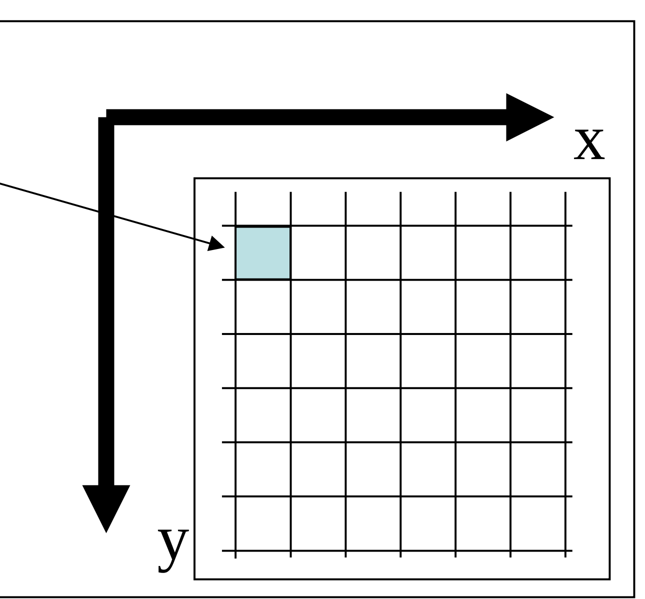
Algoritmi rasterske grafike

Risanje primitivov

- Vector displays “paint” lines across the smooth phosphor coating of the screen and generate smooth curves or straight lines.
- Raster-scan based displays work on a grid principle and are inherently unable to represent smooth curves.
- Mass-market computer displays are based on the raster-scan method.

Piksel NI majhen kvadrat

- Little Square Model pretends to represent a pixel (picture element) as a geometric square.
  - e.g. pixel (i,j) is assumed to be:

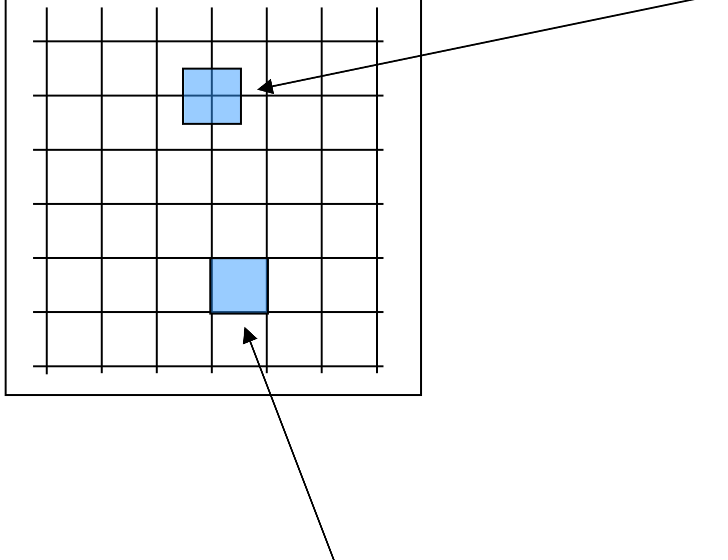


where it is bounded by the square:

$$\{(x, y) \mid i - 0.5 \leq x \leq i + 0.5,\ j - 0.5 \leq y \leq j + 0.5\}$$

Piksel NI majhen kvadrat

- Does the pixel center lie on the integers?

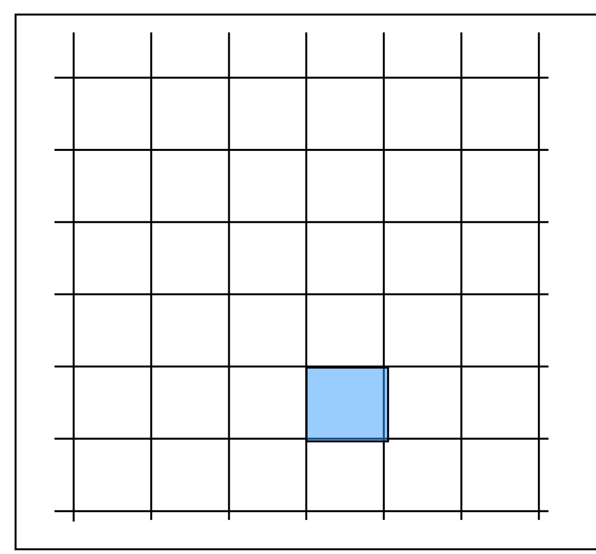


or does it lie on the half-integer?

Piksel NI majhen kvadrat

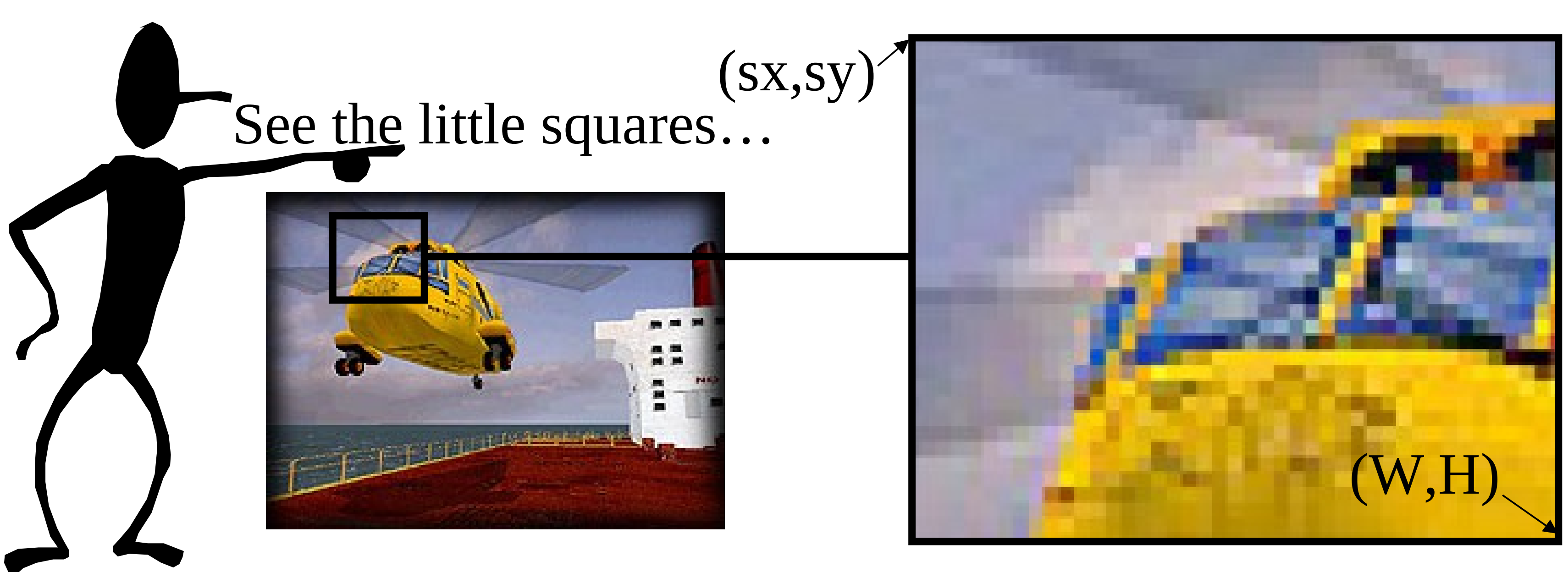
- “Half-Integerists” would place (i,j) correspond to the area of a plane.

$$\{(x,y) \mid i-0.5 \le x \le i+0.5,\ j-0.5 \le y \le j+0.5\}$$



Piksel NI majhen kvadrat

- And the resolution-independent coordinate system for an image is:



$$\{(x, y) \mid sx \leq x \leq Width, sy \leq y \leq Height\}$$

Piksel NI majhen kvadrat

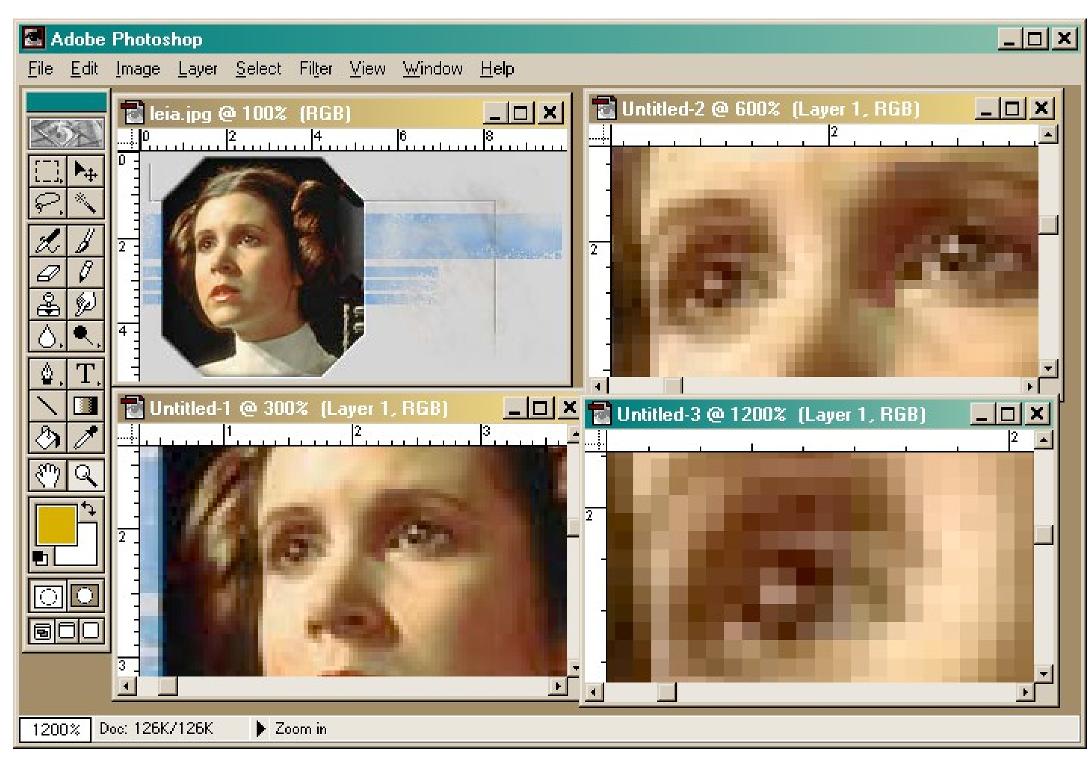
- A pixel is a *point* sample.
- It only exists at a point.
- A colour pixel will actually contain 3 samples: red, green and blue
- A pixel is not a little square.
- An image is a rectilinear array of point samples (discrete not continuous)

Piksel NI majhen kvadrat

- Why is the “little square model” popular:
  - Rendering (conversion of abstract geometry into viewable pixels)
  - The mathematics is easier if we assume a continuum.

Piksel NI majhen kvadrat

- Why shouldn't the "little square model" be identified with a single pixel?
  - –Magnification
    - a zoom looks like little squares, but the actual pixel hasn't been enlarged.



Piksel NI majhen kvadrat

- Why shouldn’t the “little square model” be identified with a single pixel?
  - Scanner Digitising a Picture
    - a light source illuminates the paper
    - light reflected is collected and measured by a colour sensitive device
    - the collected light is passed through a filtering shape (never a square)
    - pixels are determined by averaging overlapping shapes.

Uvod v 2D upodabljanje

- 2D primitives
  - Line segments
  - Ellipses and circles
  - Polygons
  - Curves
- Rasterization (Scan-Conversion)
  - Turn 2D primitives into sets of pixels
  - A Pixel Is Not A Little Square (Digital Signal Processing)
  - Antialiasing
- Clipping
  - Compute the intersection of a primitive and a shape
  - Primitive: line segment, polygon
  - Shape: rectangle, convex polygon

Nekaj matematike

- Coordinate system: y axis upward or downward?
- Pixels are at the centres of integer coordinates
- **Line segments**
  - Equation of a (2D) line: ax + by + c = 0
  - Direction: (-b a)
  - Normal vector: (a b)
  - Parametric equation of a segment [P1-P2]
    x(t) = x1 + t*(x2-x1) = (1-t)*x1 + t*x2
    y(t) = y1 + t*(y2-y1) = (1-t)*y1 + t*y2
    t in [0..1]

Nekaj matematike

- **Polygons**
  - Closed sequence of line segments (P1 P2 .. Pn)
  - Different types
  - Convex
  - Concave = not convex
  - Self-intersecting (8-shape)
  - With holes

Rasterizacija

- Converting mathematical definitions to pixels
  - We can only *approximate* the mathematical definition
    - A Pixel is Not a Little Square
    - Avoid holes
    - Draw each pixel exactly once
  - Naive (expensive) approach
    - Evaluate formulas on the pixel grid
  - Clever approach
    - Use integer calculations
    - Avoid divides and multiplies
    - Use incremental computations
    - Use spatial coherence

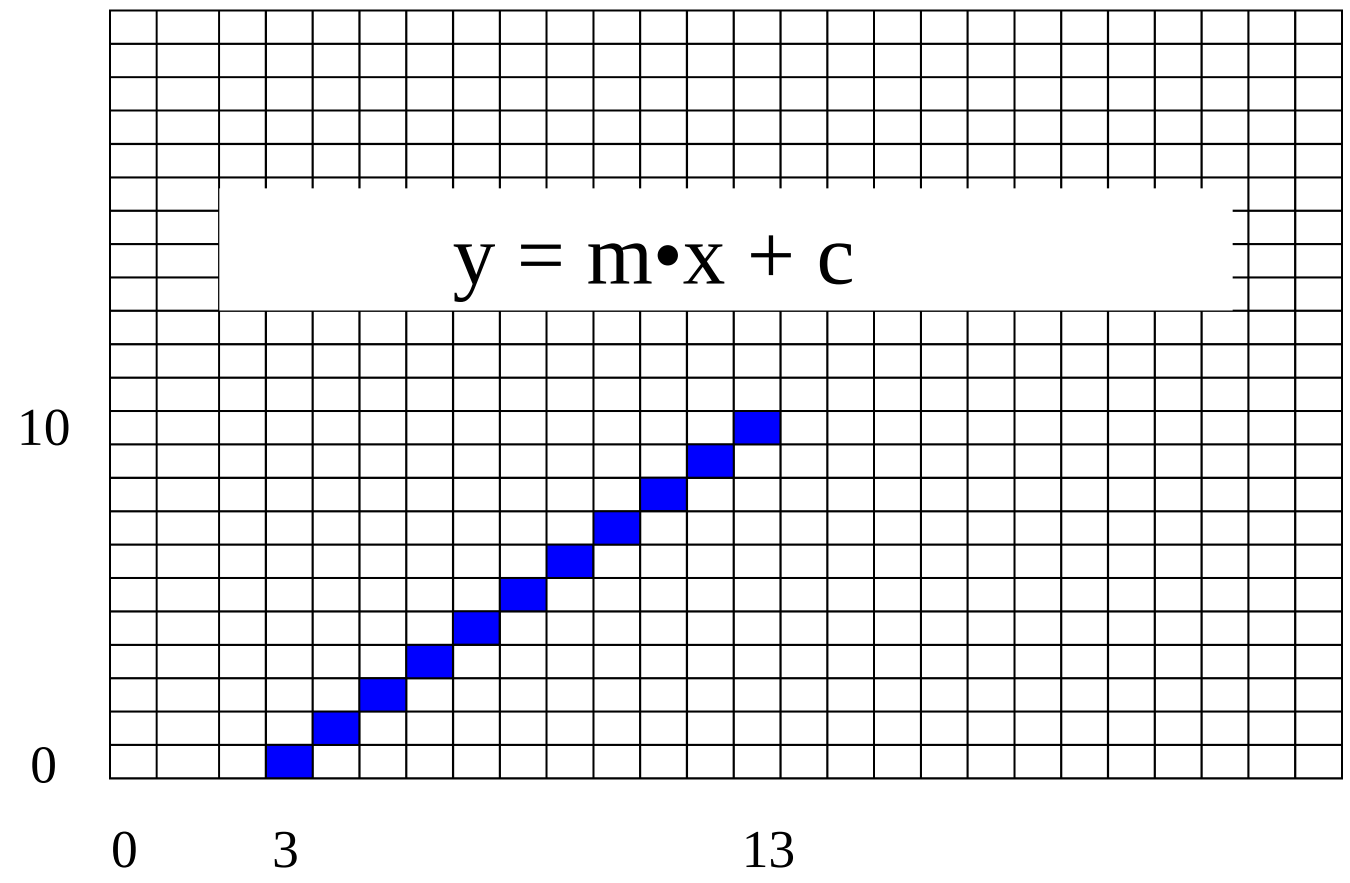
Ravne črte in krogi

- In general, straight lines are neither vertical nor horizontal.
- Pixels are roughly square and support only horizontal and vertical lines well.
- The general case of line drawing must be based upon some sort of compromise.

