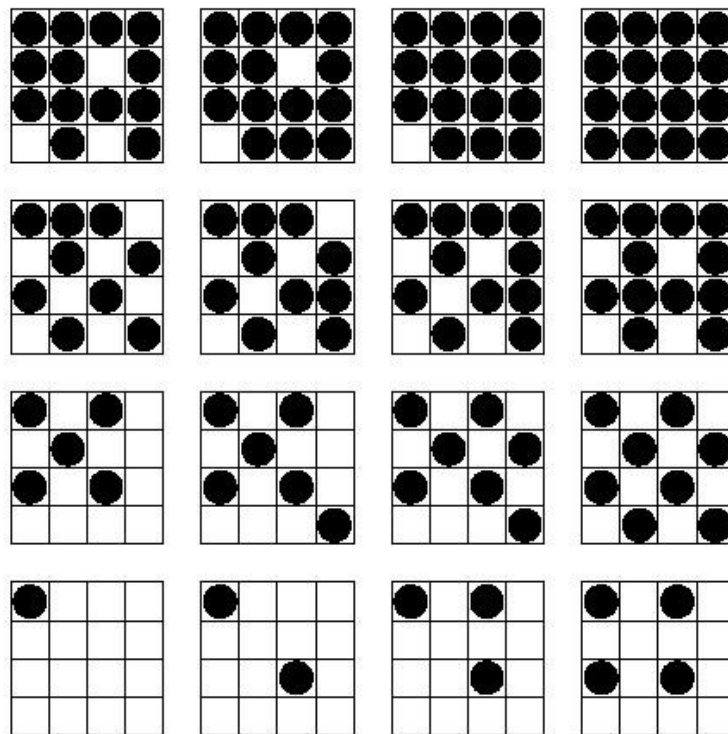


Bayerjevi vzorci urejenega razprševanja

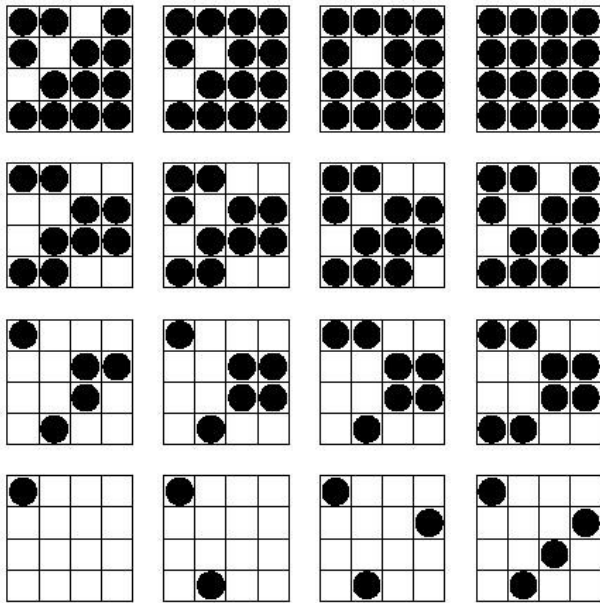
$$D_2 = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}$$

$$D_n = \begin{bmatrix} 4D_{n/2} + 0 & 4D_{n/2} + 2 \\ 4D_{n/2} + 3 & 4D_{n/2} + 1 \end{bmatrix}$$

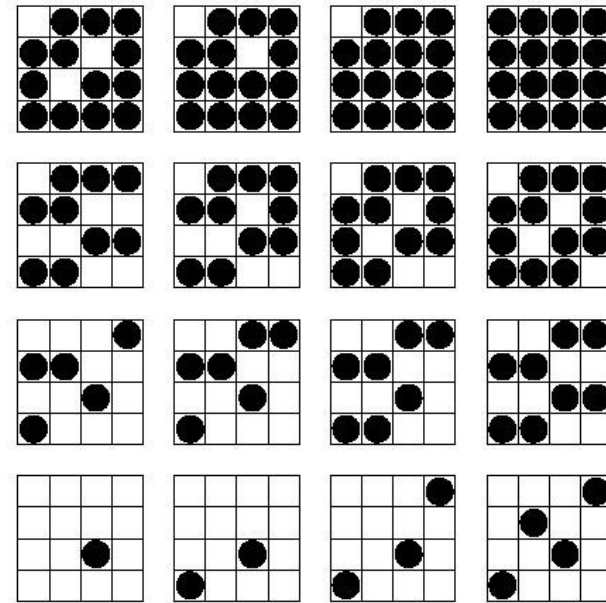
$$D_4 = \begin{bmatrix} 0 & 8 & 2 & 10 \\ 12 & 4 & 14 & 6 \\ 3 & 11 & 1 & 9 \\ 15 & 7 & 13 & 5 \end{bmatrix}$$



Druge možnosti



$$M_4 = \begin{bmatrix} 0 & 6 & 13 & 11 \\ 9 & 15 & 4 & 2 \\ 14 & 8 & 3 & 5 \\ 7 & 1 & 10 & 12 \end{bmatrix}$$

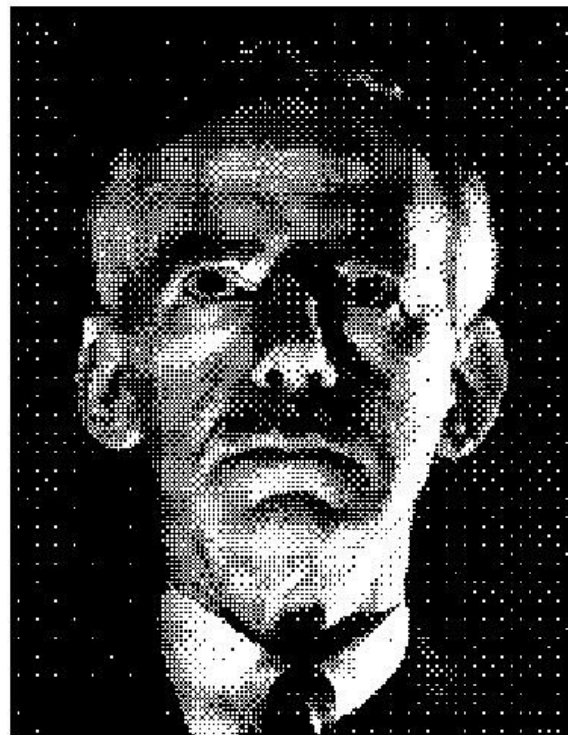


$$P_4 = \begin{bmatrix} 15 & 8 & 5 & 2 \\ 4 & 3 & 14 & 9 \\ 10 & 13 & 0 & 7 \\ 1 & 6 & 11 & 12 \end{bmatrix}$$

Primerjava



Blue Noise

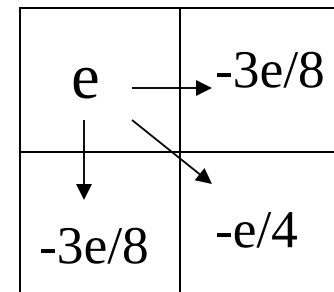


Ordered Dither

Širjenje napake

Idea: Quantize, then distribute error to neighbours

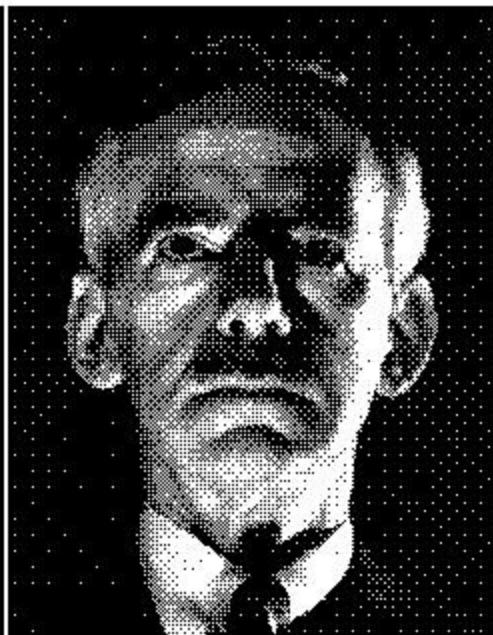
```
for(y=0; y<ny; y++)  
  for(x=0; x<nx; x++){  
    vq[x][y] = quantize(v[x][y]);  
    e = v[x][y] - vq[x][y];  
    i[x+1][y] += 3/8*e;  
    i[x][y+1] += 3/8*e;  
    i[x+1][y+1] += 1/4*e;  
  }
```



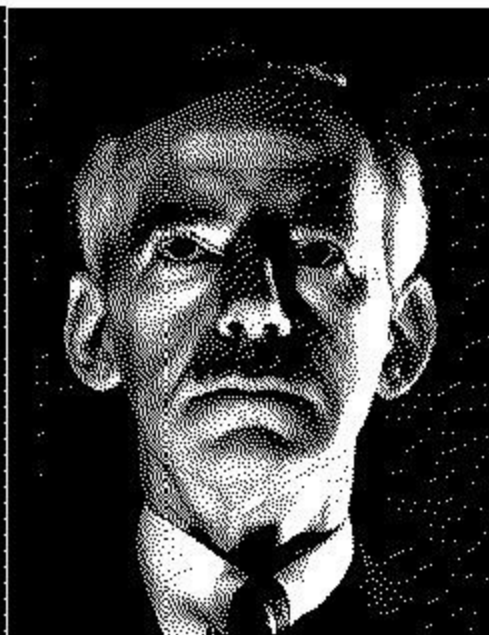
Primerjava



Blue Noise



Bayer Dither



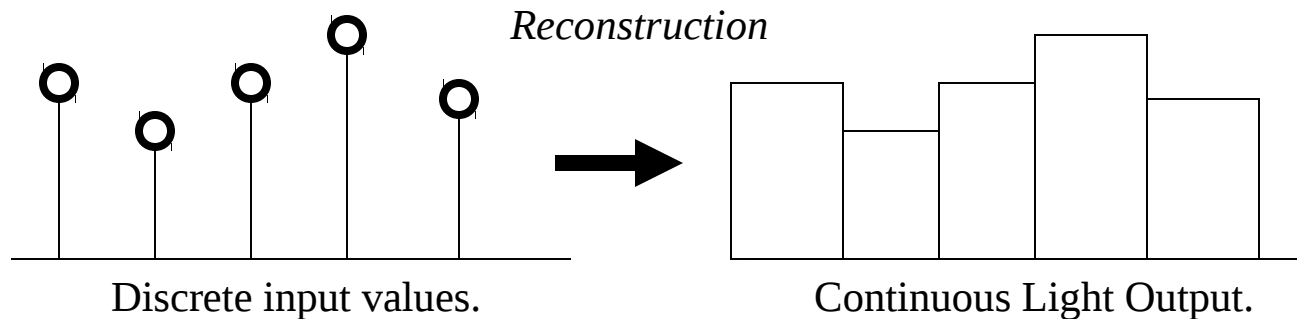
Floyd-Steinberg

Vzorčenje in aliasing

- Real world is continuous
- The computer world is discrete
- Mapping a continuous *function* to a discrete one is called *sampling*
- Mapping a continuous *variable* to a discrete one is called *quantizaion*
- To represent or render an image using a computer, we must both sample and quantize

Prikazovalniki → rekonstrukcija signala

- All physical displays recreate a continuous image from a discrete sampled image by using a finite sized source of light for each pixel.