Ravne črte

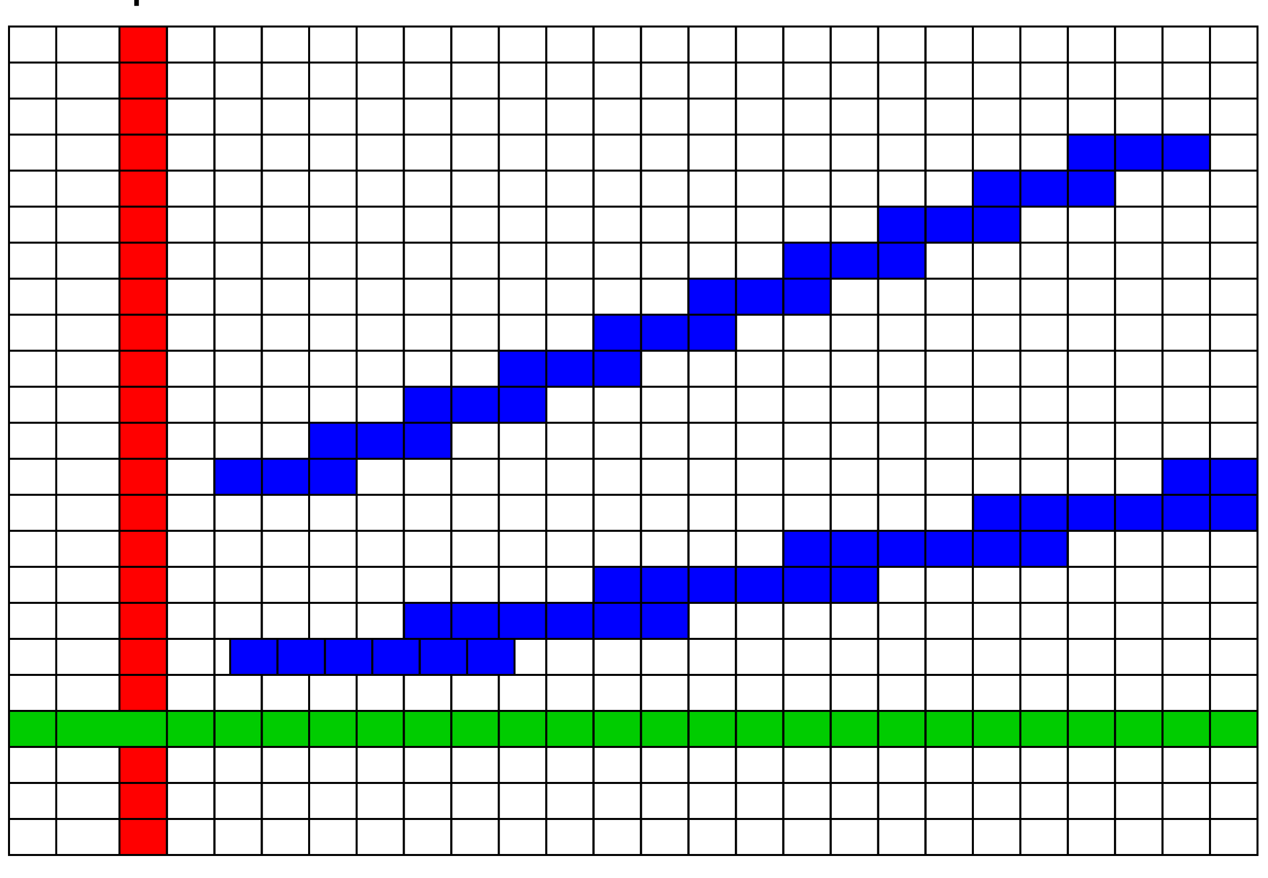
extend between two points:

(x1,y1) , (x2,y2) eg line below = (3,0) and (13, 10)



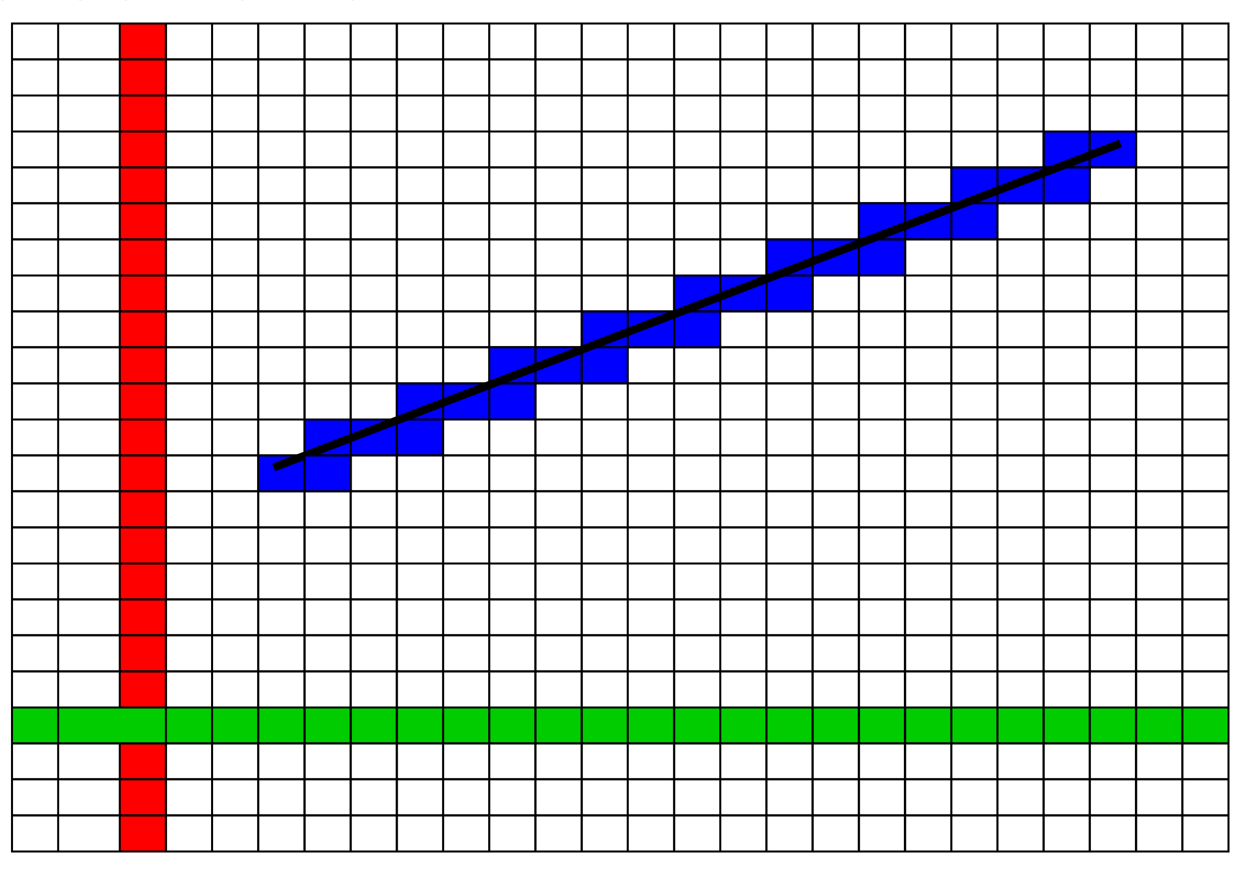
Problemi z ravnimi črtami

the stair-step effect



Problemi z ravnimi črtami

which pixels to colour?



Približno risanje črt

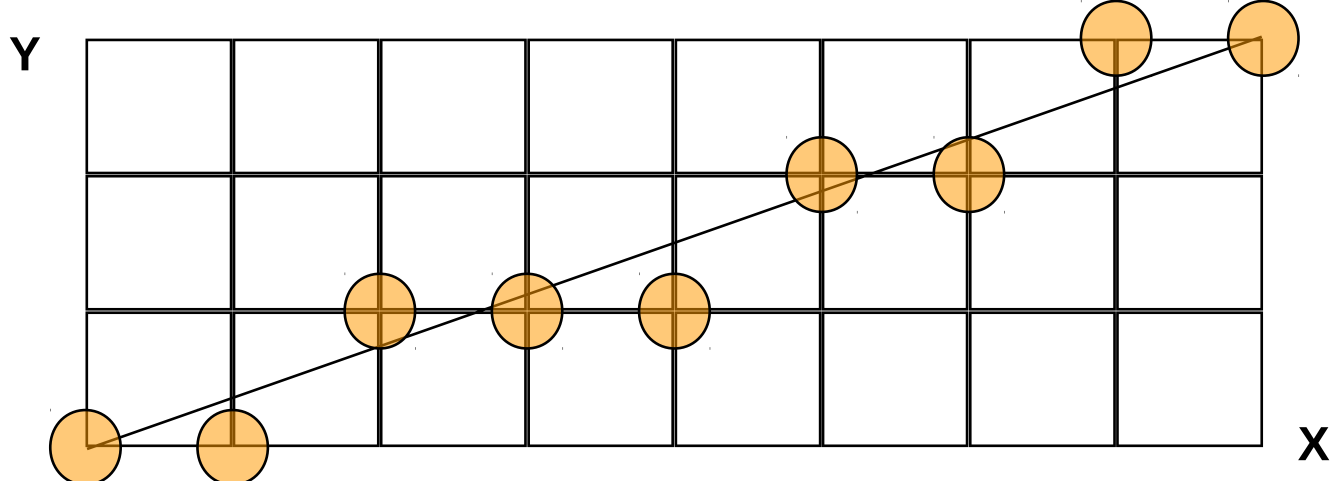
- Assume square pixels.
- Assume that the line starts at (x1, y1) and finishes at (x2, y2).
- Say that dx=x2-x1, dy=y2-y1
- If we start with the simplest non-trivial case where dx=dy, we can immediately see that a 45 degree diagonal line has one x step per y step.

Približno risanje črt

- The basic requirement for an approximation is to generate the minimum error at each step.
- The largest acceptable error must be half a pixel.
- To simplify the problem, we consider only one eighth of the possible angles, ie we choose to consider only one octant. We can generalise later using a mirroring technique.

Približno risanje črt

- For each x step the difference between the actual y position and the required y position is calculated; if it is more than half a pixel, move by one y step.

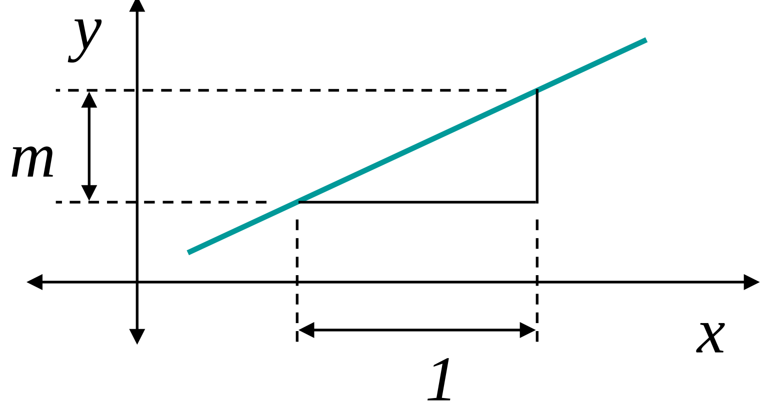


Enačba premice

- Equation of a line is ***y - m.x + c = 0***
- For a line segment joining points ***P(x1,y2)*** and ***P(x2,y2)***

$$slope \qquad m = \frac{y2 - y1}{x2 - x1} = \frac{\Delta y}{\Delta x}$$

- Slope ***m*** means that for every unit increment in *X* the increment in *Y* is *m* units



Naivni algoritem rasterizacije črt

- Line segment defined by P0 P1
- Equation of line is
  Y = mX + B
  m = (y1-y0) / (x1-x0)
  B = y0 - m*x0
- Algorithm:
  - start with the smallest of (x0,x1)
  - compute corresponding value of y
  - SetPixel(x, round(y))
  - increment x and loop until reaching max(x0,x1)
- Cost: 1 float mult + 1 float add + 1 round per loop

Inkrementalni algoritem rasterizacije črt

- Compute y using it's previous value rather than from scratch
- y[i+1] = y[i] + m*(x[i+1]-x[i]), but since we increment x by 1:
  y[i+1] = y[i] + m
- Cost: 1 float add + 1 round per loop

Minimiziranje računanj s plavajočo vejico

With straight lines one way of minimising the amount of floating point calculation is:

if $(x_1 == x_2)$ => vertical line

elseif $(y_1 == y_2)$ => horizontal line

else

$$y_{k+1} = y_k + m$$

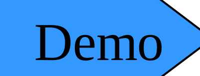
Allows the minimum of floating point calculation to be carried out.

Digitalni diferencialni analizator (DDA)

- Digital Differential Analyzer algorithm more popularly known as *DDA*
- This is an *Incremental algorithm* i.e. at each step it makes incremental calculations based on the calculations done during the preceding step
- The algorithm uses floating point operations, which are very cleverly avoided in an algorithm first proposed by J. Bresenham of IBM,. The algorithm is well known as *Bresenham's Line Drawing Algorithm*.
- A slight variation – Midpoint Line Drawing Algorithm –

Bresenham's Line Algorithm (BLA)

- Scan-converts lines using only incremental integer calculations.
- BLA again assumes the line is sampled at unit x intervals.
- BLA uses the sign (+ve/-ve) of an integer (p) whose value is proportional to:
  - *the difference between the two candidate* y *values separation from the calculated line path*
- The differential line algorithm uses floating-point values (the error value is a fraction) and floating point calculations are slow compared with integer calculations.
- BLA can be used for curves as well as straight lines.



Digitalni diferencialni analizator (DDA)

The DDA is a scan-conversion algorithm, which recognises that the x interval is always 1 (corresponding to moving to the next pixel column in the frame buffer) thus

$$y_{k+1} = y_k + m$$

and the nearest scan-line to y_{k+1} will be given by:

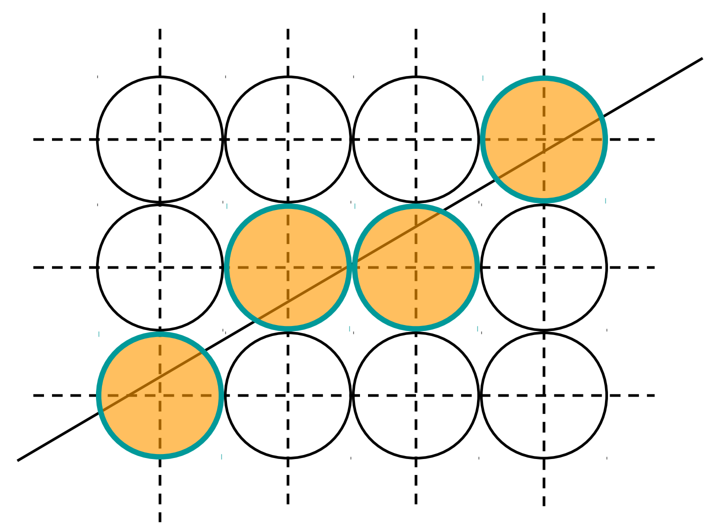
$$y = (int)\ (y_{k+1} + 0.5)$$

Digitalni diferencialni analizator (DDA)

- We consider the line in the first octant. Other cases can be easily derived.
- Uses differential equation of the line

$$y_i = m \cdot x_i + c$$

$$where, \qquad m = \frac{y2 - y1}{x2 - x1}$$

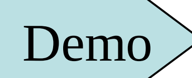


- Incrementing X-coordinate by 1

$$x_i = x_{i_prev} + 1$$
$$y_i = y_{i_prev} + m$$

$$[x_i, round(y_i)]$$

- Illuminate the pixel



Algoritem DDA

```
differential := (y2-y1) / (x2-x1);
x := x1; y := y1;              { initialise start position }
error := 0;                          { no error yet }

setpixel(x,y);

while x<x2 do                        { while not past line end }
begin
   error := error + differential;    {accumulate error }
     if error >= 1/2 then            { largest error = 1/2 pixel }
     begin
          y := y+1;                  { step up one line }
          error := error-1; { error changed by 1 line }
     end;

     x := x+1;                       { next pixel along }
     setpixel(x,y);                  { plot the pixel }
end;
```

Značilnosti algoritma DDA

Ugodnosti:

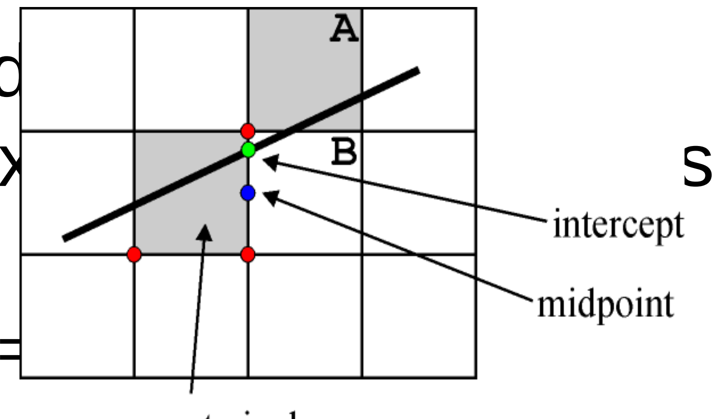
- since it uses information about raster characteristics it is faster than using **$y = mx + c$**

Možni problemi:

- accumulation of round-off error over many successive additions can result in pixel positions which drift away from the mathematically correct line
- round-off error increases with line length
- rounding-off still computationally expensive.

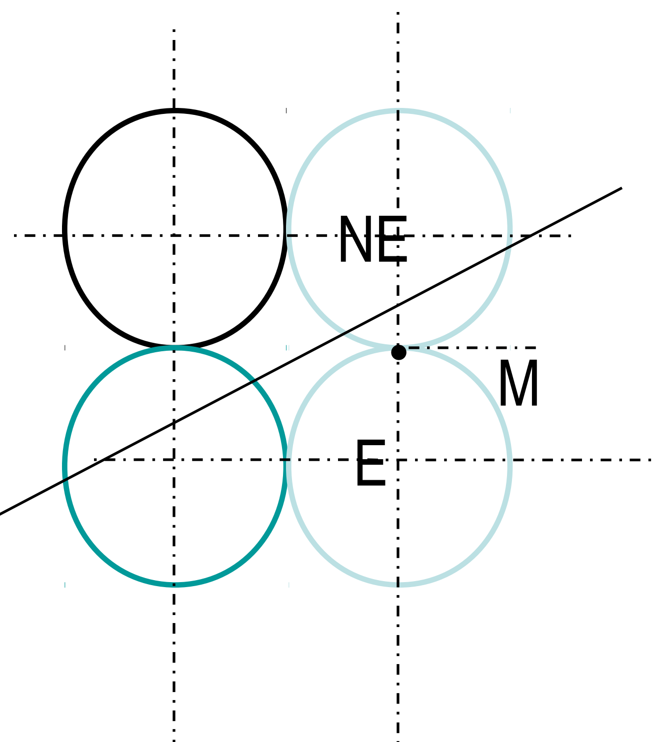
Bresenham's Algorithm: Midpoint Algorithm

- More efficient than DDA, attributed to Jack Bresenham 1965.
  - lines are *single pixel wide*
  - selects closest pixel to the line (approximating *mid-point* pixel coordinates)
  - *incremental* and *integer calculations only*
- Assume slope satisfies $0 < m < 1$
- At each iteration we d
  intersects the next pix s
  midpoint y value.
  - if above then **A** ⇒ y_{i+1} =
  - otherwise **B** ⇒ $y_{i+1} = y_i$



Midpoint Line Algorithm

- The same incremental method for scan converting lines can be derived using an integer formulation. In this the mid-point between the East (E) and NorthEast (NE) pixels is checked to see on which side of the line it lies. For this instead of y = mx + c, the line equation of the form
- F(x) = (ax + by +c = 0) is used.
- **Principle**: If F(mid-point) is above (<=0) the line then E is chosen, otherwise NE is chosen.



Demo

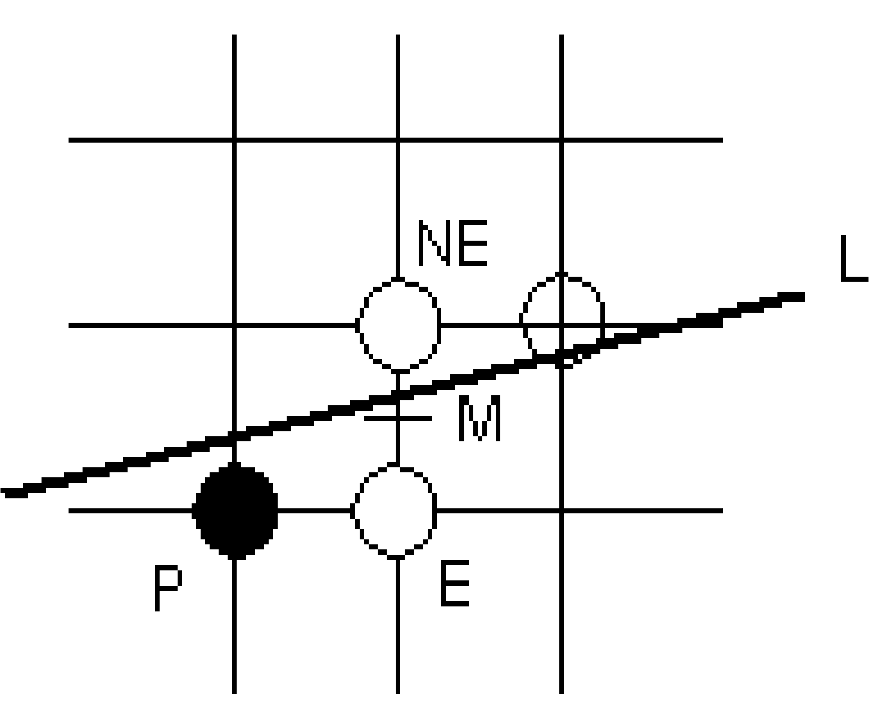
Midpoint Line Algorithm

$d = a(x_p+1) + b(y_p+1/2) + c$
is the decision variable.

if d > 0 then choose NE else
if d <=0 then choose E.

For an incremental algorithm,
we must compute d
incrementally. For that, let us
see what happens to M and d
for the next grid line.

We have two cases –
old choice was E or NE



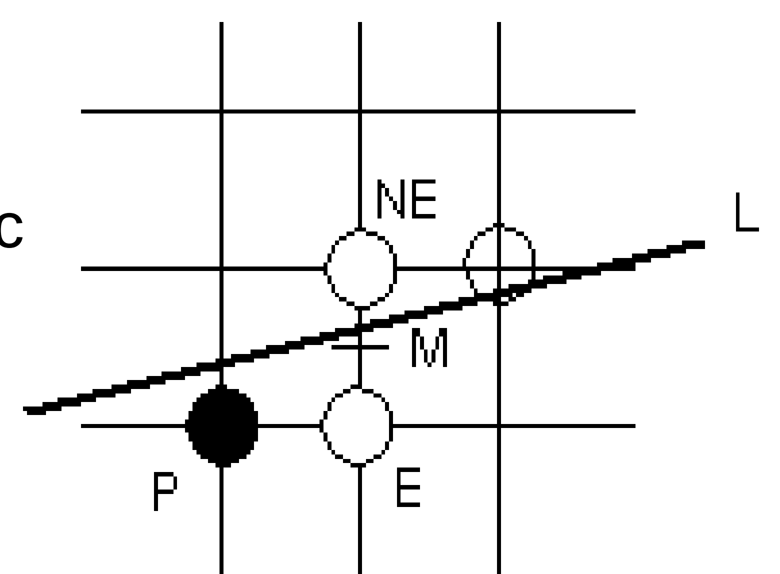
Midpoint Line Algorithm (nadaljevanje)

If the old choice is E, then

$d_{new}= a(x_p+2) + b(y_p+1/2) + c$

But $d_{old}= a(x_p+1) + b(y_p+1/2) + c$

Hence $d_{new}= d_{old} + a$.



If the old choice is NE, then

$d_{new}= a(x_p+2) + b(y_p+3/2) + c$

Now $d_{new}= d_{old} + a + b$.

Midpoint Line Algorithm (nadaljevanje)

Consider the line segment from (x_1,y_1) to (x_2,y_2).

$(y_2-y_1)x - (x_2-x_1)y + c = 0$ is the equation.

$a = dy = (y_2-y_1)$, $b = -dx = -(x_2-x_1)$

And $a + b = dy - dx$

What should be d be to start with?

The first midpoint $M_1 = (x_1+1, y_1+1/2)$

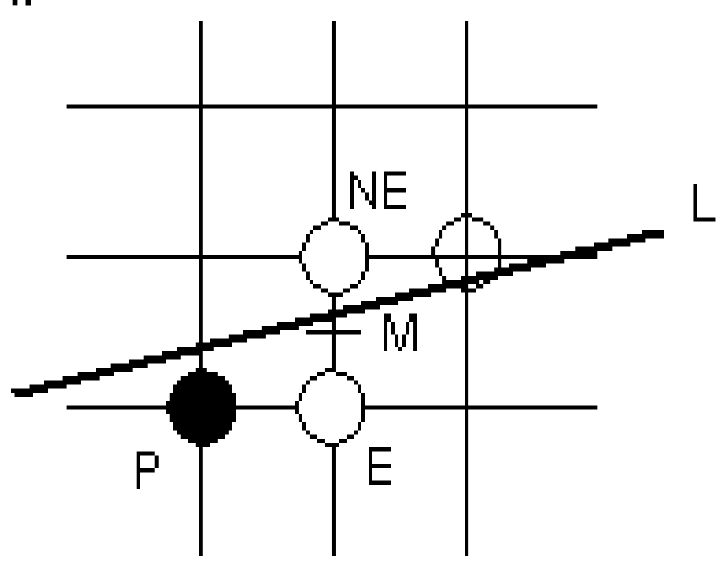
$F(M_1) = d_1 = ax_1 + by_1 + c + a + b/2$

$= F(x_1,y_1) + a + b/2$

x_1,y_1 is on the line, so $F(x_1,y_1) = 0$

Thus $d_1 = a + b/2$

In order to avoid division by 2, we choose to make our decision using $2d_1 = 2a + b$, which does not change sign of d.



Midpoint Line Algorithm

- Input line end points *(x1, y1), (x2, y2)*
- Set *x = x1 and y = y1* and SetLineColour*(x,y)*
- Calculate *dX = x2 – x1* and *dY = y2 – y1*
- Calculate *incrNE = 2*(dY - dX)* and *incrE = 2*dY*
- Calculate d = *2*dY - dX*
- While (x < x1)

```
{ x = x + 1;
  If d>0 then y = y + 1 and d = d + incrNE  else d = d + incrE;
  SetLineColour(x,y)
}
```

Midpoint Line Algorithm

```
dx = x_end-x_start
dy = y_end-y_start
d = 2*dy-dx
x = x_start
y = y_start
while x < x_end
    if d <= 0 then
        d = d+(2*dy)
        x = x+1
    else
        d = d+2*(dy-dx)
        x = x+1
        y = y+1
    endif
    SetPixel(x,y)
endwhile
```

initialisation

choose **B**

choose **A**

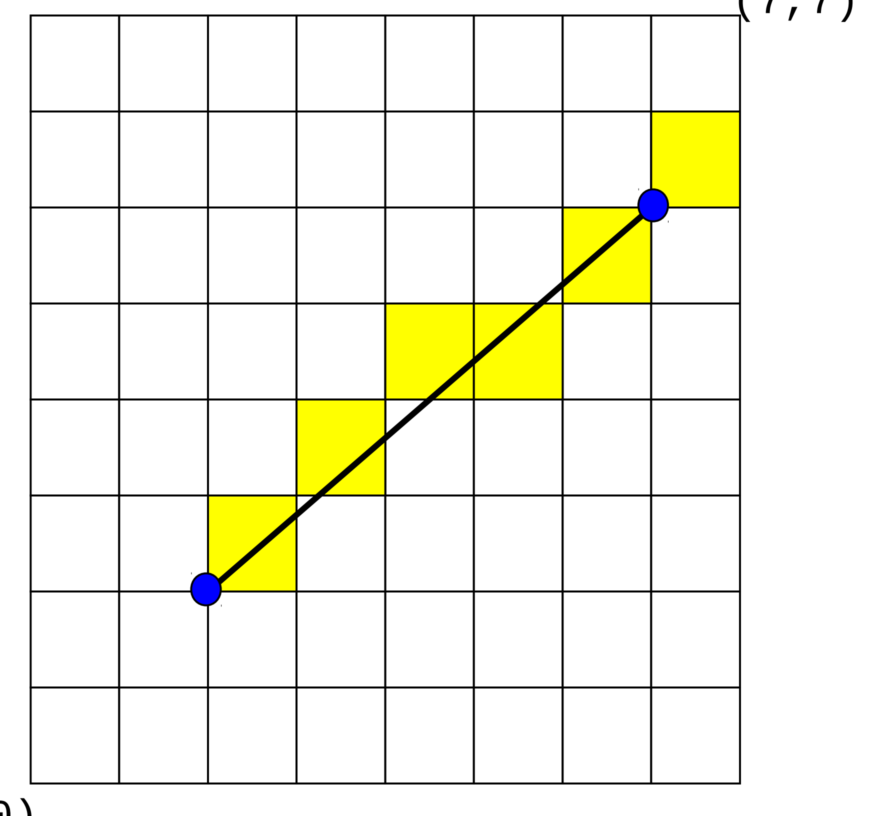
Midpoint Line Algorithm

Line: (2,2) → (7,6)

dx = 5
dy = 4
d = 3

if **d>0** then **d=d-2, y=y+1**
if **d<0** then **d=d+8**

x	y	d
2	2	3
3	3	1
4	4	-1
5	4	7
6	5	5
7	6	3



Advantages of Incremental Midpoint Line Algorithm

- It is an incremental algorithm
- It uses only integer arithmetic
- Provides the best fit approximation to the actual line

Algoritmi za kroge

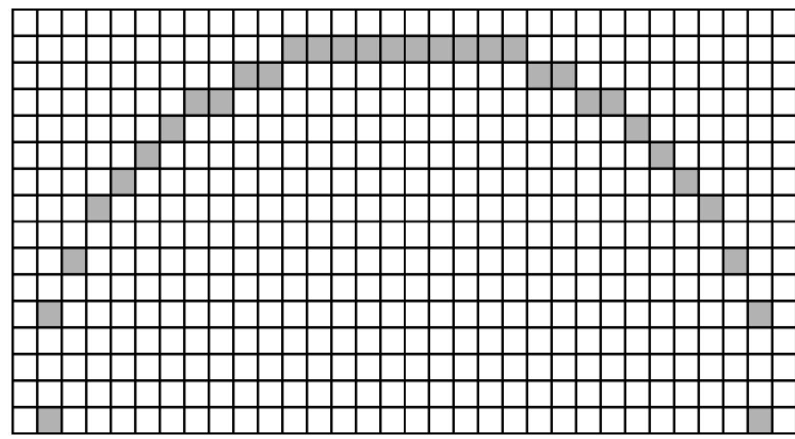
- Circle with radius r and center (x_c, y_c) is defined parametrically as:

$$x = x_c + r\cos\theta$$

$$y = y_c + r\sin\theta$$

∴ could step through θ from 0 to 2π plotting co-ordinates:

– difficult to effectively control step-size to eliminate gaps and minimise pixel overdrawing



Bresenham's Circle Algorithm

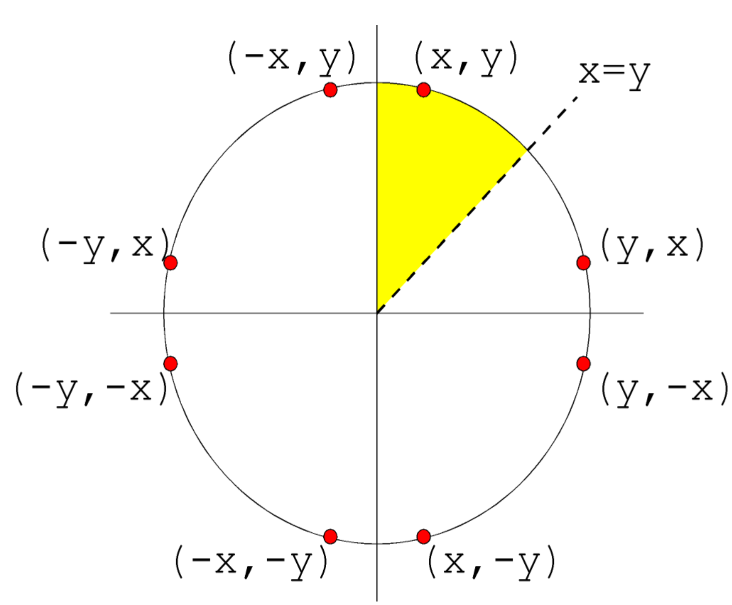
- Another "least error" method
- The same simplifications are used as for line drawing - solution is for one octant.
- The equation of a circle is
  $x^2 + y^2 = R^2$ or $x^2 + y^2 - R^2 = 0$
- If x and y are not precisely on the circle,
  $x^2 + y^2 - R^2$ will be non-zero (an error value).
- Error > 0 means (x,y) is outside the circle,
  Error < 0 means (x,y) is inside the circle.

Midpoint Circle Algorithm

- Implicit form of the circle:
- Employ a similar scheme to the midpoint line algorithm:

$$(x - x_c)^2 + (y - y_c)^2 - r^2 = 0$$

  - need only determine pixels for *one octant*, other octants are related via simple symmetries
  - maintain decision variable d_i which takes on values as follows:



$$d_i = \begin{cases} < 0 \text{ if } (x_i, y_i) \text{ is inside circle} \\ = 0 \text{ if } (x_i, y_i) \text{ is on circle} \\ > 0 \text{ if } (x_i, y_i) \text{ is outside circle} \end{cases}$$

Demo

Midpoint Circle Algorithm

- As with the line, we determine the value of the decision variable by substituting the mid-point of the next pixel into the implicit form of the circle:

$$d_i = (x_i + 1)^2 + \left(y_i - \frac{1}{2}\right)^2 - r^2$$

  – If $d_i < 0$ we choose pixel **A** otherwise we choose pixel **B**
  – <u>Note</u>: we currently assume the circle is centered at the origin

Midpoint Circle Algorithm

- Again, as with the line algorithm, the choice of A or B can be used to determine the new value of d_{i+1}
- If **A** chosen then next midpoint has the following decision variable:

$$\left(x_i + 2, y_i - \frac{1}{2}\right) \Rightarrow d_{i+1} = (x_i + 2)^2 + \left(y_i - \frac{1}{2}\right)^2 - r^2$$
$$= d_i + 2x_i + 3$$

- Otherwise if **B** is chosen then the next decision variable is given by:

$$\left(x_i + 2, y_i - \frac{3}{2}\right) \Rightarrow d_{i+1} = (x_i + 2)^2 + \left(y_i - \frac{3}{2}\right)^2 - r^2$$
$$= d_i + 2x_i - 2y_i + 5$$

Midpoint Circle Algorithm

- If we assume that the radius is an *integral value*, then the first pixel drawn is *(0, r)* and the initial value for the decision variable is given by:

$$\left(1, r-\frac{1}{2}\right) \Rightarrow \; d_0 = 1+\left(r^2 - r + \frac{1}{4}\right) - r^2$$

$$= \frac{5}{4} - r$$

- Although the initial value is *fractional*, we note that all other values are *integers*.

$\Rightarrow$ we can round down: $d_0 = 1 - r$

Midpoint Circle Algorithm

```
d = 1-r
x = 0
y = r
while y < x
    if d < 0 then
        d = d+2*x+3
        x = x+1
    else
        d = d+2*(x-y)+5
        x = x+1
        y = y-1
    endif
    SetPixel(cx+x,cy+y)
endwhile
```

initialisation

stop at diagonal ⇒ end of octant

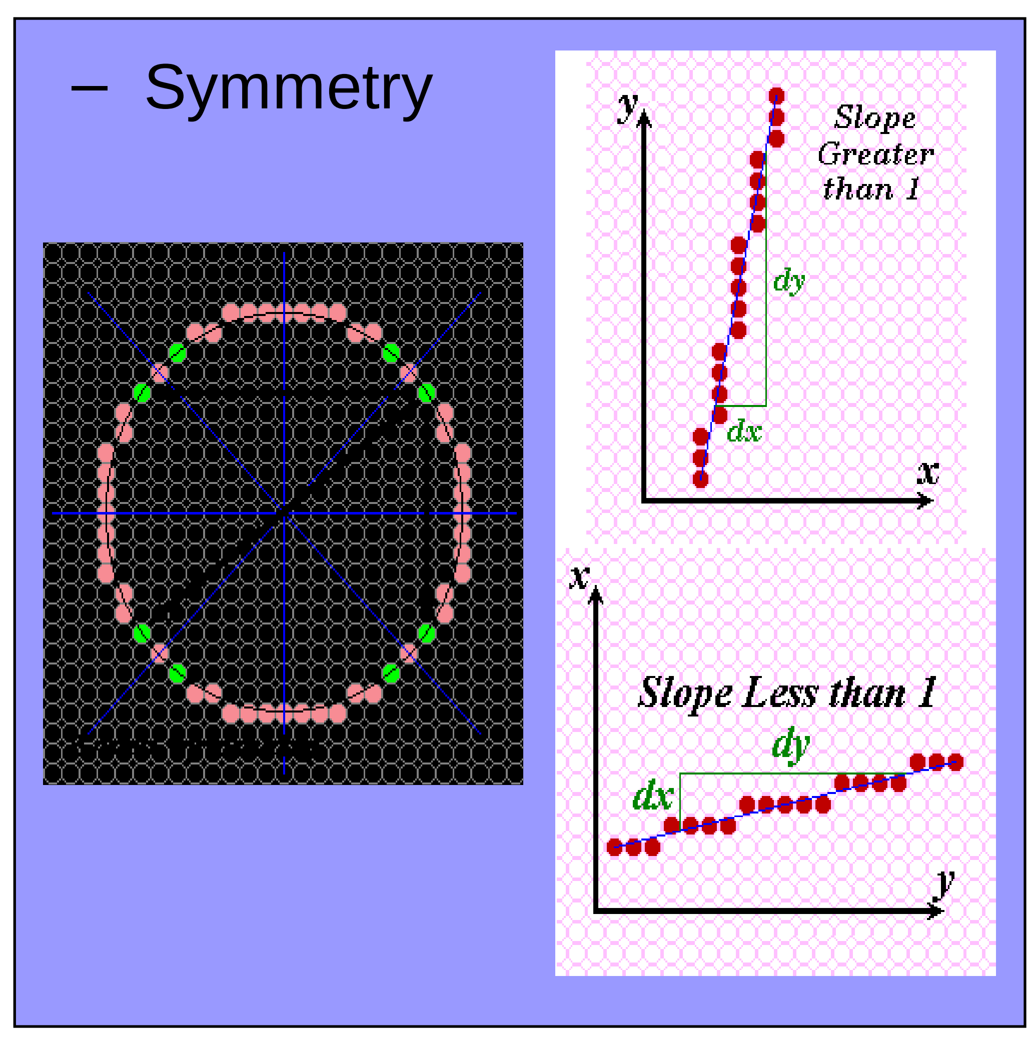
choose **A**

choose **B**

Translate to the circle center

Tehnike optimizacije

– Symmetry



Rasterizacija kroga

- Version 1 – really bad

For x = -R to R

  y = sqrt(R • R – x • x);

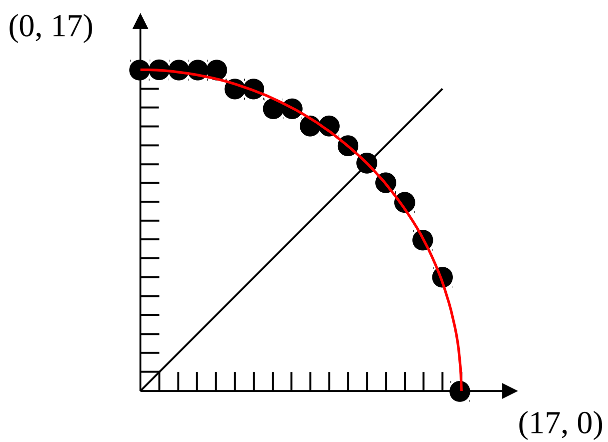
  Pixel (round(x), round(y));

  Pixel (round(x), round(-y));

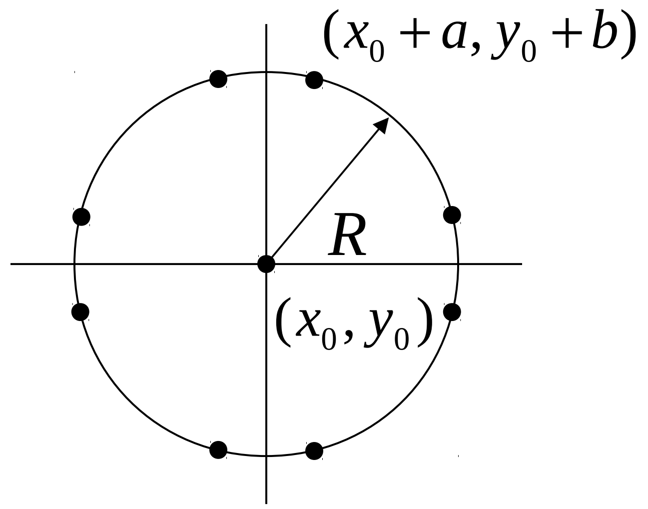
- Version 2 – slightly less bad

For x = 0 to 360

Pixel (round (R • cos(x)), round(R • sin(x)));



Uporabimo simetrijo

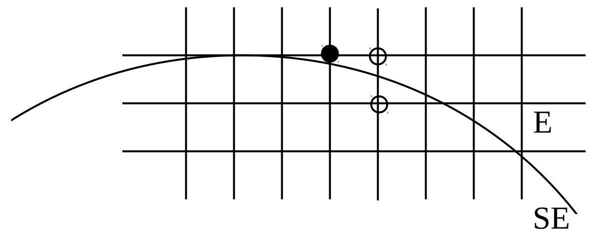


- Symmetry: If (x_0 + *a*, y_0 + *b*) is on the circle, so are (x_0 ± *a*, y_0 ± *b*) and (x_0 ± *b*, y_0 ± *a*); hence there's an 8-way symmetry.