Spomnimo se: piksel je točka...

- It is NOT a box, disc or teeny wee light
- It has no dimension
- It occupies no area
- It can have a coordinate
- *More* than a point, it is a *SAMPLE*

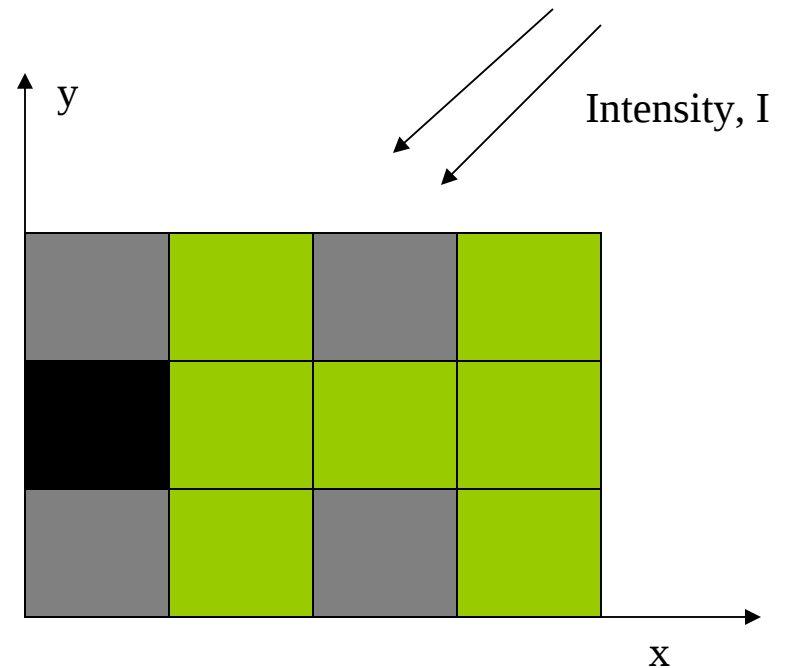
Vzorčenje v video kamerah

- Video camera : CCD array.

Regular array of imaging sensors.

Value sensed is an area integral over a pixel.

$$V = k \iint_{x,y} I dx dy$$

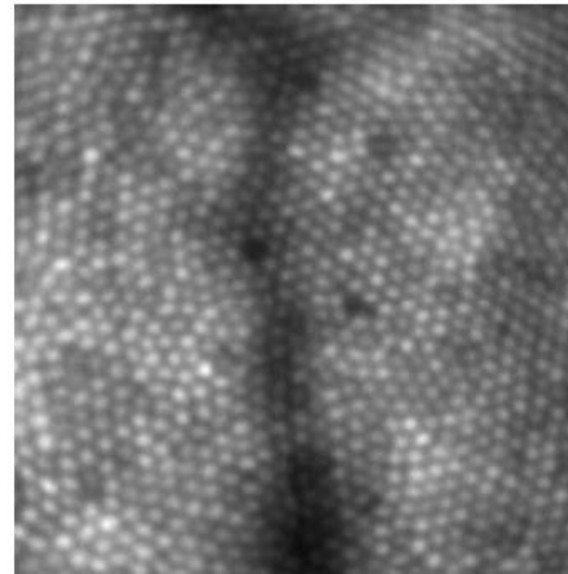


Vzorčenje v očesu

- The eye : photoreceptors

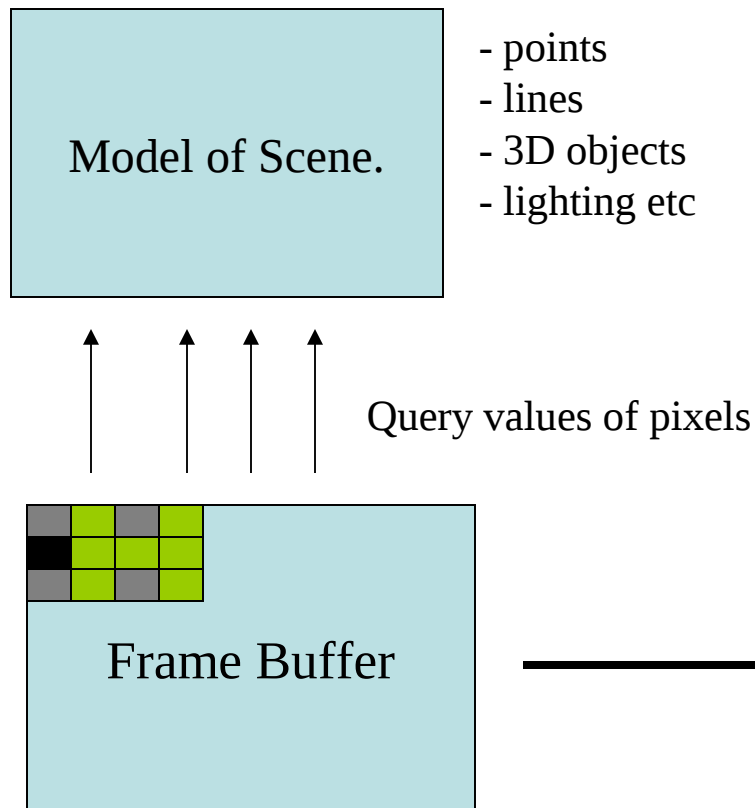
Film is similar : irregular array of receptors.

~ 0.47 deg.

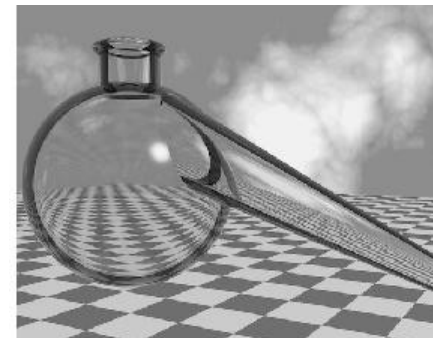


J. Liang, D. R. Williams and D. Miller, "Supernormal vision and high- resolution retinal imaging through adaptive optics," J. Opt. Soc. Am. A 14, 2884- 2892 (1997)

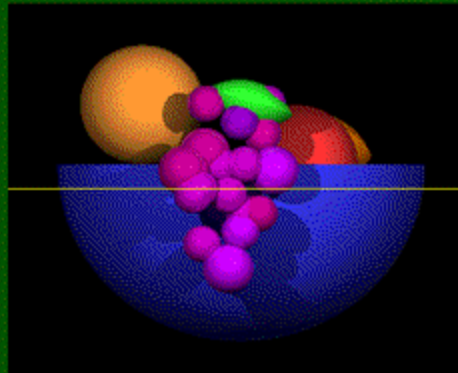
Ali je pri računalniški grafiki drugače ?



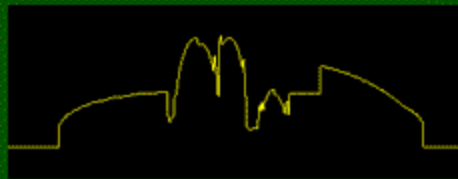
Point Sampling.



Zvezen svetlostni signal

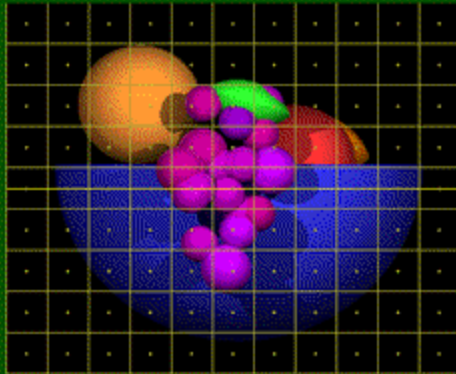


**Original
scene**

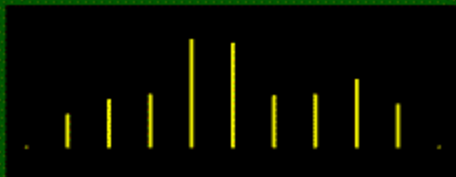


**Luminosity
signal**

Vzorčena svetlost

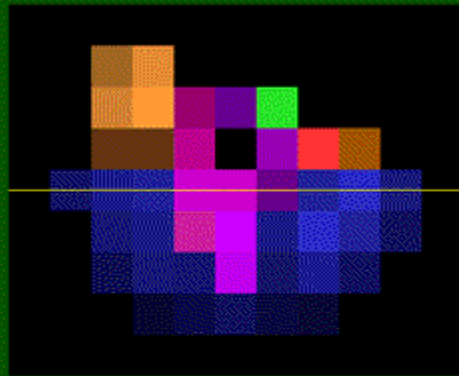


**Sampling at
pixel centers**

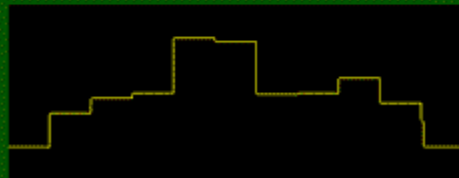


**Sampled
signal**

Rekonstruirana svetlost

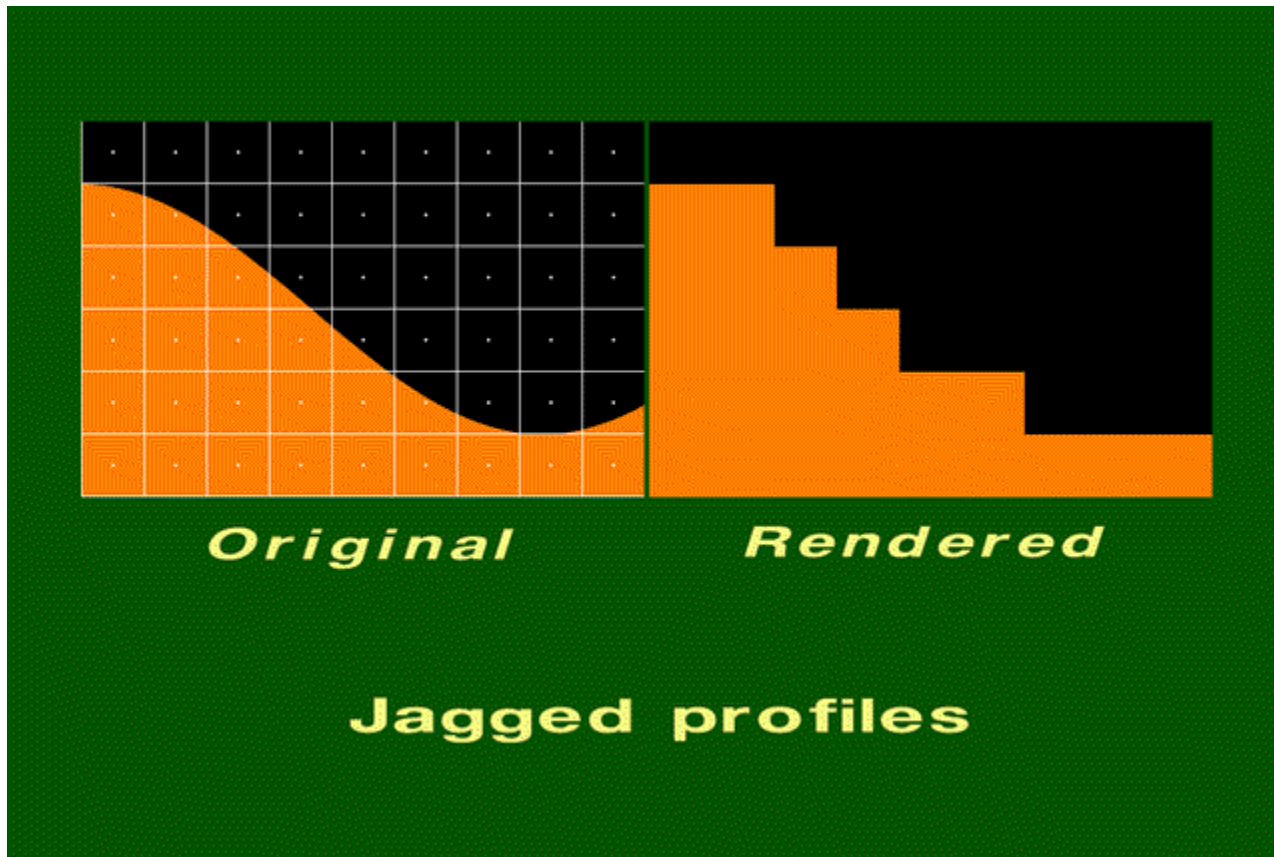


**Rendered
image**



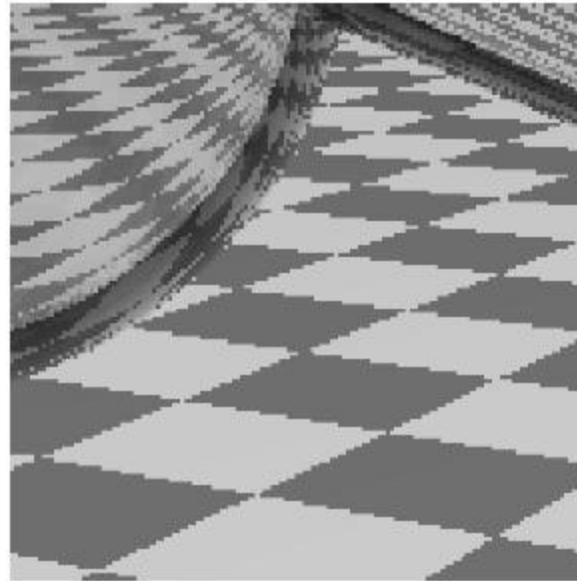
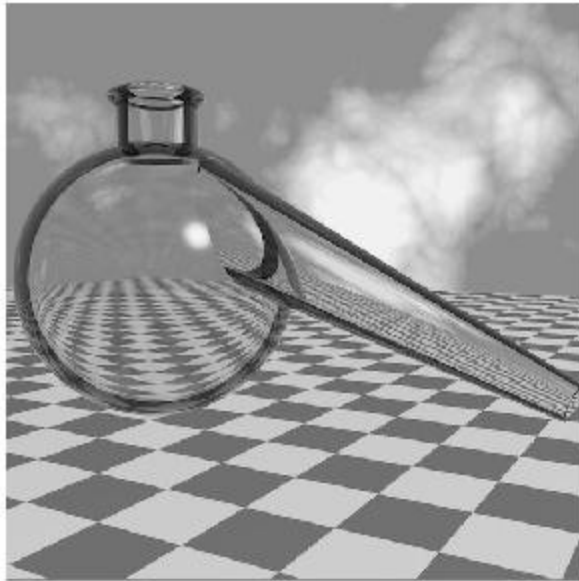
**Luminosity
signal**

Rezultat rekonstrukcije



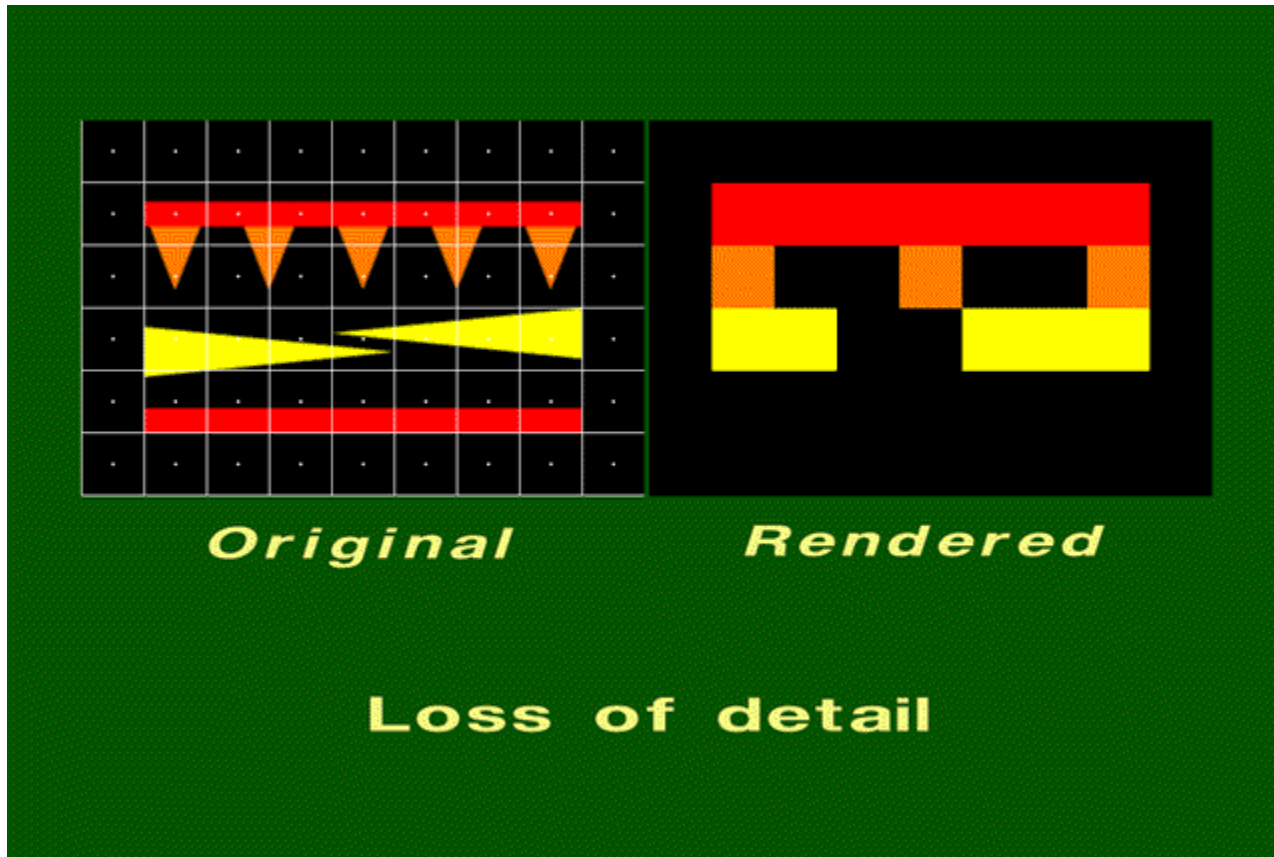
Nazobčanost

The raster *aliasing* effect – removal is called *antialiasing*



Images by Don Mitchell

Je lahko resen problem...