- But in a practical setting of considering pixel values, it depends on the fact that x_0 and y_0 are integers.

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

Skica inkrementalnega algoritma




```
y = y0 + R; x = x0; Pixel(x, y);
For (x = x0+1; (x – x0) < (y – y0); x++) {
    if (decision_var < 0) {
    /* move east */
    update decision_var;
    }
    else {
    /* move south east */
    update decision_var;
    y--;
    }
    Pixel(x, y);
}
```

- Note: can replace all occurrences of x_0 and y_0 with 0, 0 and Pixel ($x_0 + x, y_0 + y$) with Pixel (x, y)
- Essentially a *shift of coordinates*

Poligoni

Poligoni

- Objects in 3D are made out of polygons

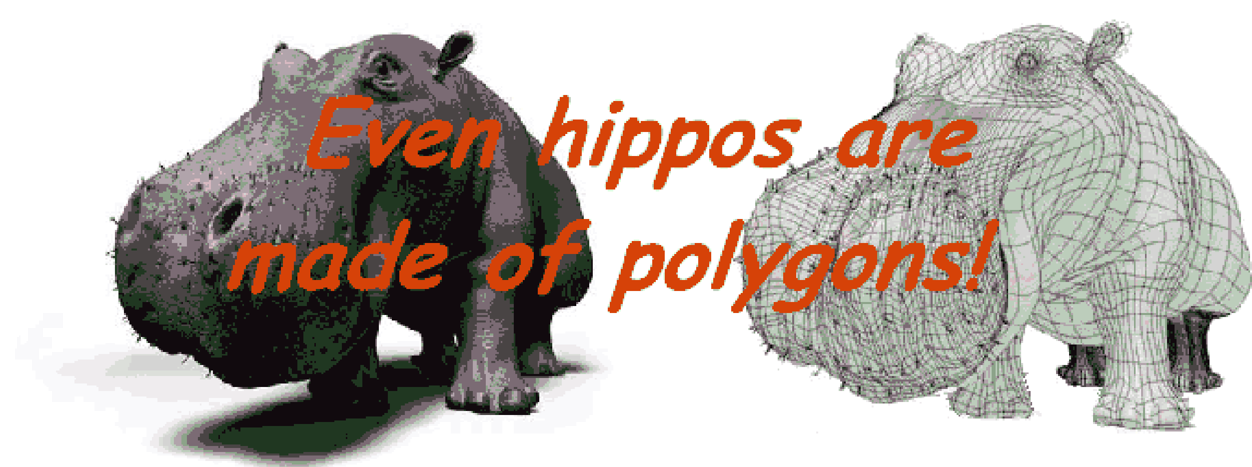
- Polygons are a fundamental building block in graphics!

Rasterizacija poligonov

- In interactive graphics, polygons rule the world
- Two main reasons:
  - Lowest common denominator for surfaces
    - Can represent any surface *with arbitrary accuracy*
    - Splines, mathematical functions, volumetric isosurfaces…
  - Mathematical simplicity lends itself to simple, regular rendering algorithms
    - Like those we're about to discuss…
    - Such algorithms embed well in hardware

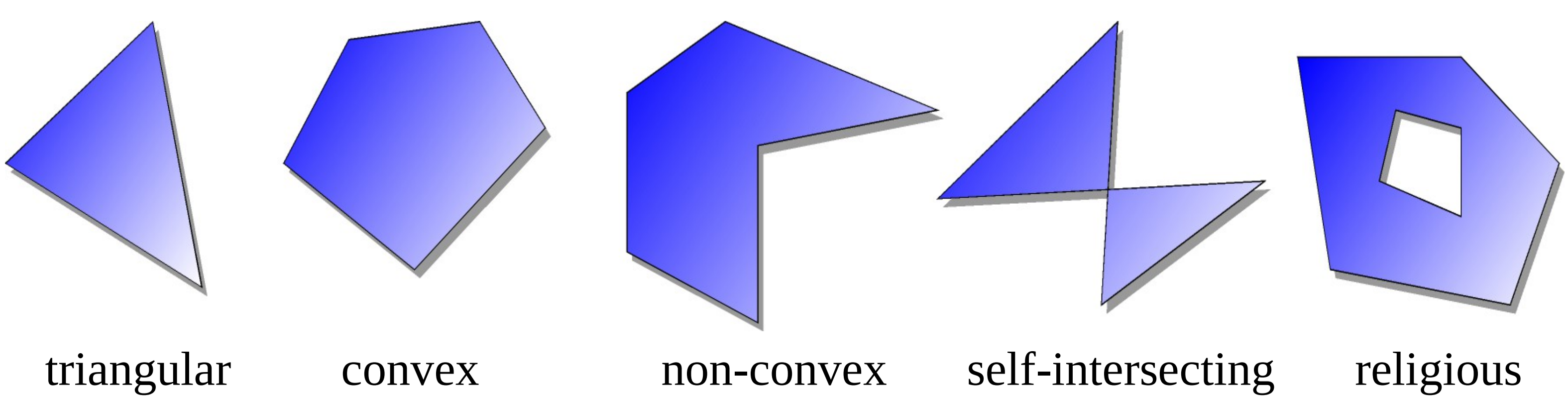
Rasterizacija poligonov

- Triangle is the *minimal unit* of a polygon
  - All polygons can be broken up into triangles
  - Triangles are guaranteed to be:
    - Planar
    - Convex



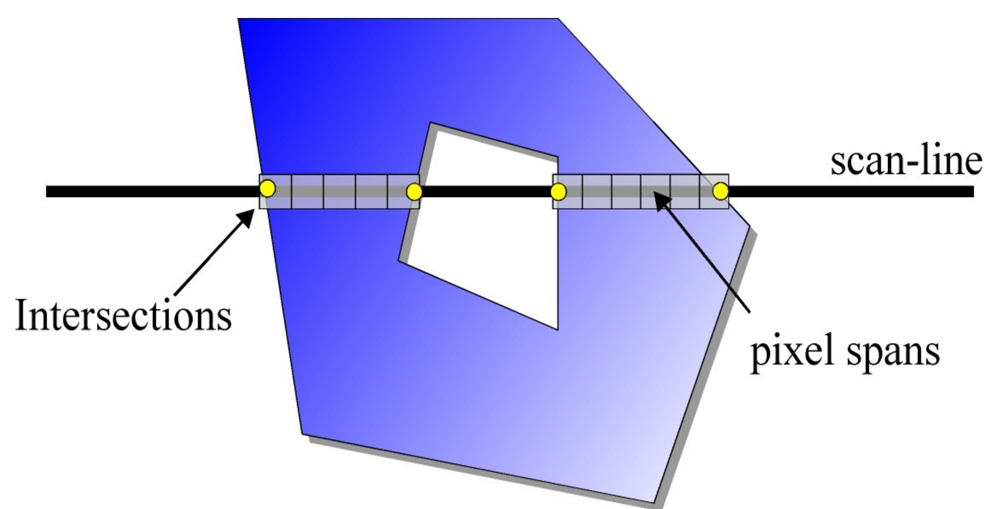
Rasterizacija poligonov

- There are a large number of algorithms for displaying polygons on raster displays.
- Each exploits some aspect of the types of polygons to be displayed:
  - some algorithms allow *triangular polygons* only
  - others require that the polygons are *convex* and *non self-intersecting* and have no *holes*



Rasterizacija poligonov

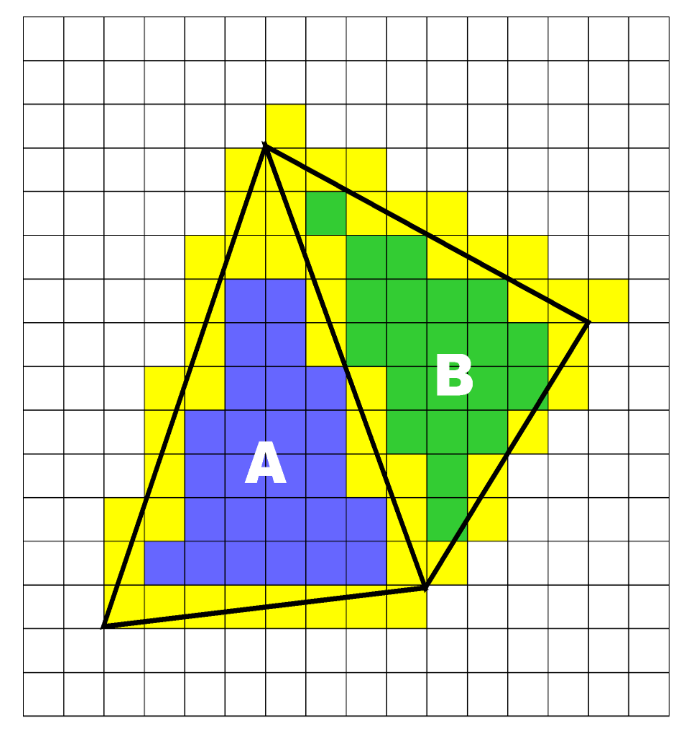
- *Polygon scan conversion* is a classic general purpose algorithm.
- For each *scan-line* we determine the *polygon edges* that intersect it, compute *spans* representing the interior portions of the polygons along this *scan-line* and fill the associated pixels.



- This represents the heart of a *scan-line rendering algorithm* used in many commercial products including Renderman and 3D Studio MAX.

Rasterizacija poligonov

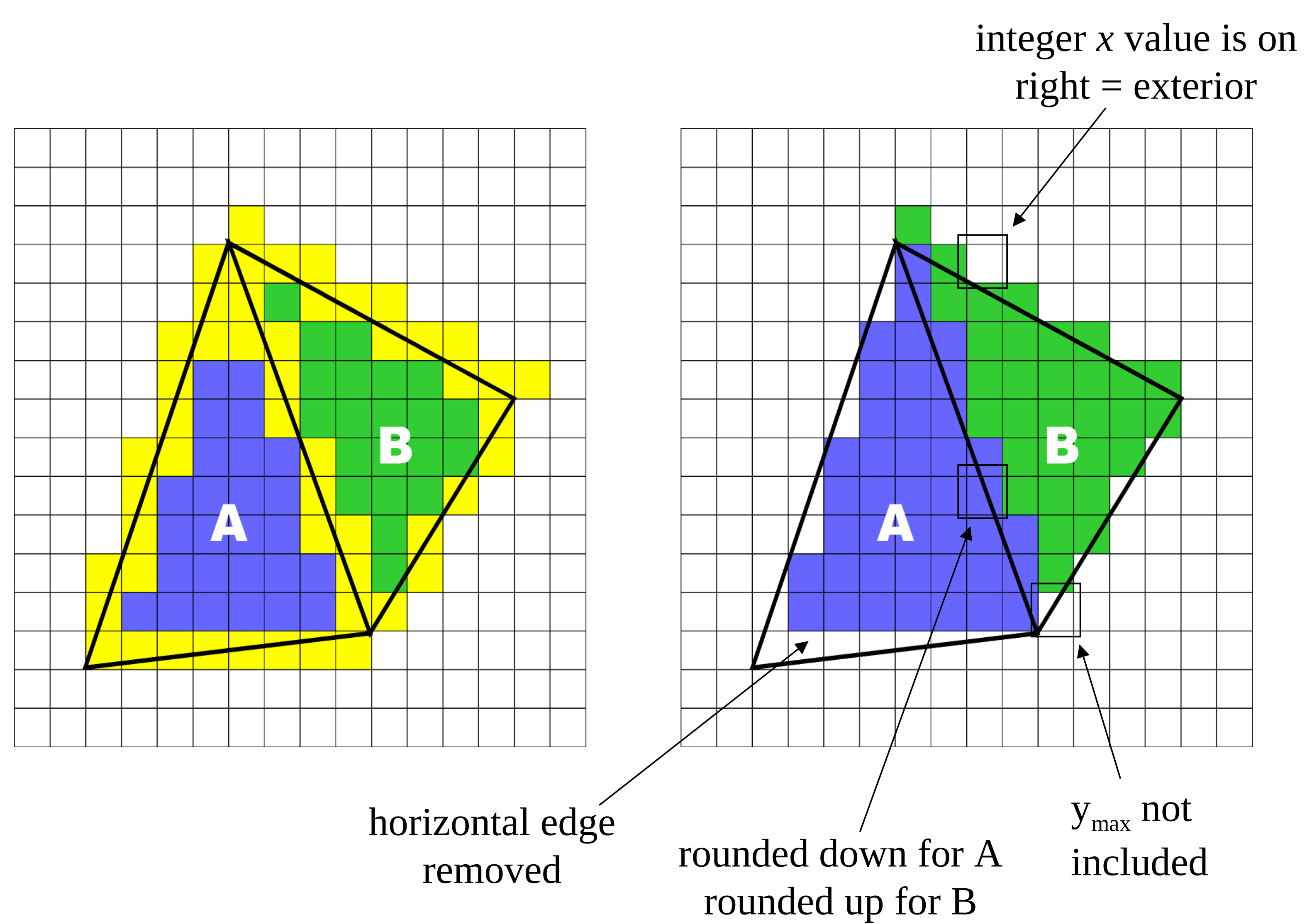
- We might choose to use midpoint line algorithms to determine the boundary pixels at each edge incrementally.
- This will not work:
  - pixels will be shared by neighbouring polygons
  - particularly bad if polygons are semi-transparent
- Must ensure that polygons which share an edge do not share pixels.



Rasterizacija poligonov

- General Procedure:
  - determine intersection of scan-line with polygon edges
  - sort intersections according to increasing *x* value
  - fill pixels between successive pairs of *x* values
- Need to handle 4 cases to prevent pixel sharing:
  - if intersection has fractional *x* value, do we round up or down?
    - if inside (on left of span) round up, if outside (on right) round down
  - what happens if intersection is at an integer *x* value?
    - if on left of span assume its interior otherwise exterior
  - how do we handle shared vertices?
    - ignore pixel associated with y_{max} of an edge
  - how do we handle horizontal edges?
    - handled as a result of previous rule (lower edges not drawn)

Rasterizacija poligonov



Rasterizacija poligonov

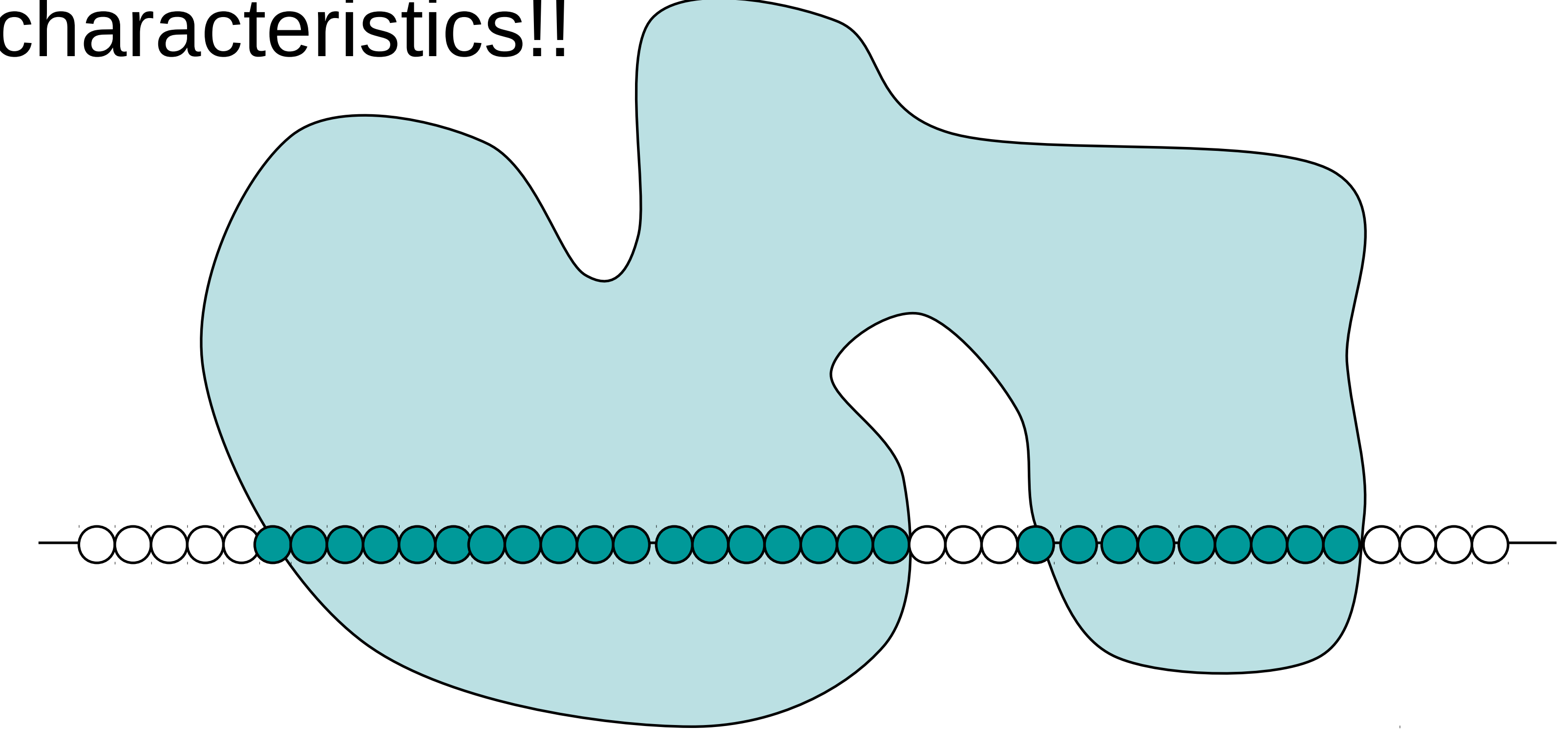
- Determining intersections with polygon edges is expensive
  - rather than re-computing all intersections at each iteration, use *incremental calculations*
  - i.e. if we intersect edge *e* on scan-line *i* then it is likely we will intersect the edge on scan-line *i+1* (this is known as *edge-coherence*)
- Assume slope of the edge > 1 (other edges obtained via symmetries)
  - incremental DDA calculation was: $y_{i+1} = y_i + 1, \quad x_{i+1} = x_i + \frac{1}{m}$
  - slope m is given by $$m = \frac{\left(y_{end} - y_{start}\right)}{\left(x_{end} - x_{start}\right)}$$
  - note that *numerator* and *denominator* are integral ⇒ we can use integer DDA.