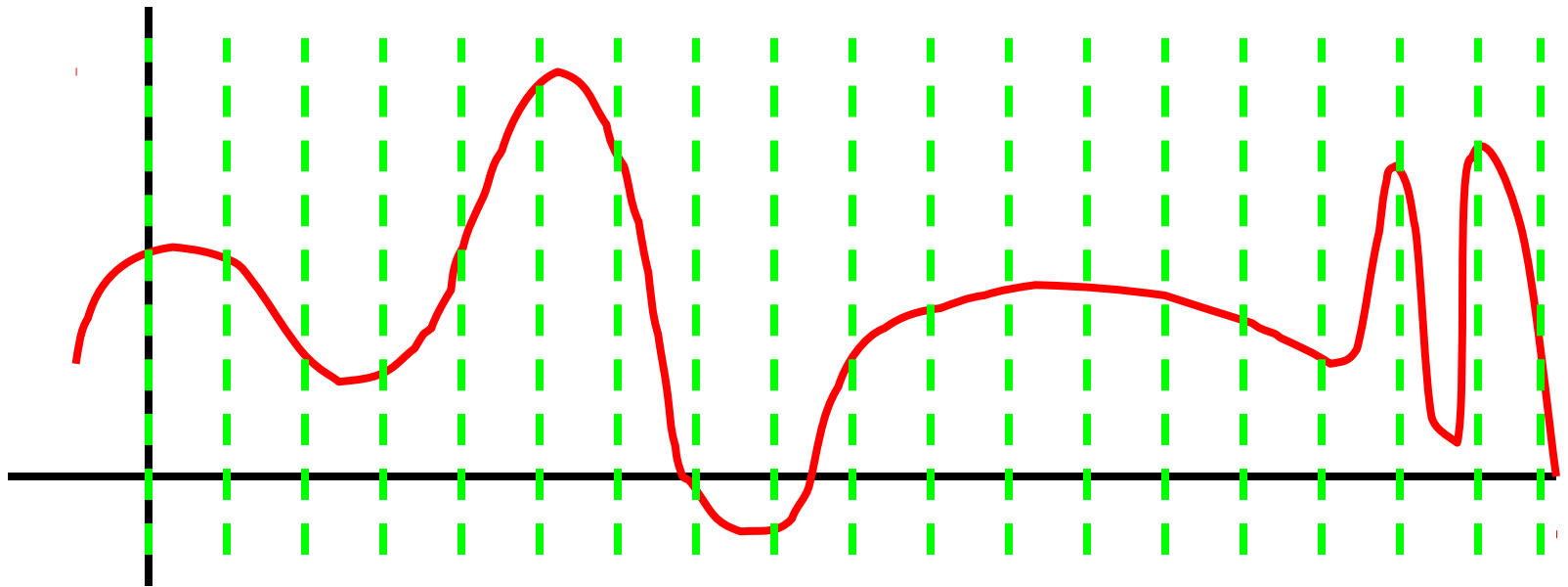


...zelo resen problem!



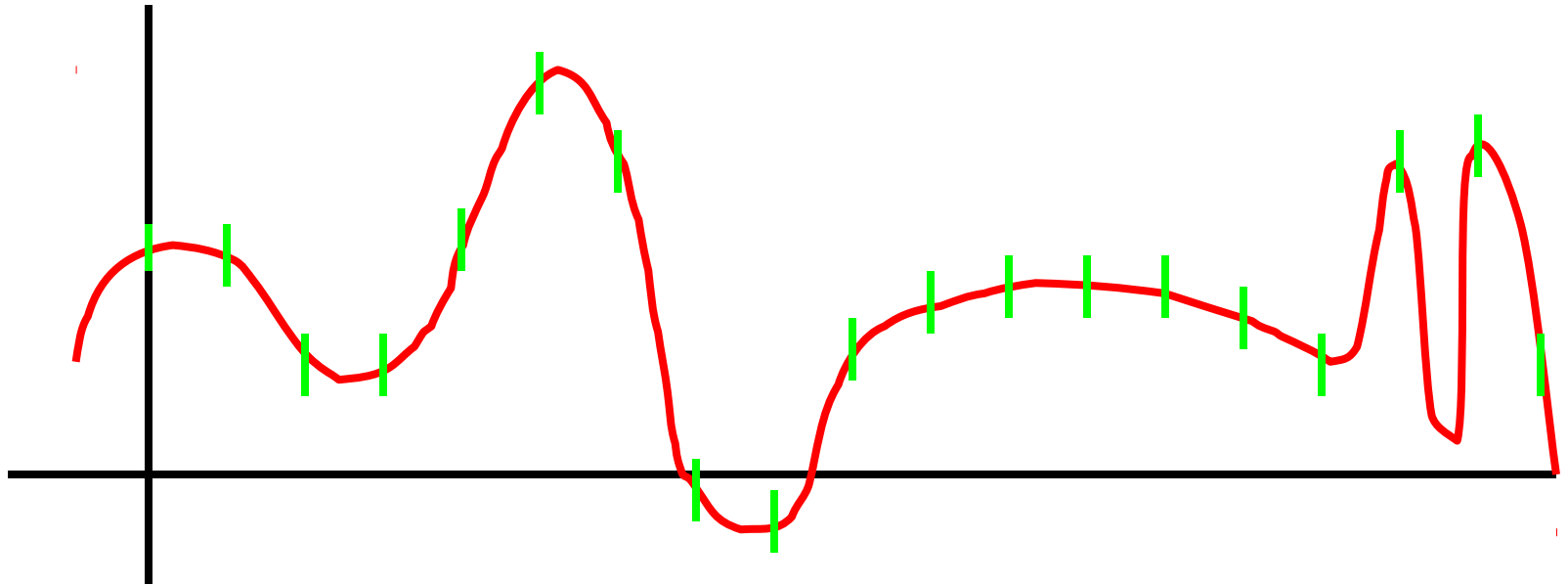
Podvzorčenje v 1D prostoru

- Sampling a 1-D function:



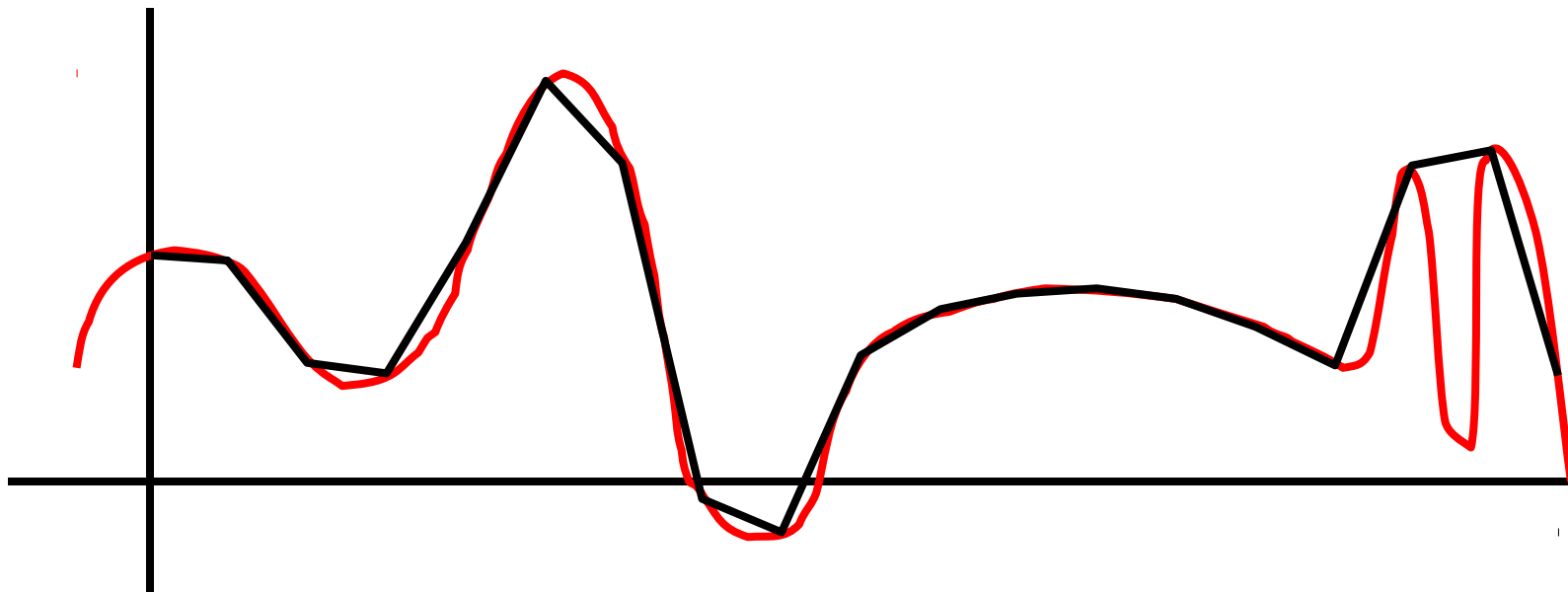
Podvzorčenje v 1D prostoru

- Sampling a 1-D function:



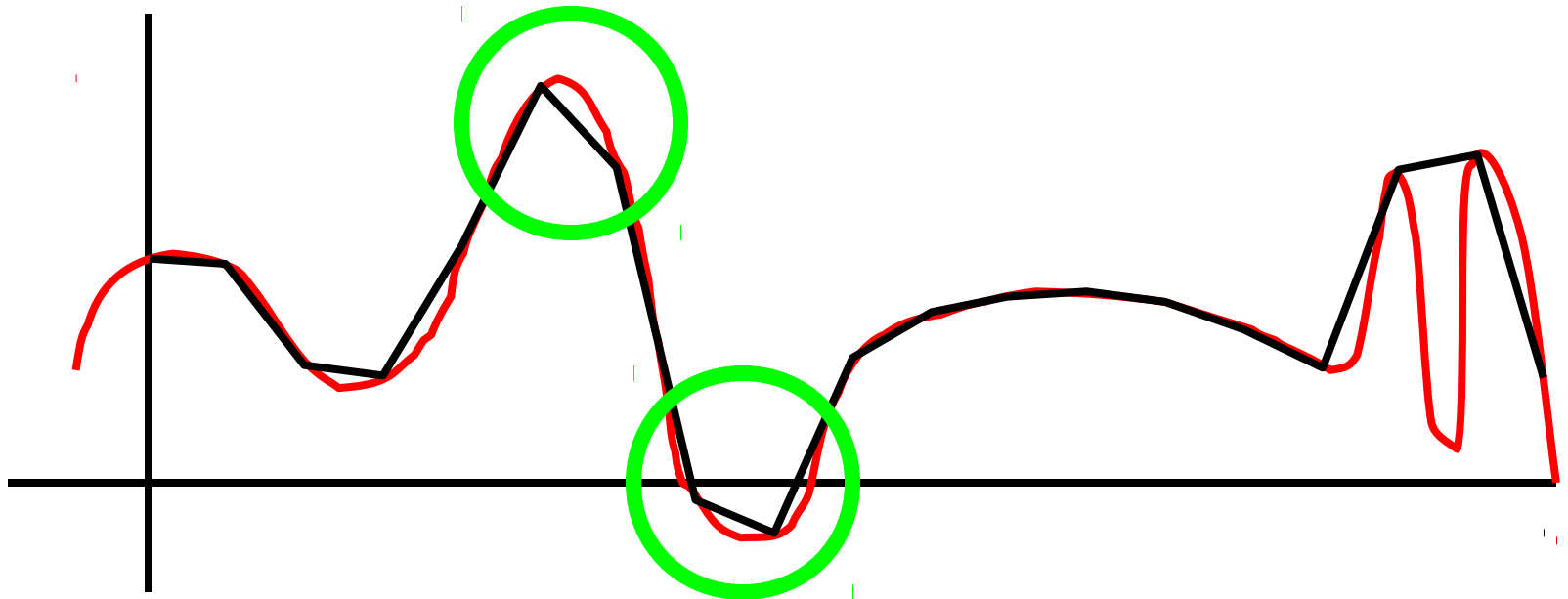
Podvzorčenje v 1D prostoru

- Sampling a 1-D function:
 - What do you notice?



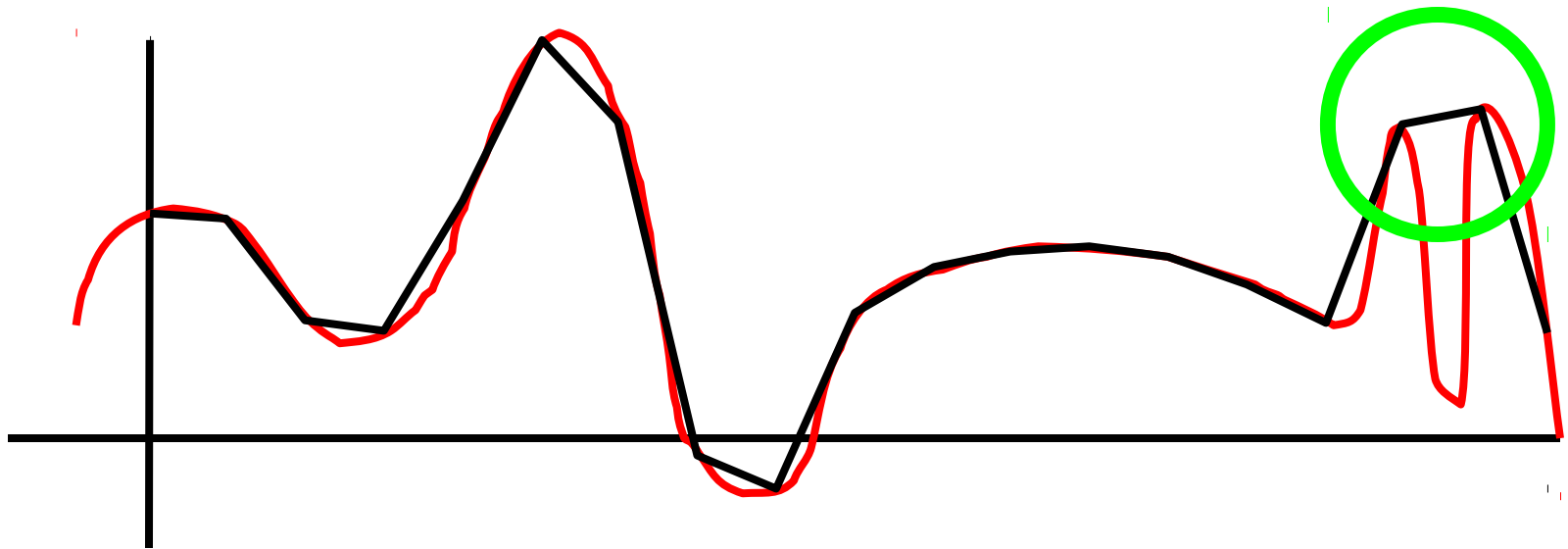
Podvzorčenje v 1D prostoru

- Sampling a 1-D function: what do you notice?
 - Jagged, not smooth



Podvzorčenje v 1D prostoru

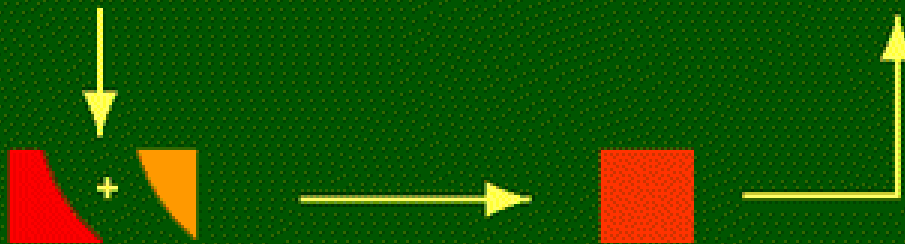
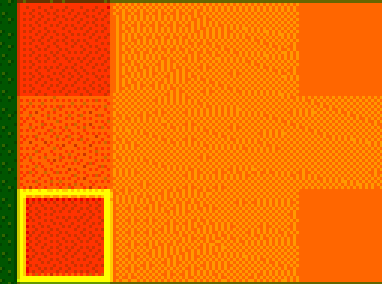
- Sampling a 1-D function: what do you notice?
 - Jagged, not smooth
 - Loses information!



Original



Rendered



Prefiltering methods examine areas of color within a pixel.

$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$
$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

*Combines
nine
samples*

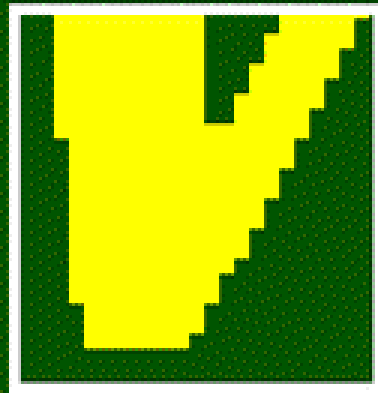
**Filters combine samples
to find a pixel's color.**

Hello World

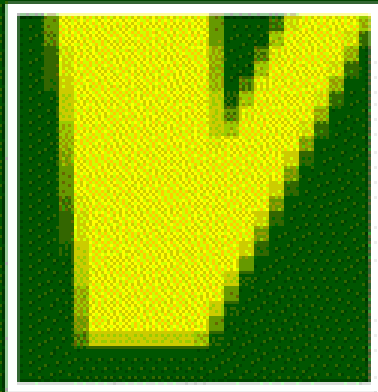
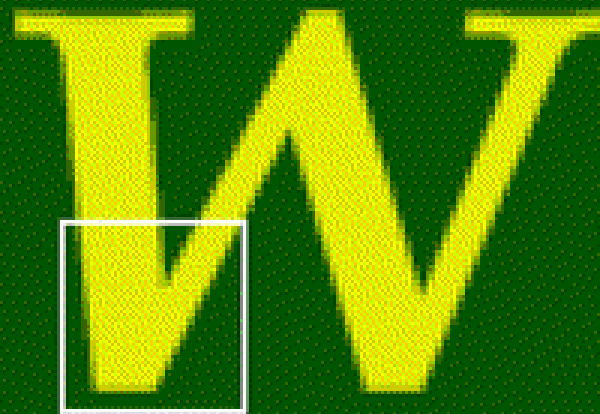
Hello World

A demonstration

W

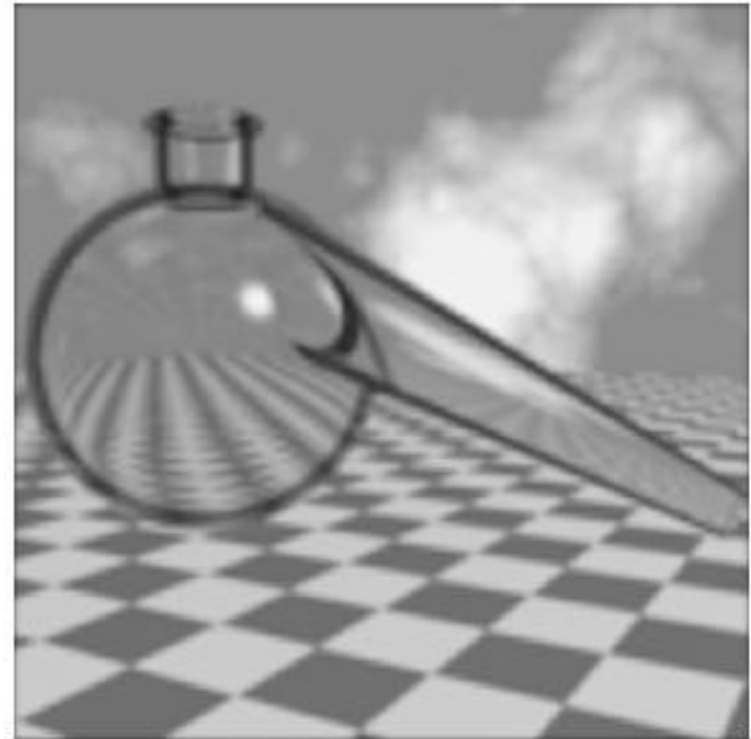
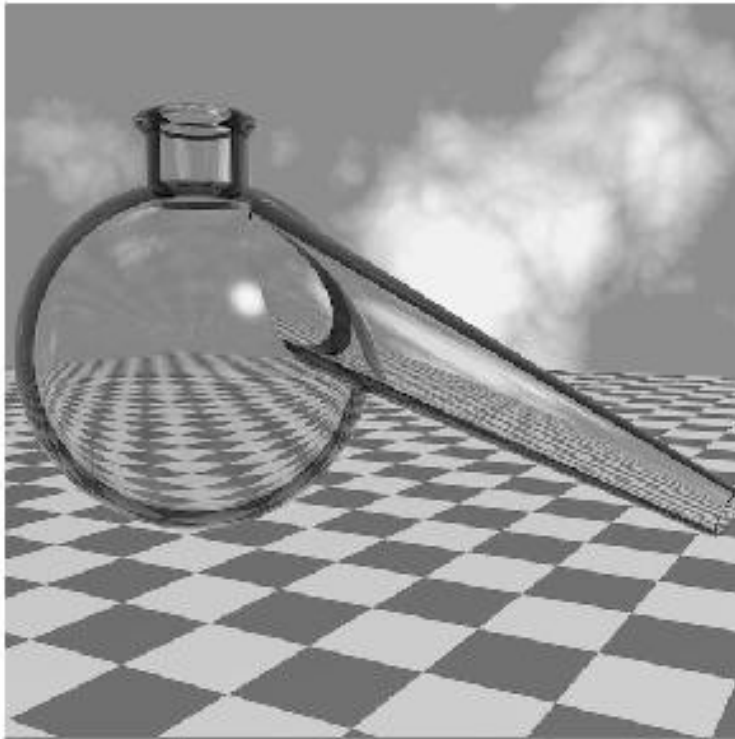