Metode rasterizacije

- Makes use of the *coherence* properties
  - Spatial coherence : Except at the boundary edges, adjacent pixels are likely to have the same characteristics
  - Span coherence : Pixels in a scan line will be set to same values for solid shaded primitives
  - Scan line coherence : Pixels in the adjacent scan lines are likely to have the same characteristics
- Uses intersections between area boundaries and scan lines to identify pixels that are inside the area

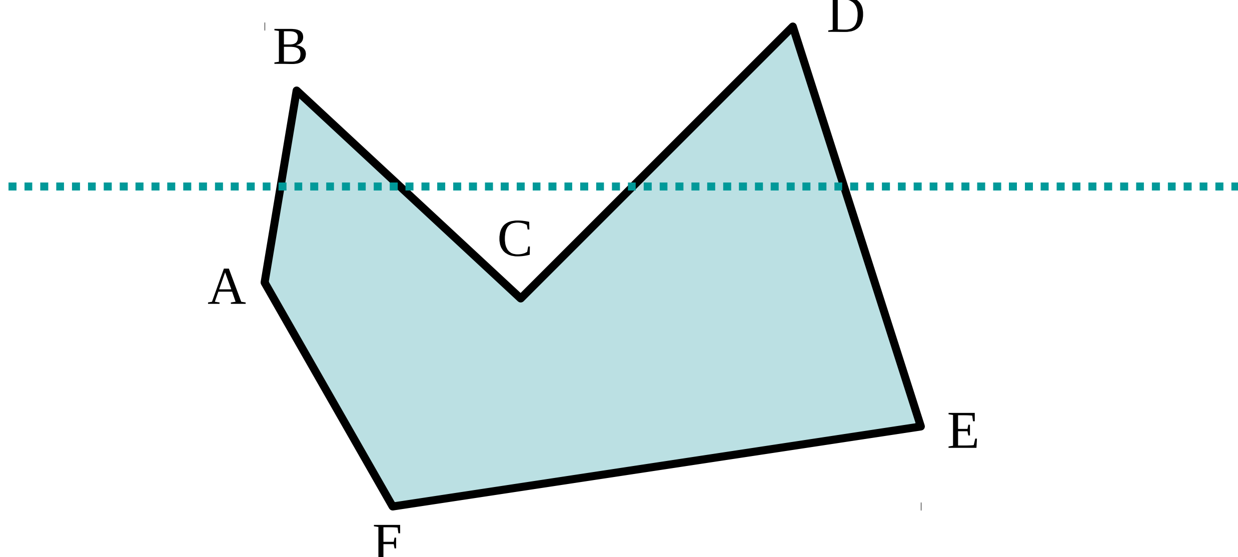
Prostorska koherenca

- Adjacent pixels are likely to have the same characteristics!!



Rasterizacija poligonov

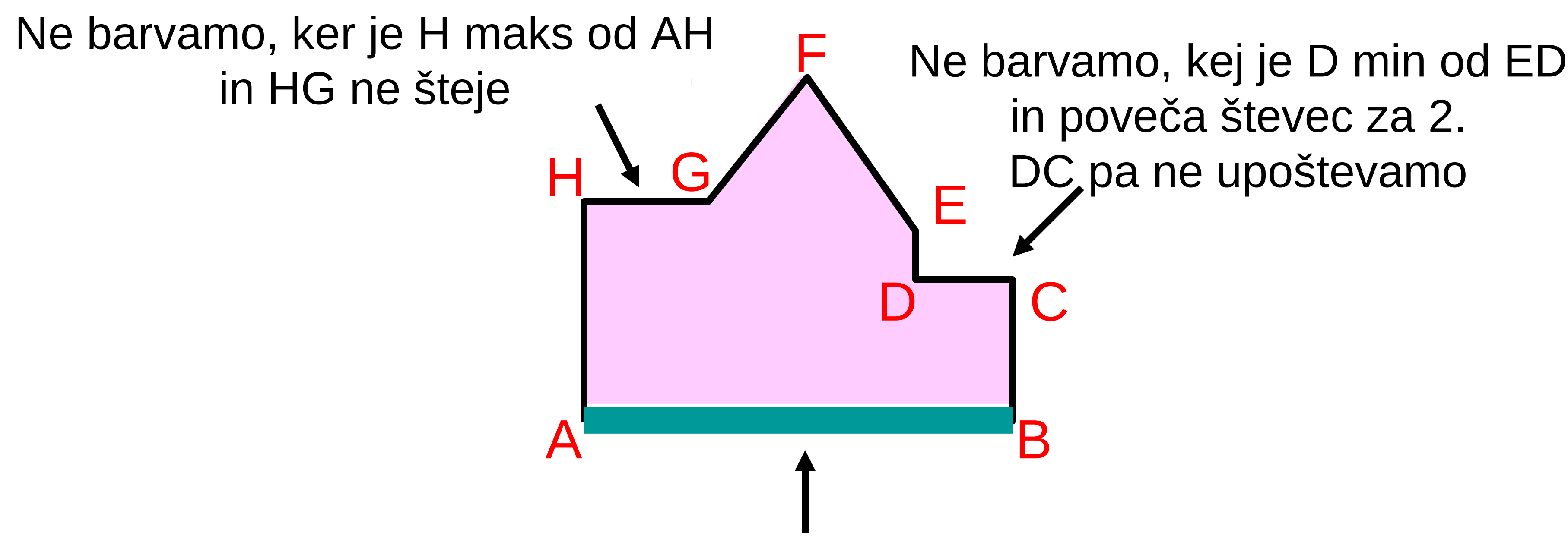
- Consider the following polygon:



- How do we know whether a given pixel on the scanline is inside or outside the polygon?

Rasterizacija poligonov

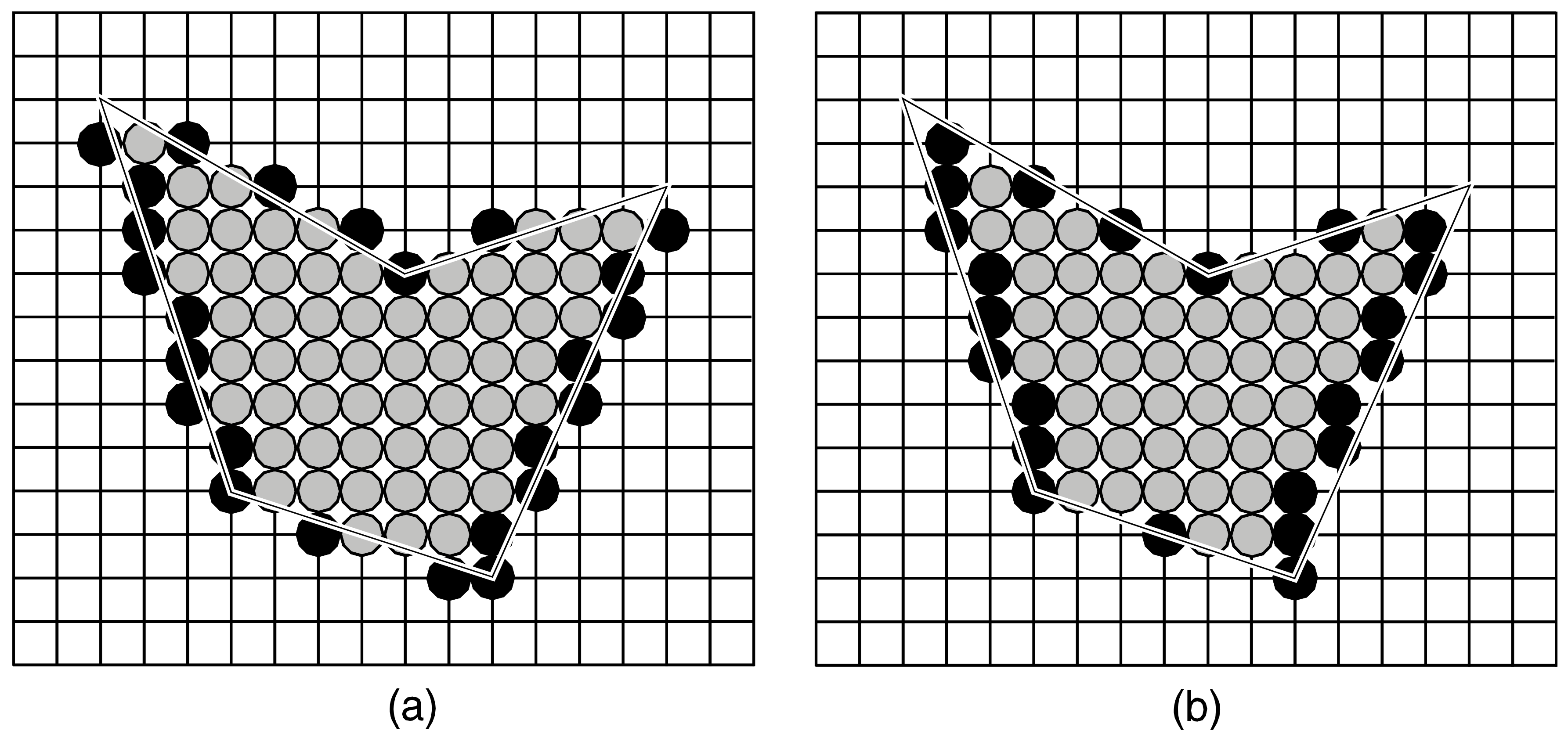
Za vrstico skeniranja določimo vse preseke poligona s to vrstico
Preseke razvrstimo od prvega do zadnjega
S štetjem parnosti ugotovimo, kdaj barvamo piksle
Vodoravnih črt v štetju parnosti ne upoštevamo
Končne točke Y_{min} se pri štetju parnosti upoštevajo
Končne točke Y_{max} se pri štetju parnosti ne upoštevajo



Barvanje poligonov

- Find intersections of scanline with all polygon edges
- Sort intersections by increasing x
- Fill all interior pixels between pairs of intersections (odd-parity rule)

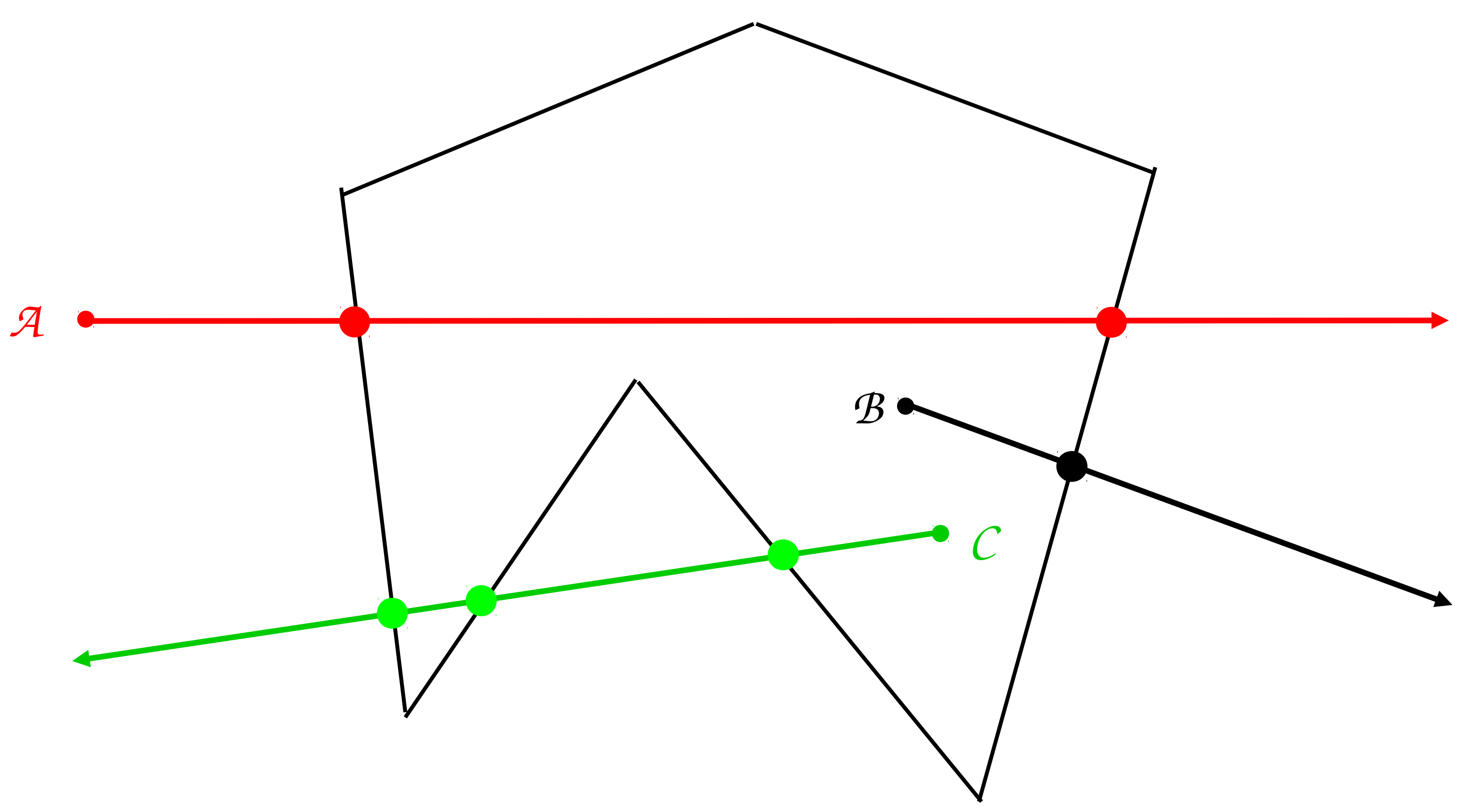
Barvanje poligonov



Prednosti metode Scan Line

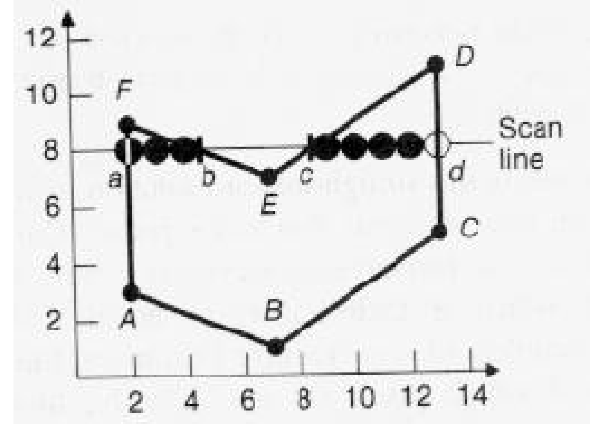
- The algorithm is efficient
- Each pixel is visited only once
- Shading algorithms could be easily integrated with this method to obtain shaded area

Pravilo parnosti



#intersections = odd: point is inside polygon

Uporaba pravila parnosti




```
for each scanline
     edgeCnt = 0;
     for each pixel on scanline (l to r)
         if (oldpixel->newpixel crosses edge)
             edgeCnt ++;
         // draw the pixel if edgeCnt odd
         if (edgeCnt % 2)
             setPixel(pixel);
```

Tabela robov in Tabela aktivnih robov

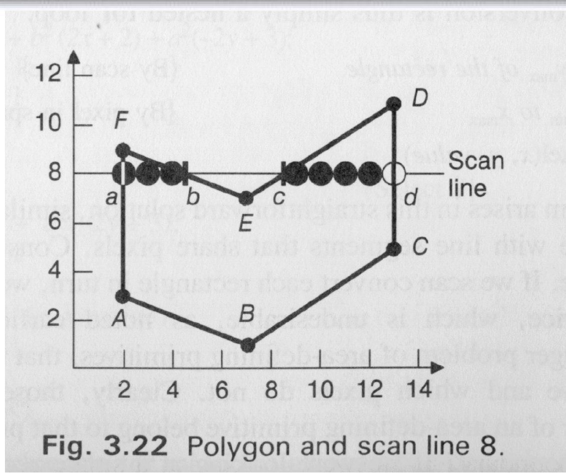


Fig. 3.22 Polygon and scan line 8.

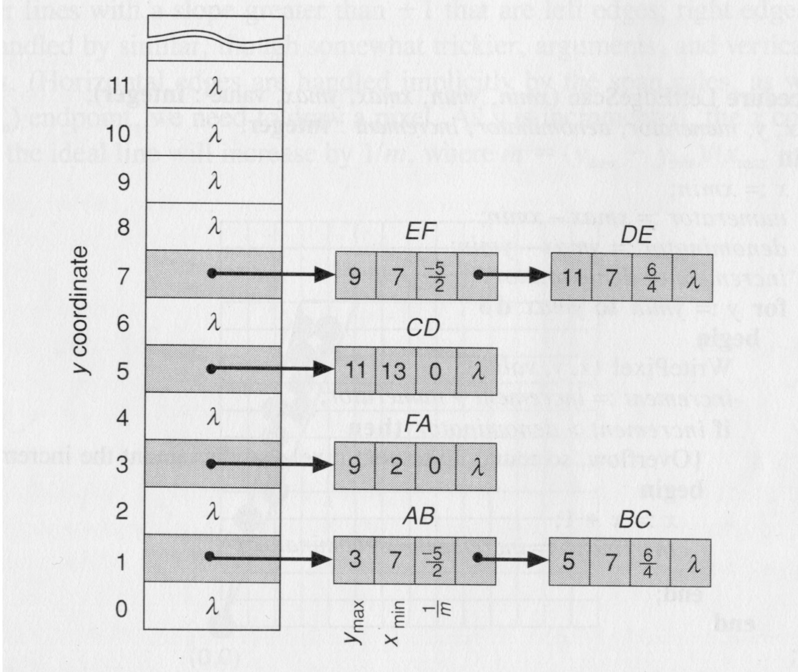


Fig. 3.27 Bucket-sorted edge table for polygon of Fig. 3.22.

Global edge table: contains all edges, sorted by minimal y-value
Active edge table: edges intersected by current scanline

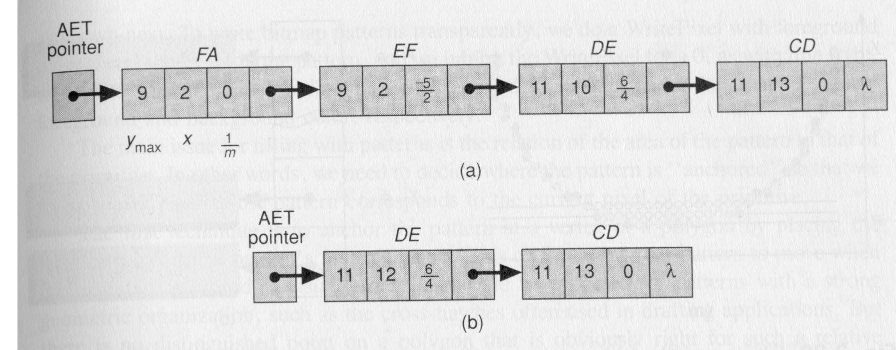


Fig. 3.28 Active-edge table for polygon of Fig. 3.22. (a) Scan line 9. (b) Scan line 10. (Note *DE*'s *x* coordinate in (b) has been rounded up for that left edge.)

Tabela robov (ET, Edge Table)

- Polje kazalcev A, dolžina enaka višini zaslona
- A[i] kaže na povezan seznam vseh robov z ymin = I
- Robovi v povezanem seznamu so razvrščeni glede na koordinato x verteksa ymin
- Rob v seznamu je predstavljen z: ymax, začetnim x, naklonom (1/m)

Tabela robov

y coordinate

11 1
10 1
9 1
8 1
7 → EF: 9 | 7 | −5/2 → DE: 11 | 7 | 6/4 | 1
6 1
5 → CD: 11 | 13 | 0 | 1
4 1
3 → FA: 9 | 2 | 0 | 1
2 1
1 → AB: 3 | 7 | -5/2 → BC: 5 | 7 | 6/4 | 1
0 1

y_{max} x_{min} $\frac{1}{m}$

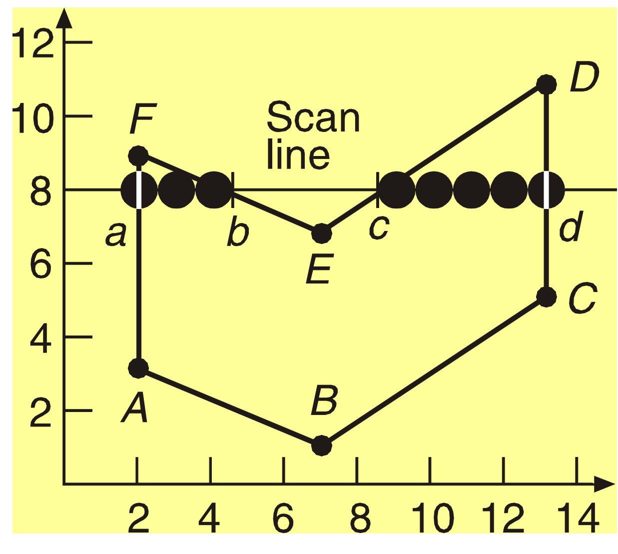
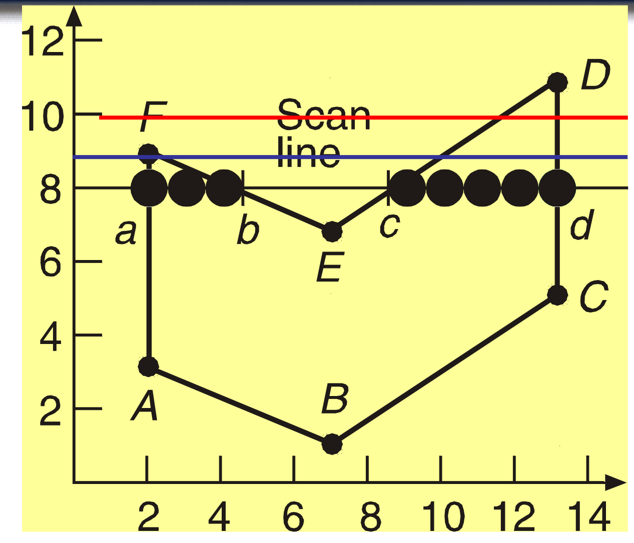
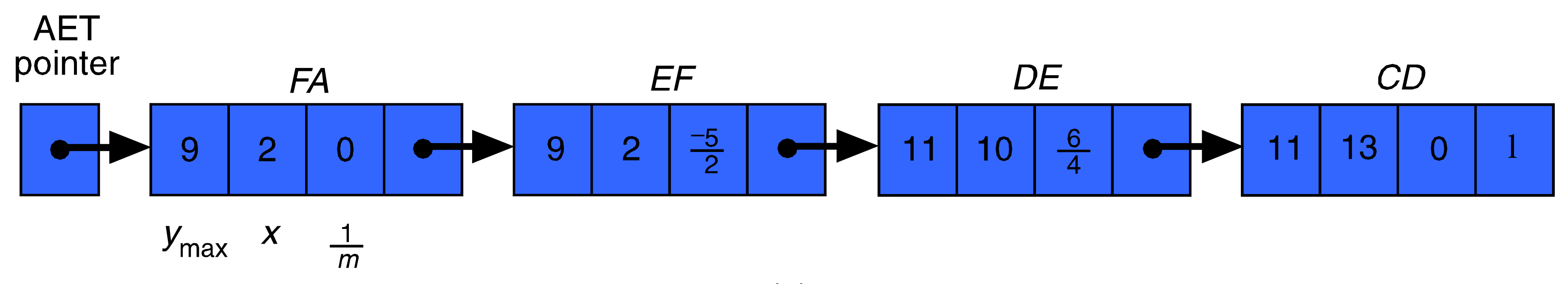


Tabela aktivnih robov

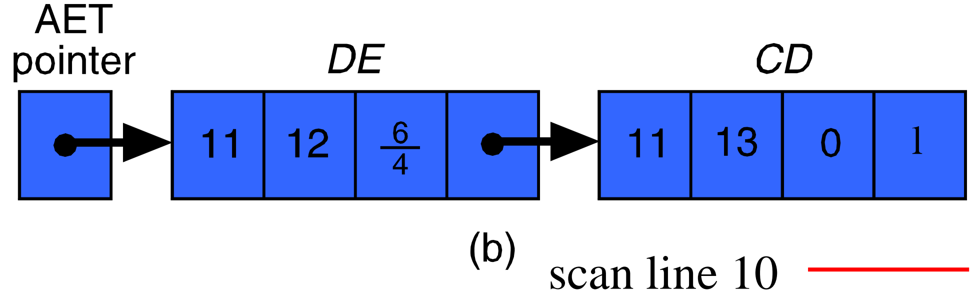
- Povezan seznam vseh robov, ki sekajo tekočo vrstico skeniranja
- Seznam vedno uredimo glede na sekanje x z vrstico skeniranja
- Najprej dodamo vse robove iz te tabele z najmanjšim y
- S testom parnosti zapolnjujemo (barvamo) piksle na vrstici skeniranja
- Vrstico skeniranja premikamo navzgor
- Dodajamo vse robove iz tabele robov, pri katerih je vrednost ymin enaka vrstici skeniranja
- Odstranimo vse robove iz tabele aktivnih robov, pri katerih je vrednost ymax enaka vrstici skeniranja
- Posodobimo vrednosti x vrednosti presečišč vseh robov v aktivni tabeli robov in prerarvrstimo

Tabela aktivnih robov (AET, Active Edge Table)





(a) scan line 9



(b) scan line 10

Algoritem

- The scan-line algorithm uses *edge-coherence* and *incremental integer calculations* for maximum efficiency:
  - create an *edge table* (ET) which lists all edges in order to their y_{min} value
  - keep track of an *active edge table* (AET) which lists those edges under the current scan-line
- As the scan progresses, edges are moved from the ET to the AET.
- An edge remains in the AET until y_{max} for that edge has been reached.
- At this point the edge is removed from the AET.

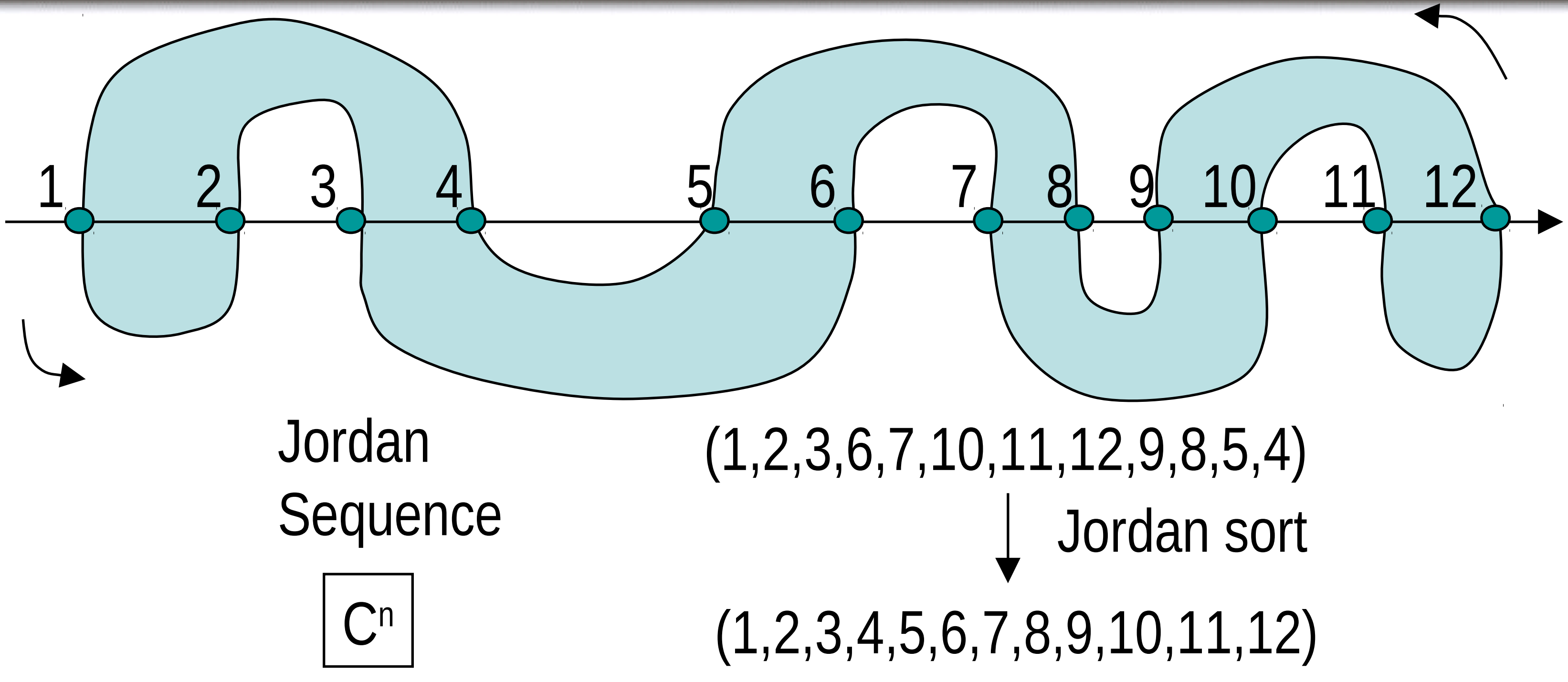
Algoritem

- Initialize edge table
- y = smallest ymin from edge table
- active edge table = empty
- Repeat:
  - Update active edge table: remove, add, sort on x
  - Fill pixels
  - Increment y
  - Update x for each span
- Untill active edge table and edge table empty

Koda algoritma

```
y = y of first non empty entry in ET
AET = null
repeat
    move all ET entries in slot y to AET
    sort AET entries according to x_min
    fill spans using pairs of AET entries
    for all AET members
        if y_max = y then remove from AET
    y = y+1
    for all AET members
        update numerator
        if numerator>denominator
            numerator=numerator-denominator
            x = x+1
until AET and ET empty
```

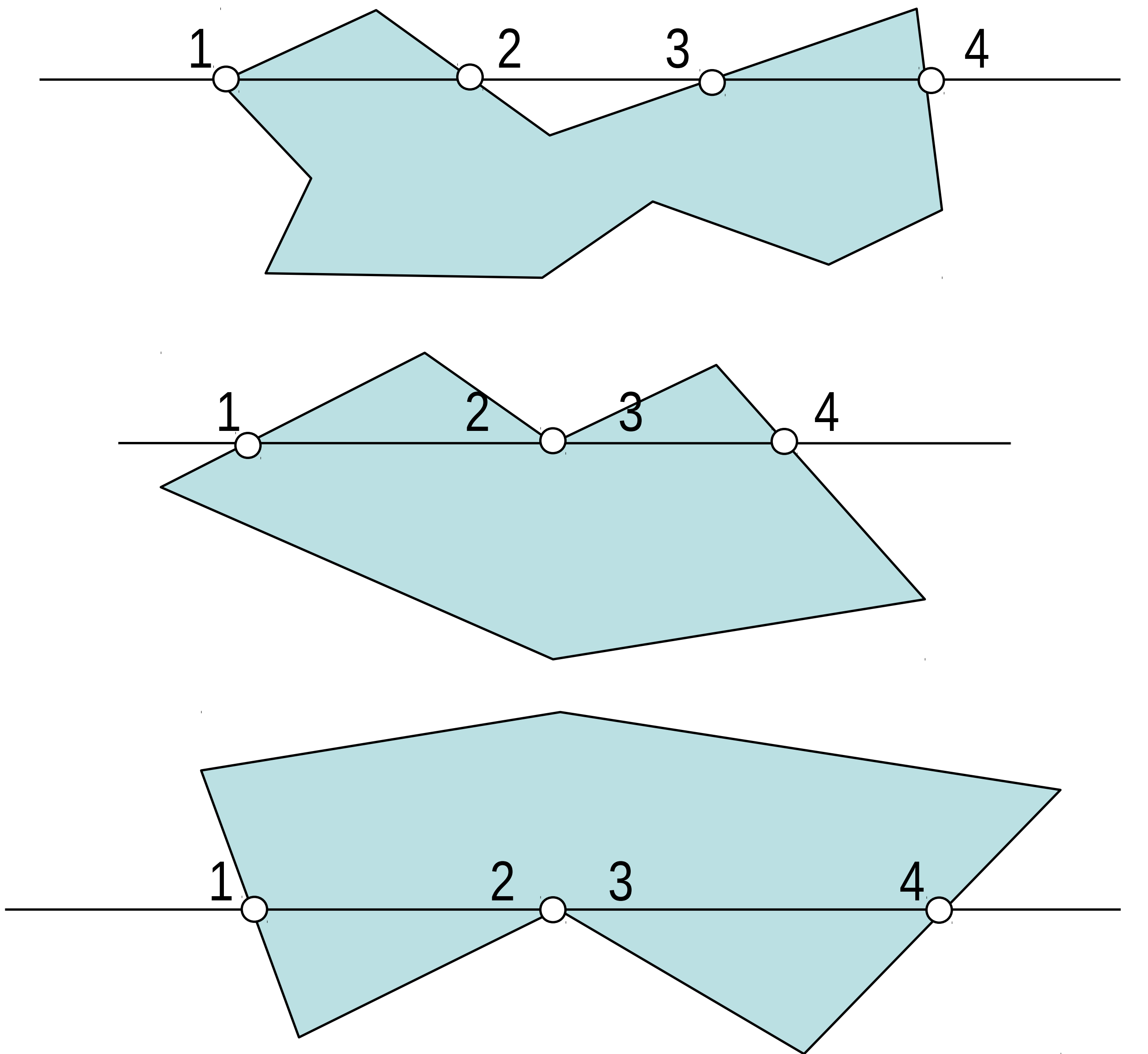
Prostorska koherenca (nadaljevanje)



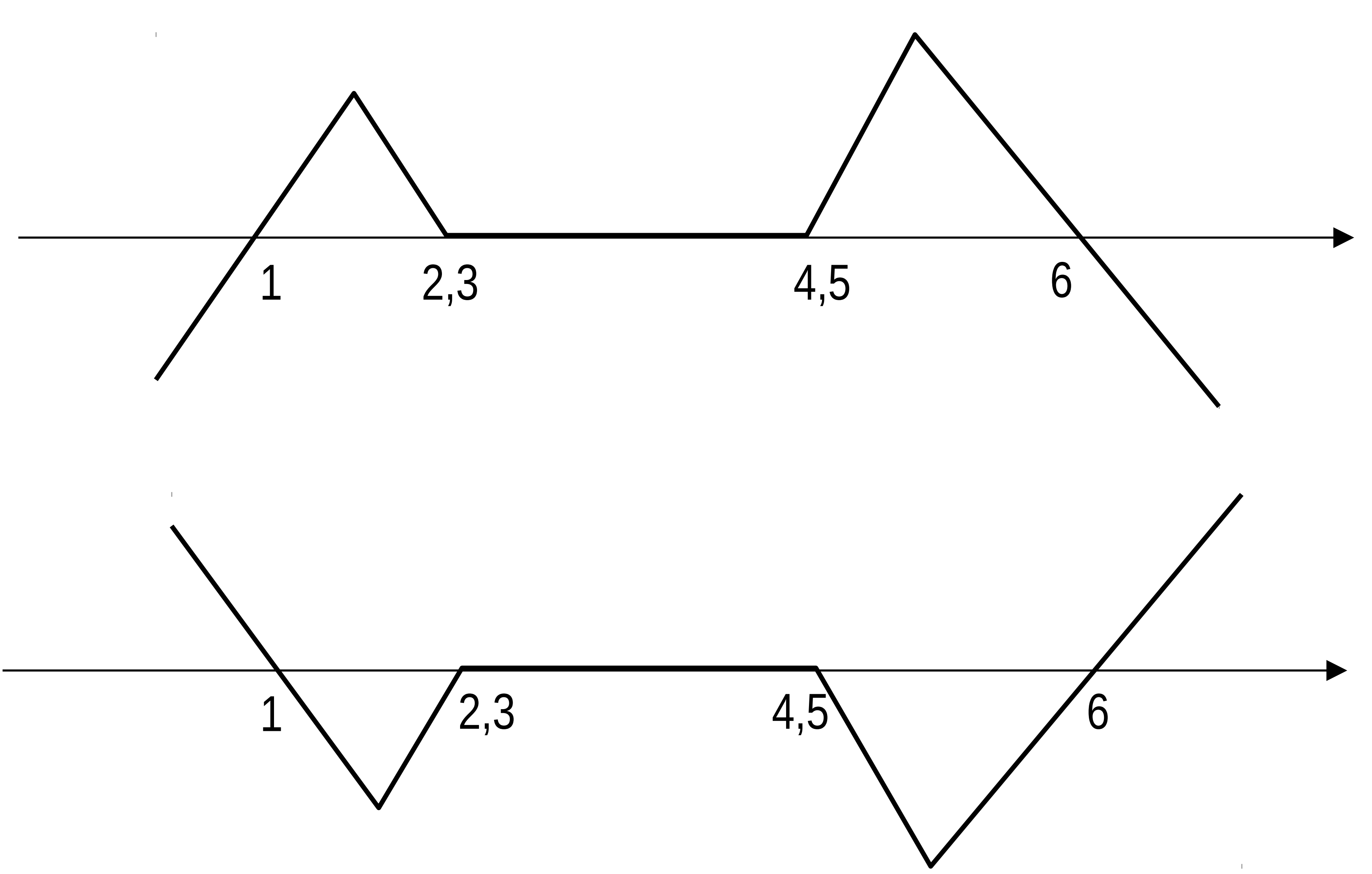
- O(n) algorithm

  Hoffman, Mehlhorn, Rosenstiehl, and Tarjan, Sorting Jordan Sequence in Linear Time.
- However, not practical !!!