No antialiasing



Prefiltering

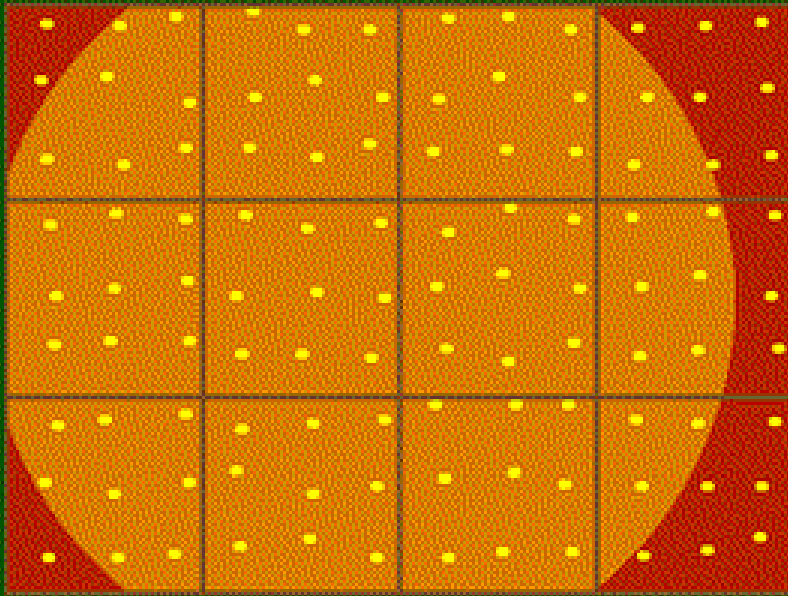
Zameglitev (blurring) ni dobra



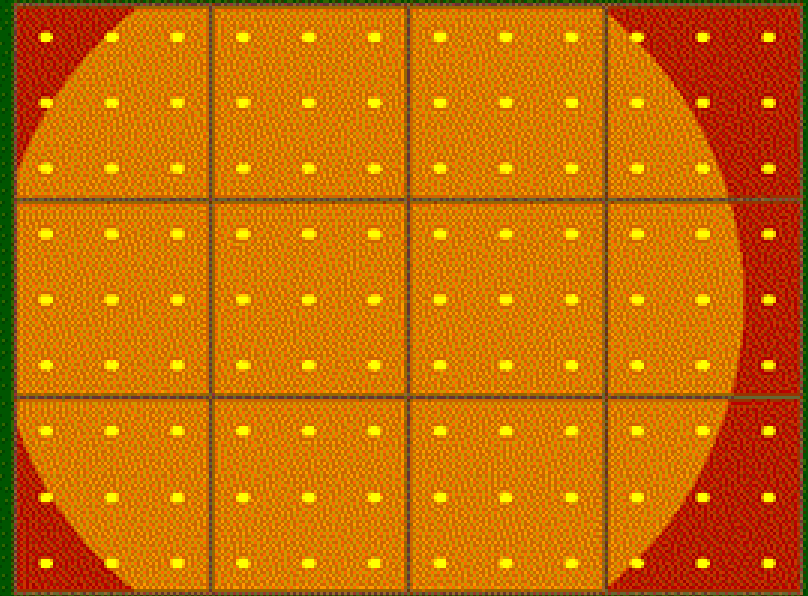
Removed the *jaggies*, but also all the detail ! → Reduction in resolution

nadvzorčenje (supersampling) ?

- solution : take multiple samples for each pixel and average them together → **supersampling**.
- Can weight them towards the centre → weighted average sampling
- Stochastic sampling



Jittered



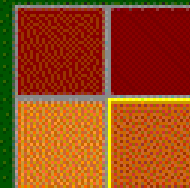
Regular

Taking 9 samples per pixel

$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$
$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$
$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$
$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

Samples

This filter
computes
a weighted
average.



Pixels

No antialiasing



3x3 supersampling
3x3 unweighted filter



3x3 supersampling
5x5 weighted filter



3x3 jittered supersampling
5x5 weighted filter

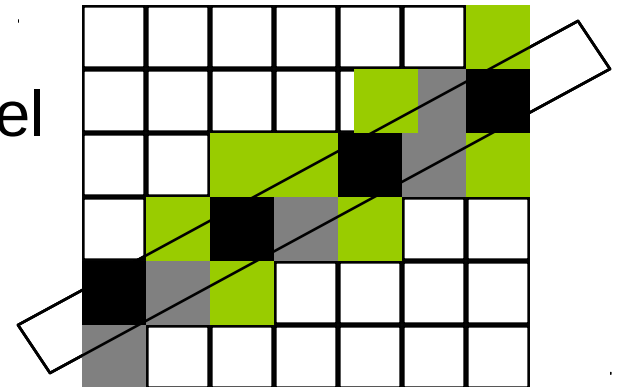


Vzorčenje prostora je drago

- Computationally expensive.
- Difficult to do analytically in a 3D environment
 - Single sample is a ray in 3D - easy computation of intersections.
- Area sample becomes a cone !
 - Reflections difficult.
- Is it an optimum solution ?

Antialiasing with Area Sampling

- A scan converted primitive occupies finite area on the screen
- Intensity of the boundary pixels is adjusted depending on the percent of the pixel area covered by the primitive. This is called weighted area sampling
- Intensities can be weighted depending on the distance of the area from the center of the pixel



Antialiasing with Area Sampling

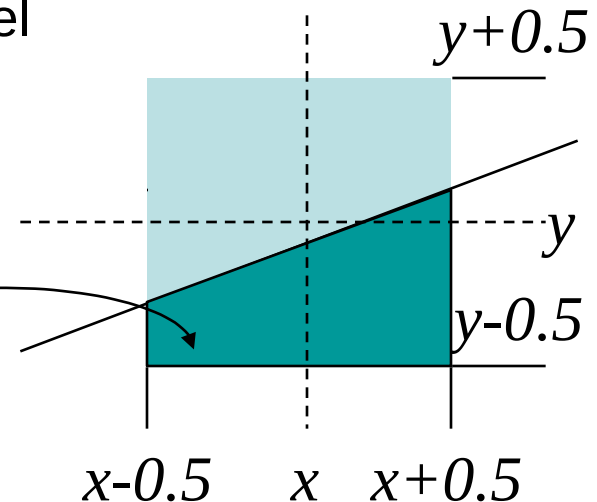
- Methods to estimate percent of pixel covered by the pixel
 - subdivide pixel into sub-pixels and determine how many sub-pixels are inside the boundary
 - Incremental line algorithm can be extended, with area calculated as

one edge
the

$$Area = m \cdot x - y + c + 0.5$$

filled area

Efficient only when
of the area passes through
pixel



Kako še naredimo antialiasing?

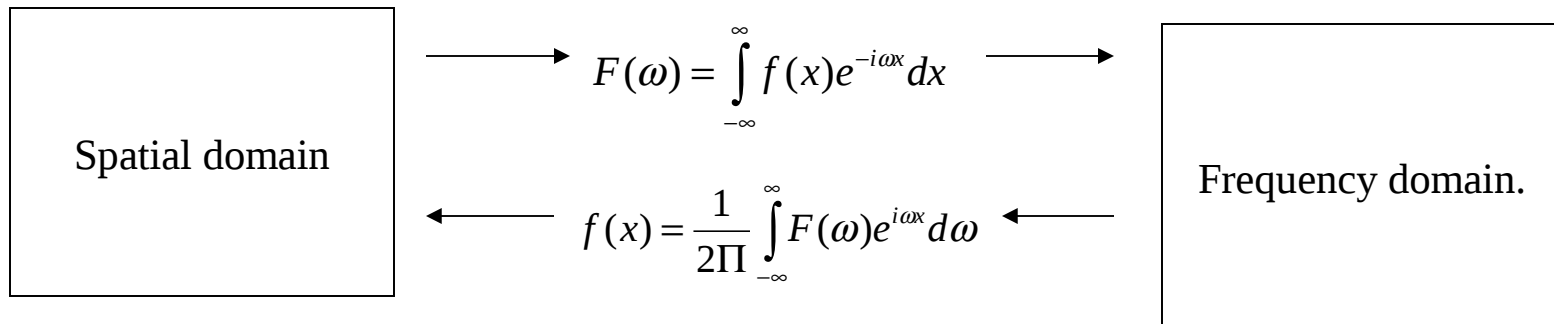
- We need some mathematical tools to
 - analyse the situation.
 - find an optimum solution.
- Tools we will use :
 - Fourier transform.
 - Convolution theory.
 - Sampling theory.