Prostorska koherenca (nadaljevanje)

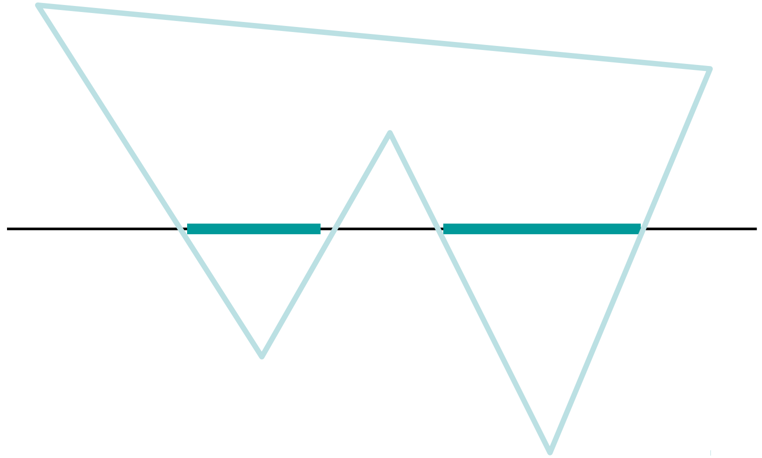


Prostorska koherenca (nadaljevanje)



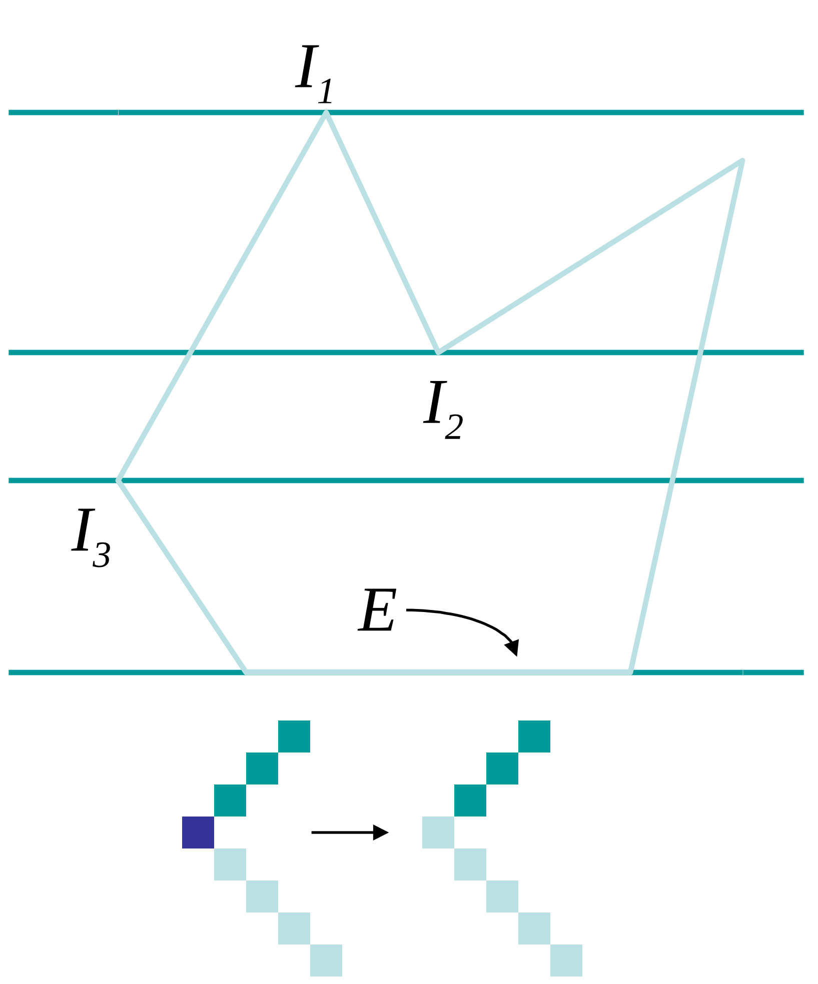
Scan Line Method

- Proceeding from left to right
  the intersections are paired and
  intervening pixels are set to the
  specified intensity
- Algorithm
  - Find the intersections of the scan line with all the edges in the polygon
  - Sort the intersections by increasing X-coordinates
  - Fill the pixels between pair of intersections



Special cases for Scan Line Method

- Overall topology should be considered for intersection at the vertices
- Intersections like I_1 and I_2 should be considered as two intersections
- Intersections like I_3 should be considered as one intersection
- Horizontal edges like E need not be considered



Efficiency Issues in Scan Line Method

- Intersections could be found using edge coherence

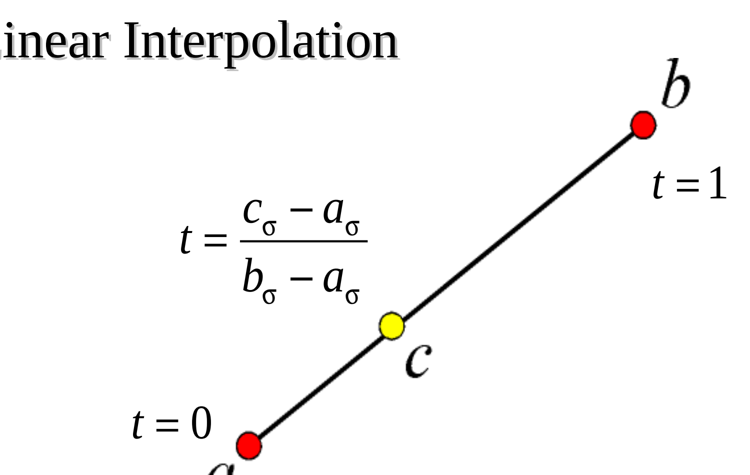
  the X-intersection value x_{i+1} of the lower scan line can be computed from the X-intersection value x_i of the preceeding scanline as

$$x_{i+1} = x_i + \frac{1}{m}$$

- List of active edges could be maintained to increase efficiency
- Efficiency could be further improved if polygons are convex, much better if they are only triangles

Bilinear Interpolation

- When scan-converting polygons we can exploit the incremental calculations to speed up vertex parameter interpolation:
  - interpolate **colour** for smoothly filled polygon
  - interpolate **texture co-ordinates** for continuous mapping of images
  - interpolate **normal** to polygon for smooth shading (*Phong Shading*)
  - interpolate **depth** for approximate depth testing

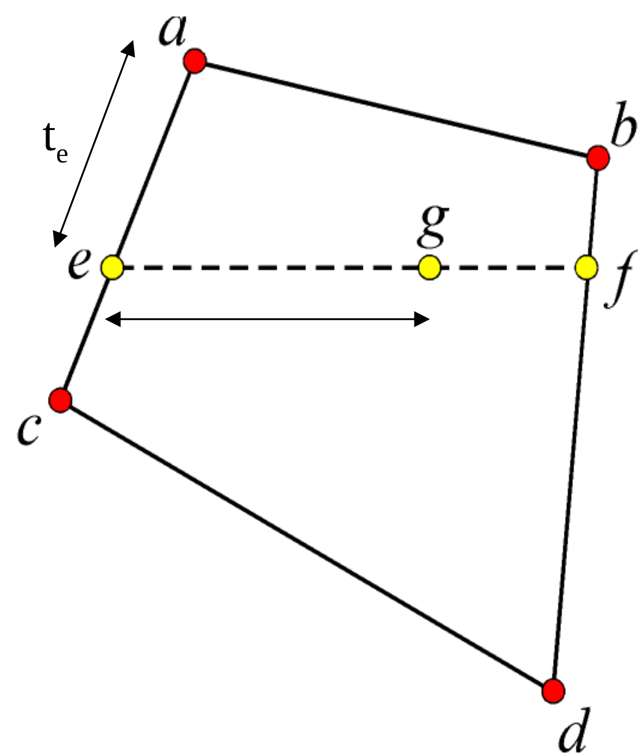


$$c_\sigma = (1 - t) a_\sigma + t b_\sigma$$

Bilinear Interpolation

$$\text{linear} \Rightarrow f(a+b) = f(a) + f(b)$$

$$\text{bilinear} \Rightarrow f(sa+tb) = sf(a) + tf(b)$$



$$e_\sigma = (1-t_e)a_\sigma + tc_\sigma$$

$$f_\sigma = (1-t_f)b_\sigma + td_\sigma$$

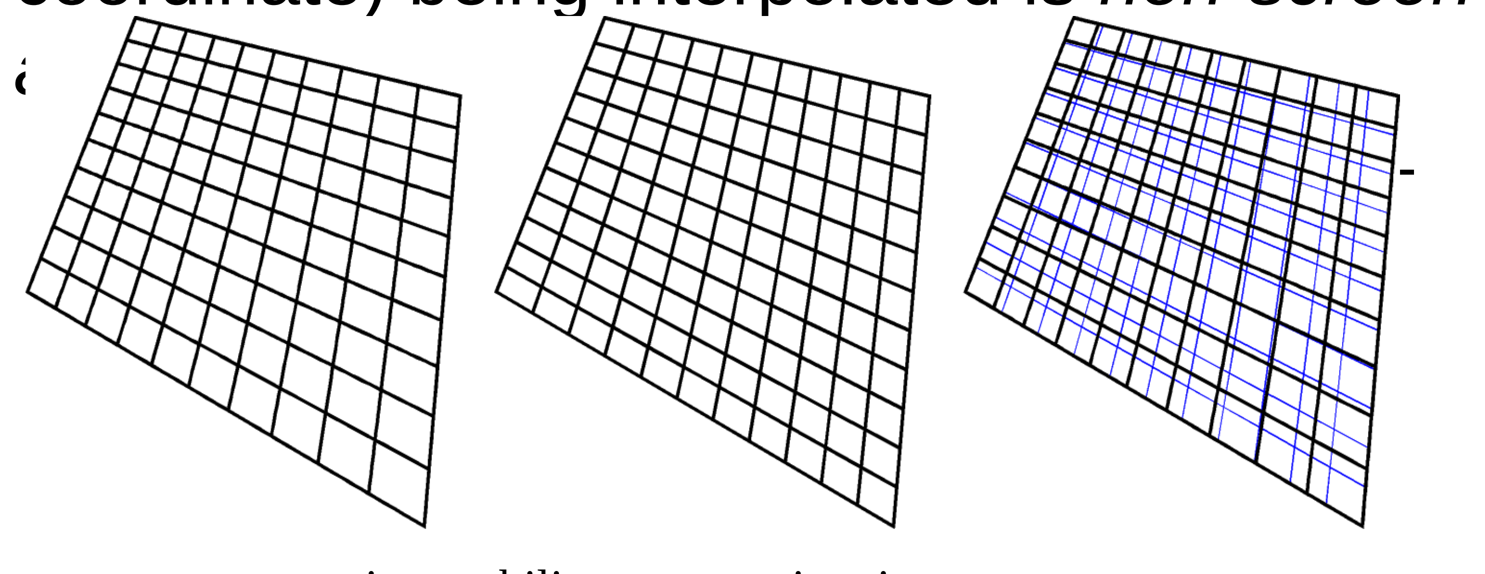
$$g_\sigma = (1-t_g)e_\sigma + tf_\sigma$$

If we know the 2D co-ordinates then:

$$t_e = \frac{e_y - a_y}{c_y - a_y}, t_f = \frac{f_y - b_y}{d_y - b_y}, t_g = \frac{g_x - e_x}{f_x - e_x}$$

Bilinear Interpolation

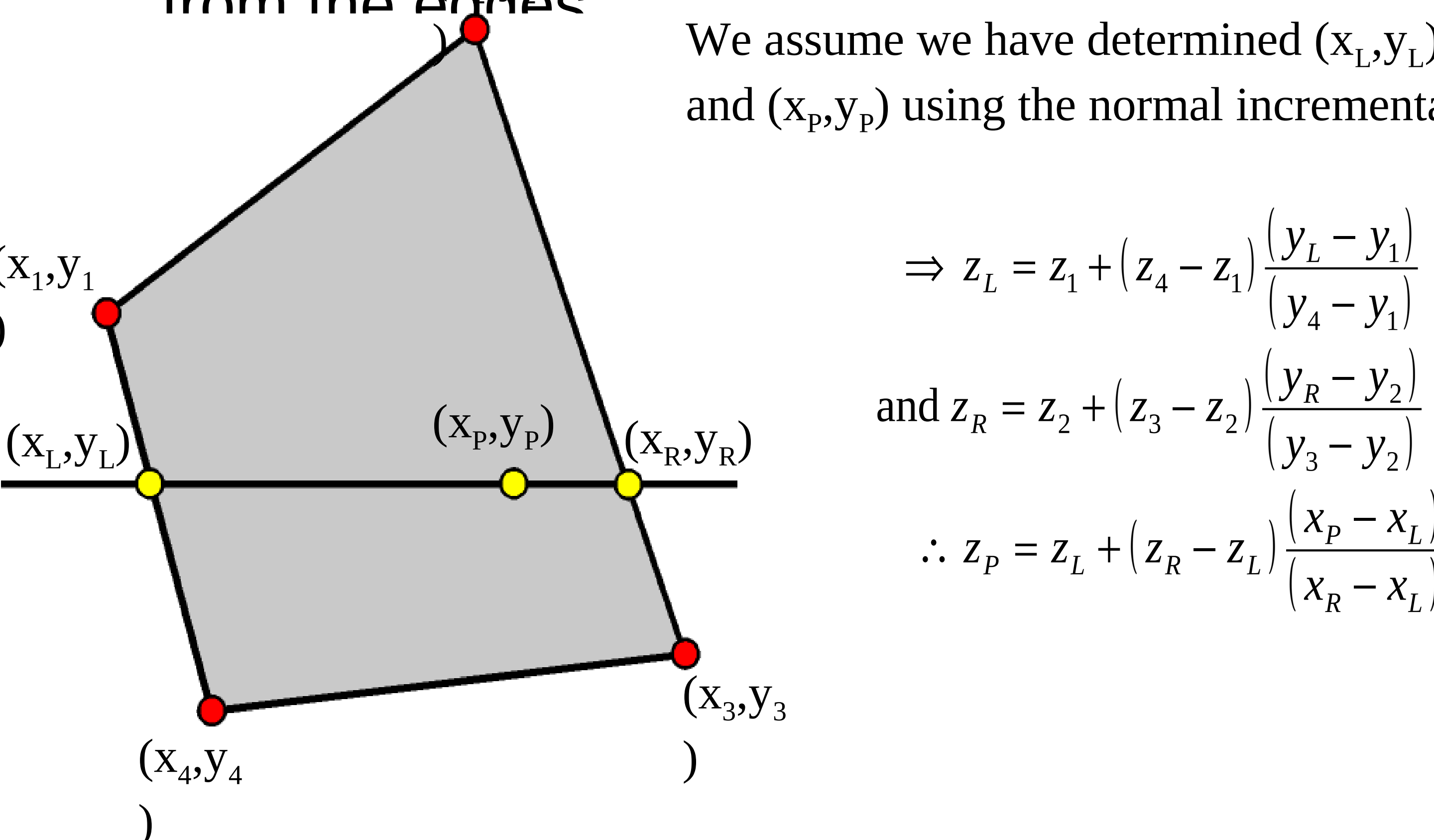
- We can build the calculations into the incremental scan-conversion process.
- For many applications this is sufficient, however if will introduce errors if the parameter (e.g. colour, depth, texture coordinate) being interpolated is *non-screen-*



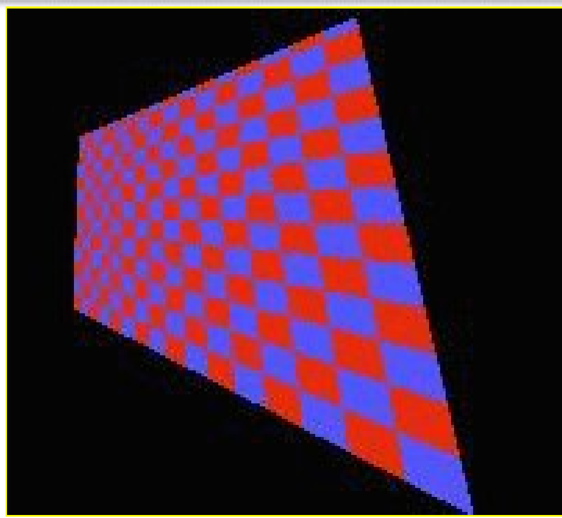
correct perspective bilinear approximation error