Nekaj matematike – spektralna analiza

- Spectral representation treats the function as a weighted sum of sines and cosines
- Every function has two representations
 - Spatial (time) domain - normal representation
 - Frequency domain - spectral representation
- The \mathcal{F} converts between the spatial and frequency domains.

Nekaj matematike – spektralna analiza

- The $\diamond \square \blacklozenge \square \ast \ast \square \blacktriangledown \square \ast \blacktriangle \blacktriangle \ast \square \square \circ$ converts between the spatial and frequency domain.

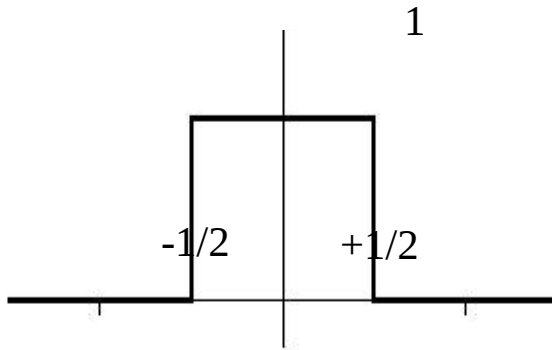


$$e^{it} = \cos t + i \sin t$$

- Note the Euler formula :
- Real and imaginary components.
- Forward and reverse transforms very similar.
 - Reversal in sign of imaginary component, scale constant.

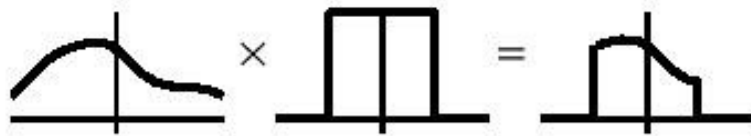
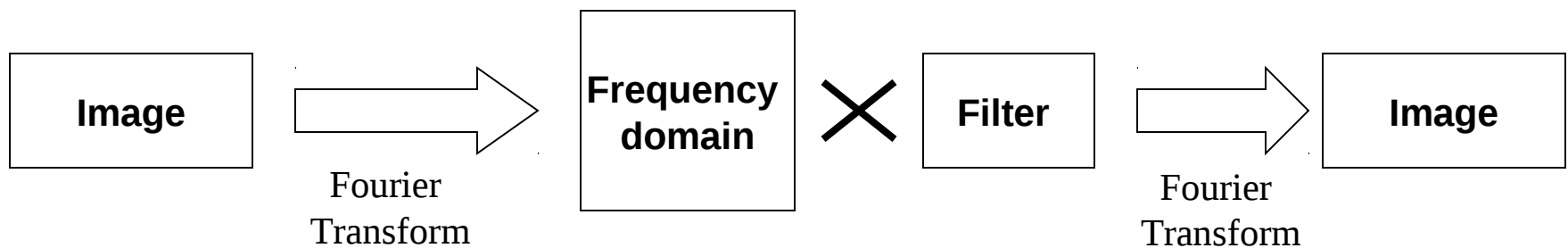
Some important properties of Fourier transforms.

- Finite function \Leftrightarrow Infinite function.
 - eg. single sine wave \Leftrightarrow single point or 'delta' function.
- Square or *boxcar* function - corresponds to 1 pixel.



Function called :
Square(x)

Filtering in the frequency domain



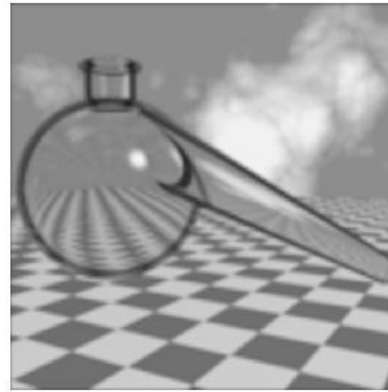
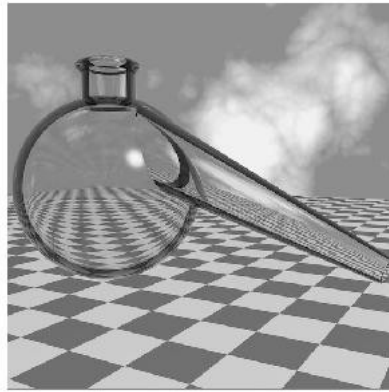
Lowpass filter



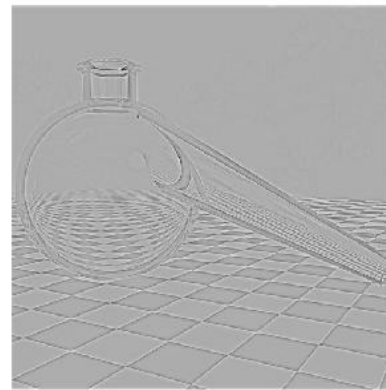
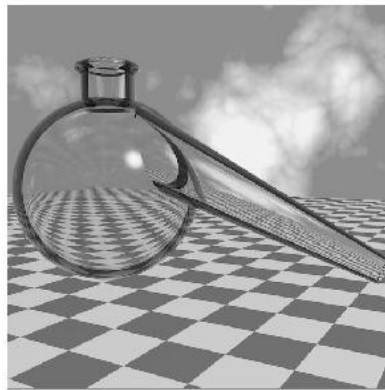
Highpass filter

Filtriranje

- Low pass

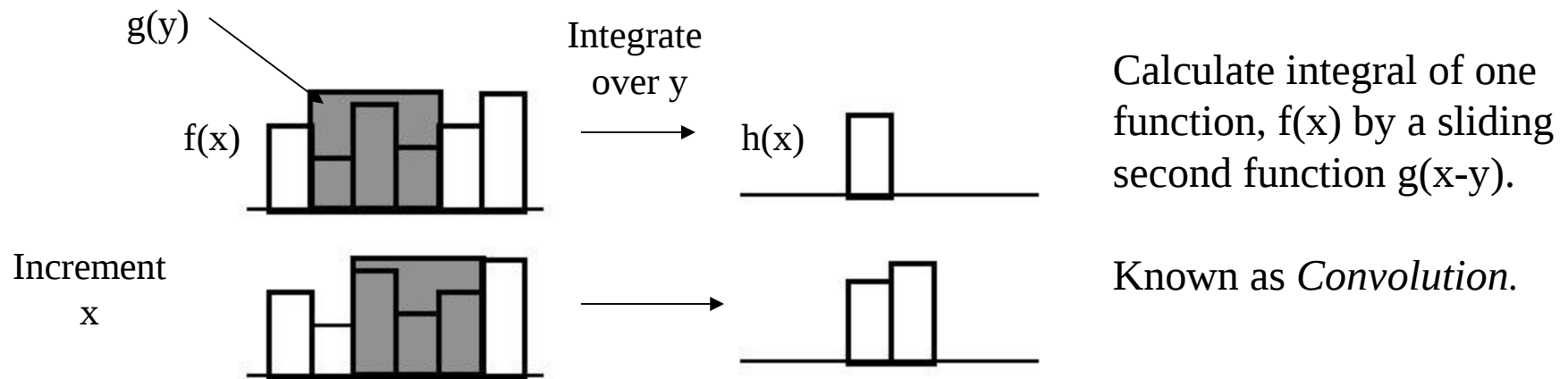


- High pass



Filtriranje v časovnem prostoru

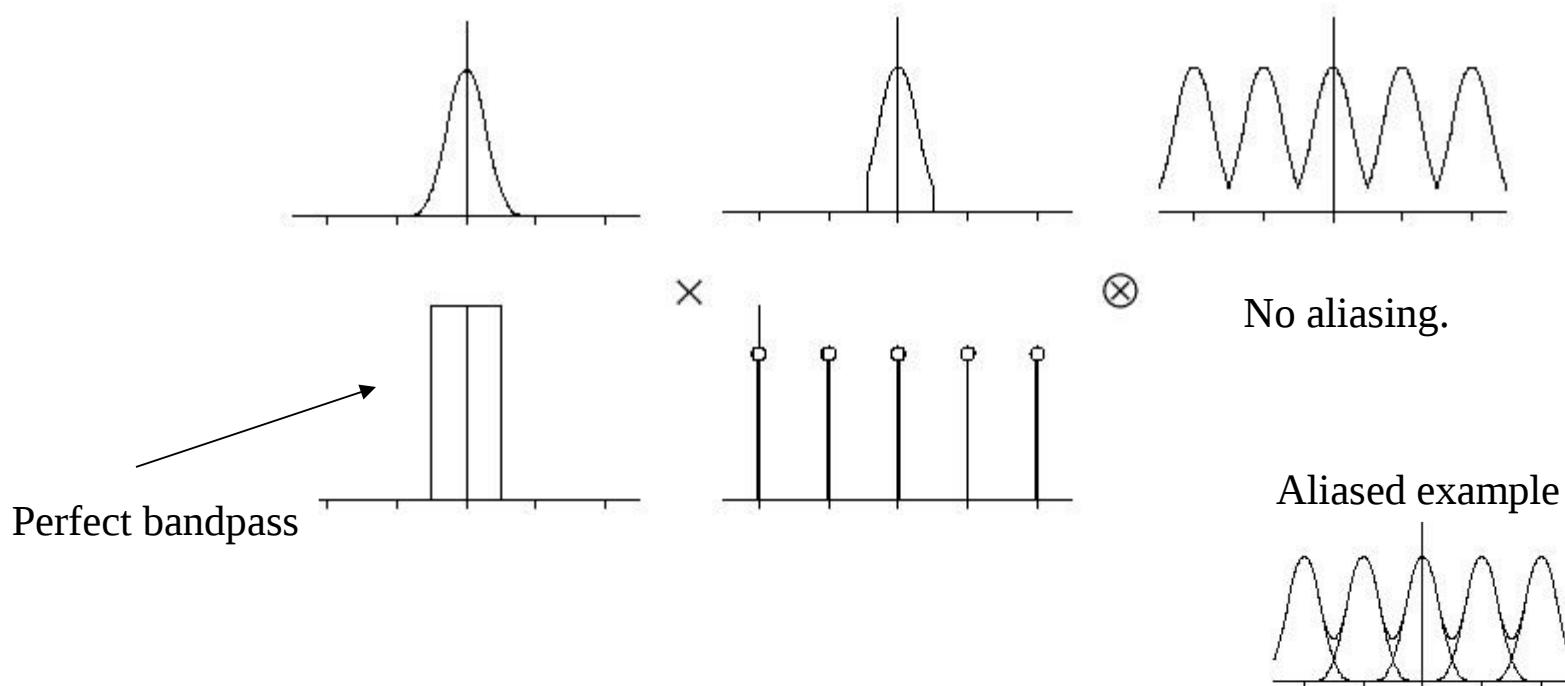
- Blurring or averaging pixels together.



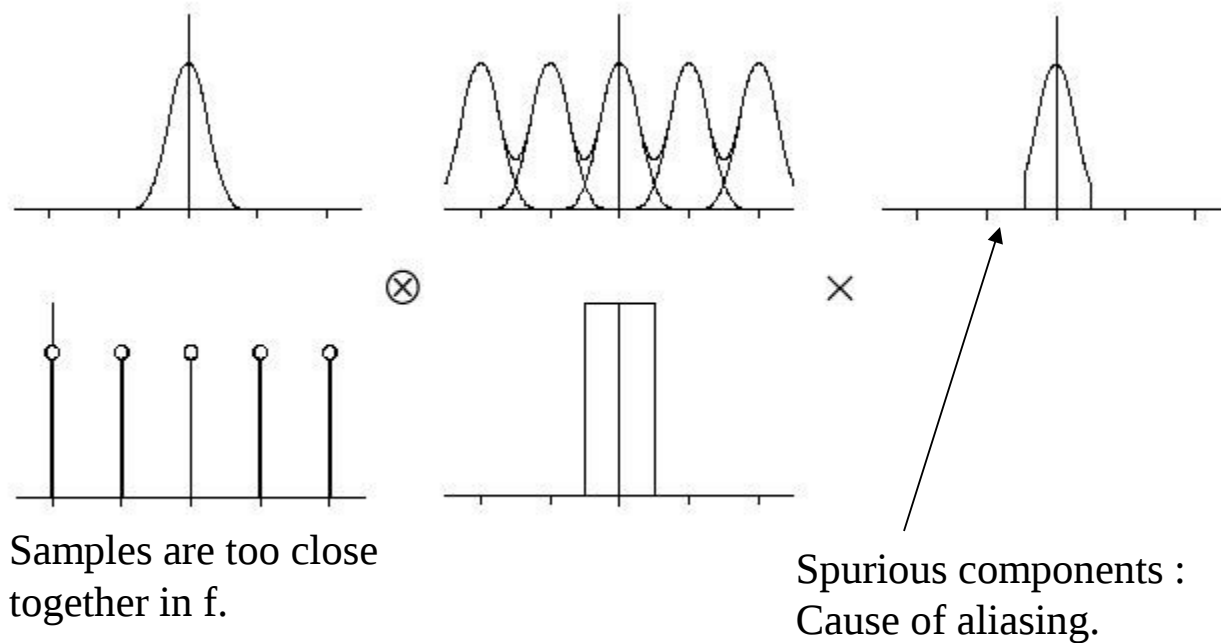
$$h(x) = f \otimes g = \int f(x)g(x-y)dy$$

Kako odstraniti aliasing ?

- Perfect solution - prefilter with perfect bandpass filter.

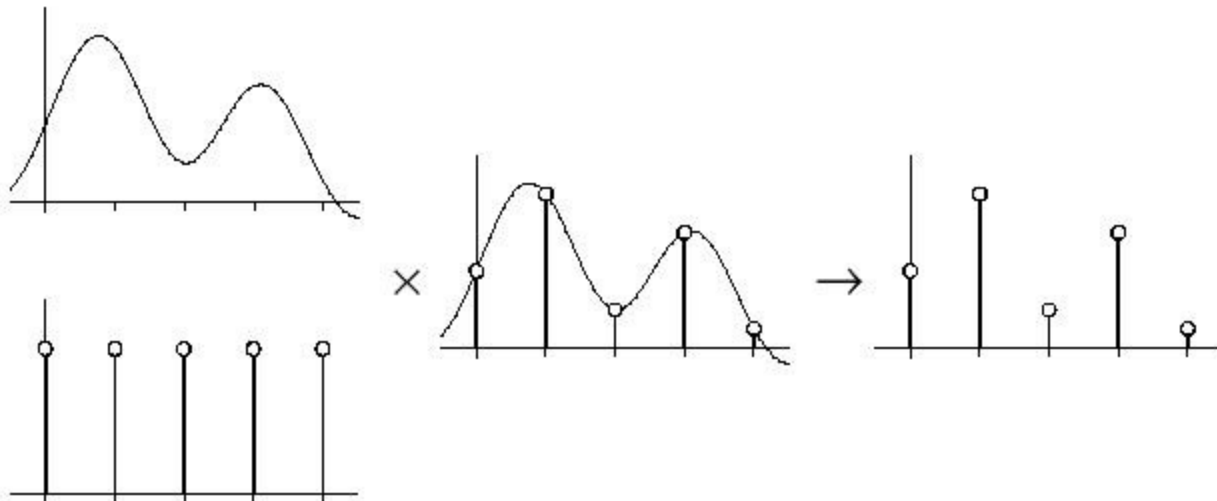


Do aliasing pripelje podvzorčenje



Kako predstavimo vzorčenje ?

- Multiplication of the sample with a regular train of delta functions.

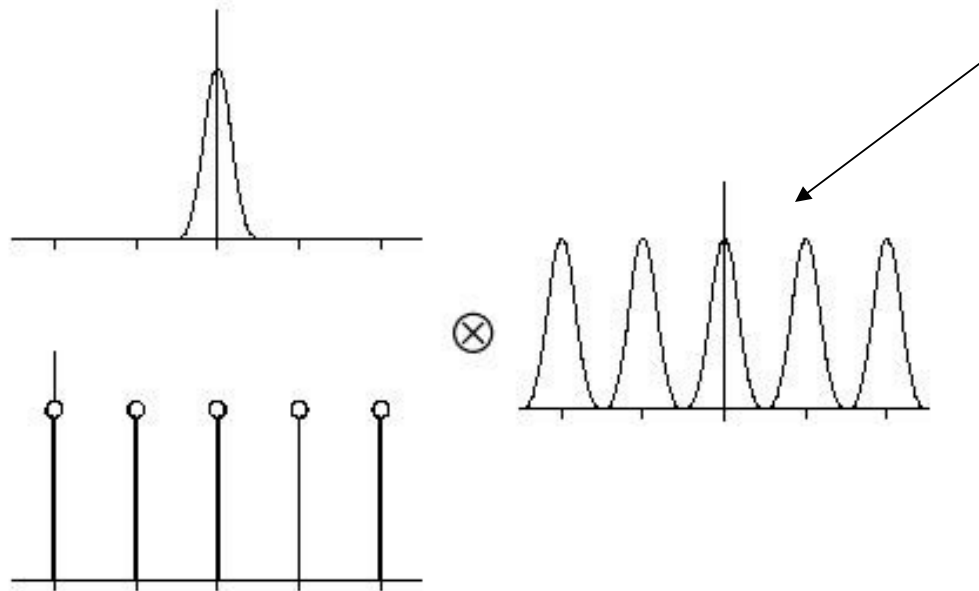


Vzorčenje – frekvenčni prostor

Need Fourier transform of regular train of delta functions.

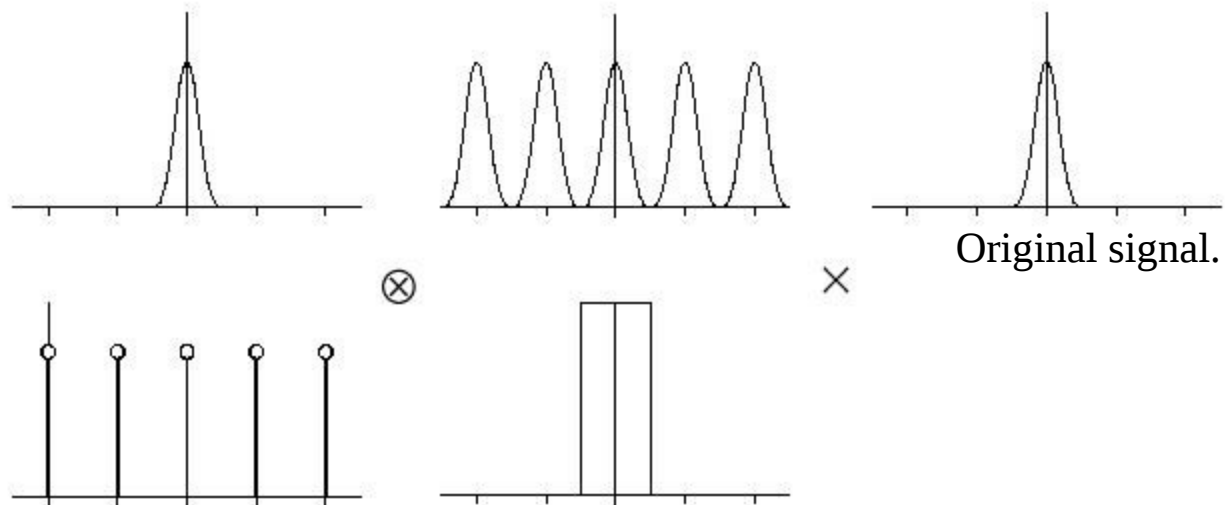
- A regular train of delta functions - spacing is inversely proportional

So convolve with this function.



Multiple solutions at regularly increasing values of f

Rekonstrukcija v frekvenčnem prostoru



Bandpass filter due to regular array of pixels.

Kako odstranimo aliasing ?

- Perfect solution - prefilter with perfect bandpass filter.
 - Difficult/Impossible to do in frequency domain
- Convolve with sinc function in space domain
 - Optimal filter - better than area sampling.
 - Sinc function is infinite !!
 - Computationally expensive.