Bilinear Interpolation

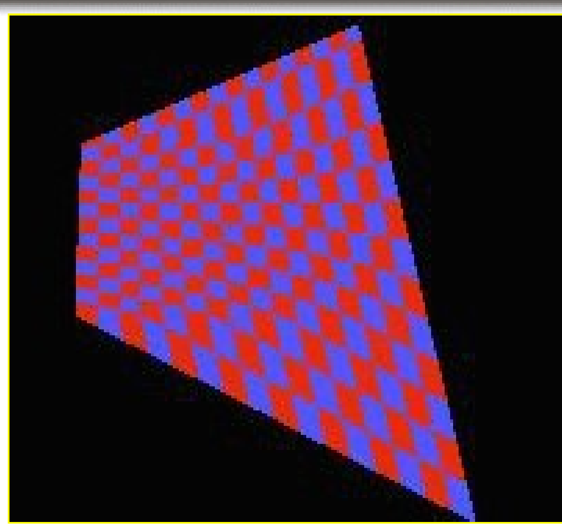
- Implementation:
  - interpolate parameter along each edge
  - interpolate within space using interpolated value from the edges



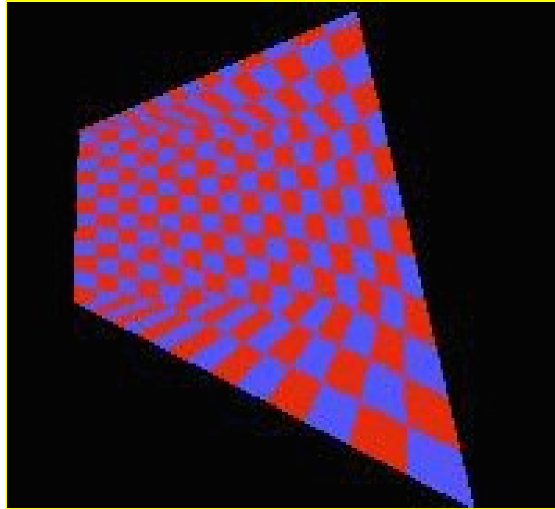
Texture Mapping with Approximate Depth Information



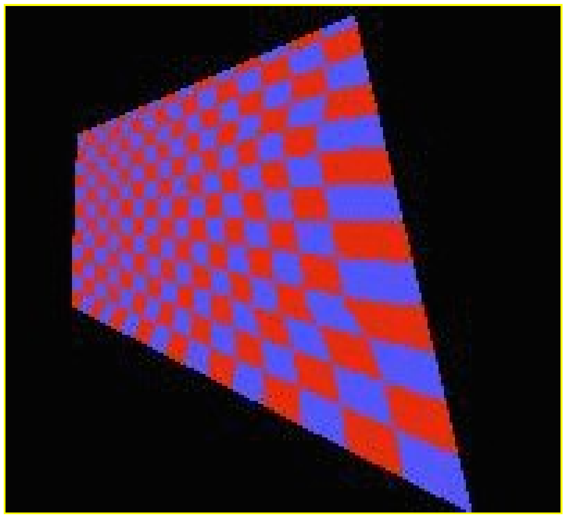
True Perspective



Bilinear Approximation



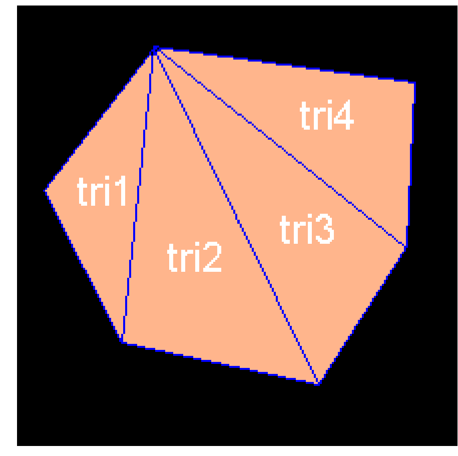
True Edges, Linear Spans



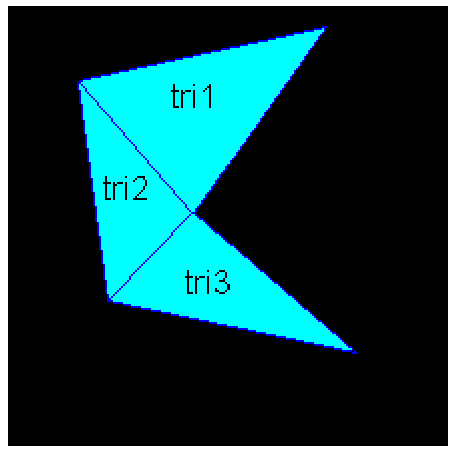
True Edges. Quadratic Spans

Triangulacija

- Convex polygons easily triangulated



- Concave polygons present a challenge

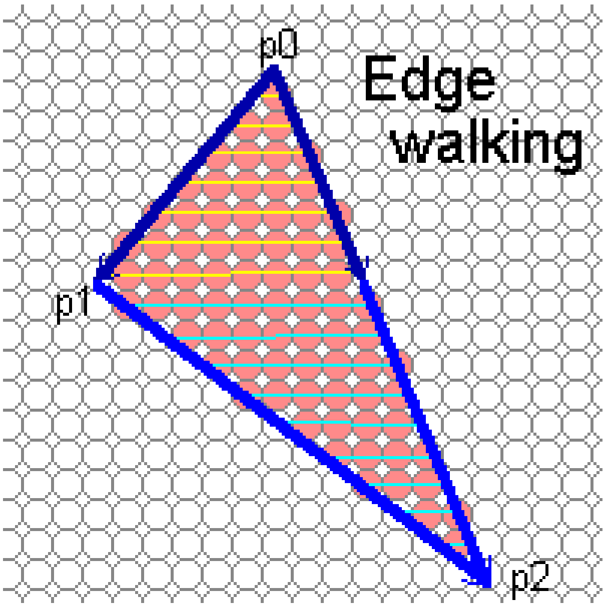


Rasterizacija trikotnikov

- Interactive graphics hardware commonly uses *edge walking* or *edge equation* techniques for rasterizing triangles

Edge Walking

- Basic idea:
  - Draw edges vertically
    - Interpolate colors down edges
  - Fill in horizontal spans for each scanline
    - At each scanline, interpolate edge colors across span

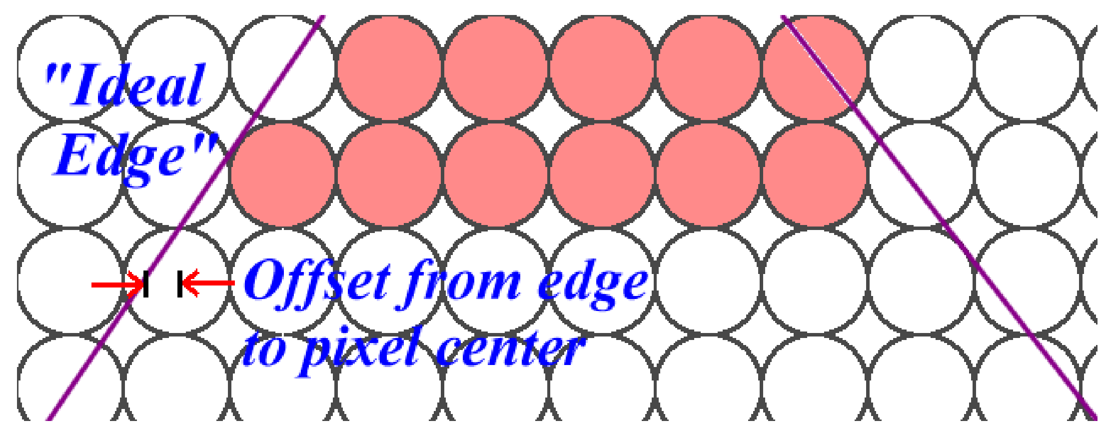


Edge Walking: Notes

- Order three triangle vertices in x and y
  - Find middle point in y dimension and compute if it is to the left or right of polygon. Also could be flat top or flat bottom triangle
- We know where left and right edges are.
  - Proceed from top scanline downwards
  - Fill each span
  - Until breakpoint or bottom vertex is reached
- Advantage: can be made very fast
- Disadvantages:
  - Lots of finicky special cases

Edge Walking: Disadvantages

- Fractional offsets:



- Be careful when interpolating color values!
- Beware of gaps between adjacent edges
- Beware of duplicating shared edges

Edge Equations

- An edge equation is simply the equation of the line defining that edge
  - Q: *What is the implicit equation of a line?*
  - A: $Ax + By + C = 0$
  - Q: *Given a point (x,y), what does plugging x & y into this equation tell us?*
  - A: Whether the point is:
    - On the line: $Ax + By + C = 0$
    - “Above” the line: $Ax + By + C > 0$
    - “Below” the line: $Ax + By + C < 0$

Edge Equations

- Edge equations thus define two *half-spaces*:

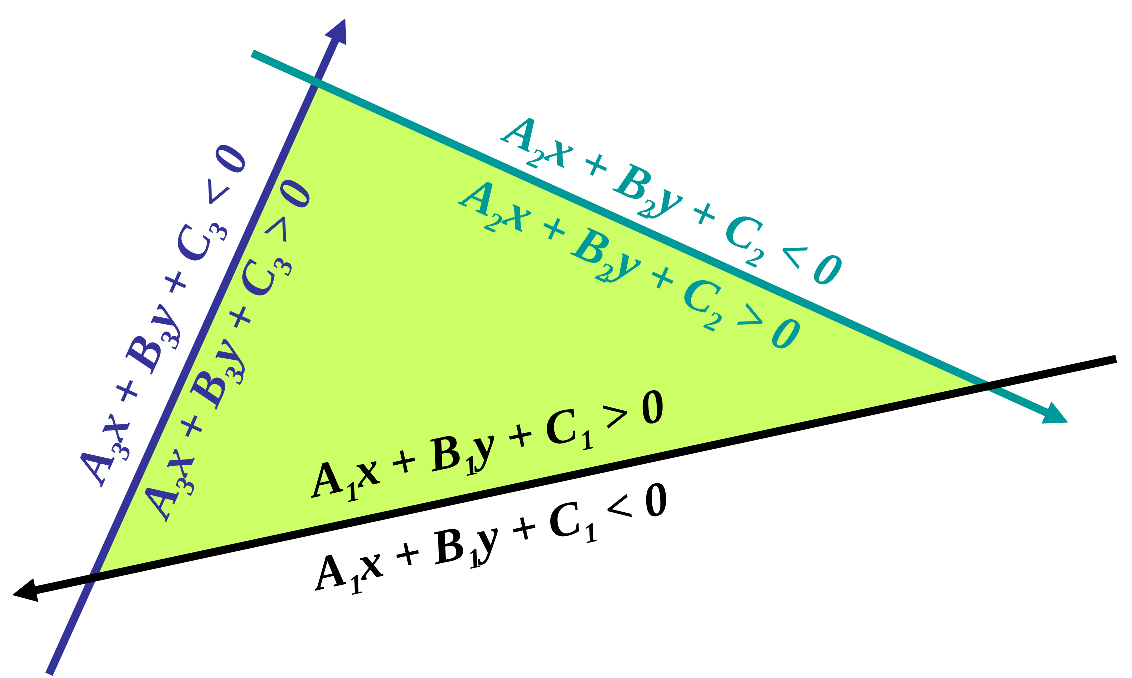
$Ax+By+C > 0$

$Ax+By+C = 0$

$Ax+By+C < 0$

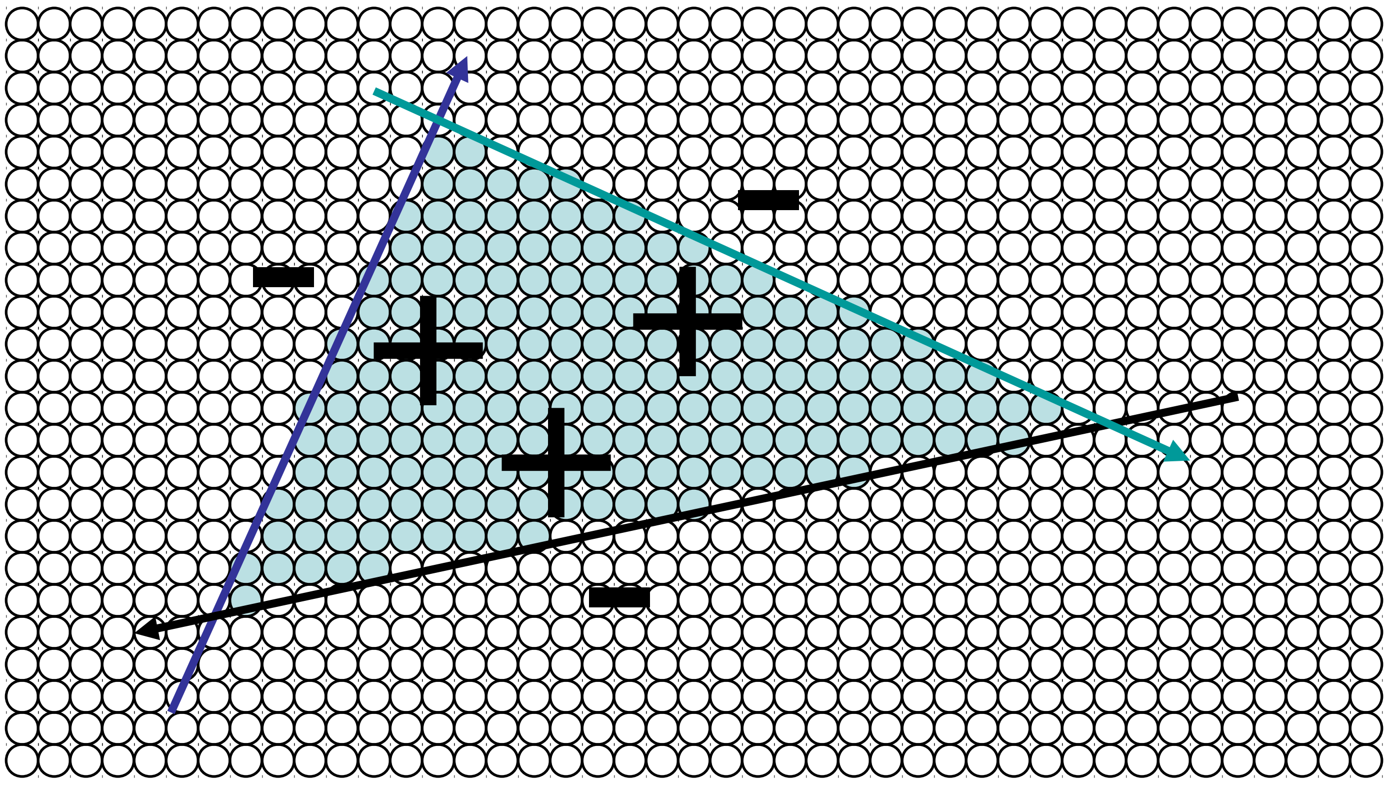
Edge Equations

- And a triangle can be defined as the intersection of three positive half-spaces:



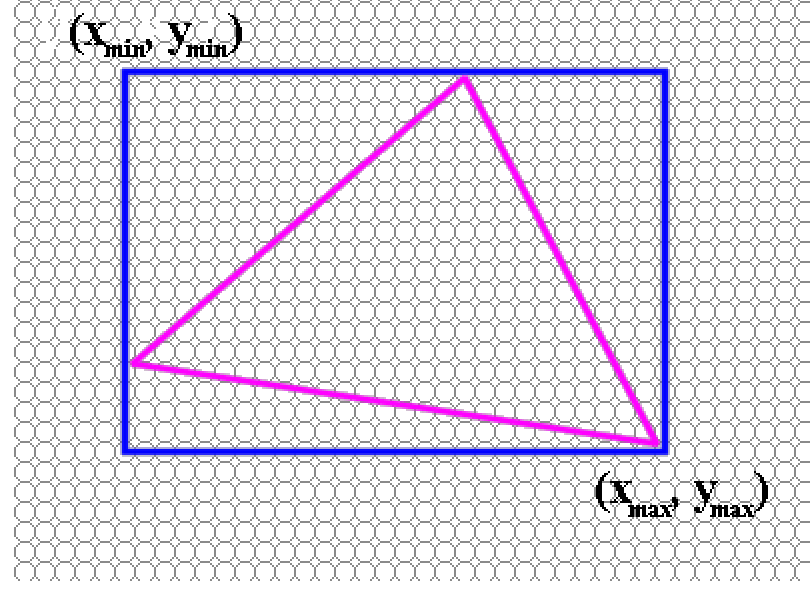
Edge Equations

- So…simply turn on those pixels for which all edge equations evaluate to > 0:



Using Edge Equations

- Which pixels: compute min,max bounding



Computing Edge Equations

- Want to calculate A, B, C for each edge from (x_1, y_1) and (x_2, y_2)
- Treat it as a linear system:

$$Ax_1 + By_1 + C = 0$$

$$Ax_2 + By_2 + C = 0$$

- Notice: two equations, three unknowns
- *What can we solve?*
- Goal: solve for A & B in terms of C

Computing Edge Equations

- Set up the linear system:

$$\begin{bmatrix} x_0 & y_0 \\ x_1 & y_1 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = -C \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

- Multiply both sides by matrix inverse:

$$\begin{bmatrix} A \\ B \end{bmatrix} = \frac{-C}{x_0 y_1 - x_1 y_0} \begin{bmatrix} y_1 - y_0 \\ x_1 - x_0 \end{bmatrix}$$

- Let $\boldsymbol{C = x_0 y_1 - x_1 y_0}$ for convenience
  - Then $\boldsymbol{A = y_0 - y_1}$ and $\boldsymbol{B = x_0 - x_1}$

Edge Equations

- So…we can find edge equation from two verts.
- Given three corners $\mathbf{P_0, P_1, P_2}$ of a triangle, what are our three edges?
- *How do we make sure the half-spaces defined by the edge equations all share the same sign on the interior of the triangle?*
- A: Be consistent (Ex: $[\mathbf{P_0 P_1}], [\mathbf{P_1 P_2}], [\mathbf{P_2 P_0}]$)
- *How do we make sure that sign is positive?*
- A: Test, and flip if needed ($A = -A$, $B = -B$, $C = -C$)

Edge Equations: Code

- Basic structure of code:
  - Setup: compute edge equations, bounding box
  - (Outer loop) For each scanline in bounding box...
  - (Inner loop) …check each pixel on scanline, evaluating edge equations and drawing the pixel if all three are positive

Triangle Rasterization Issues

- *Exactly which pixels should be lit?*
- A: Those pixels inside the triangle edges
- *What about pixels exactly on the edge?*
  - Draw them: order of triangles matters (it shouldn't)
  - Don't draw them: gaps possible between triangles
- We need a consistent (if arbitrary) rule
  - Example: draw pixels on left or top edge, but not on right or bottom edge

Polygon filling

- Simplest method to fill a polygonal area is to test every pixel in the raster to see if it lies inside the polygon.
- There are two methods to make an inside check
  - even-odd test
  - winding number test
- Bounding boxes can be used to improve performance

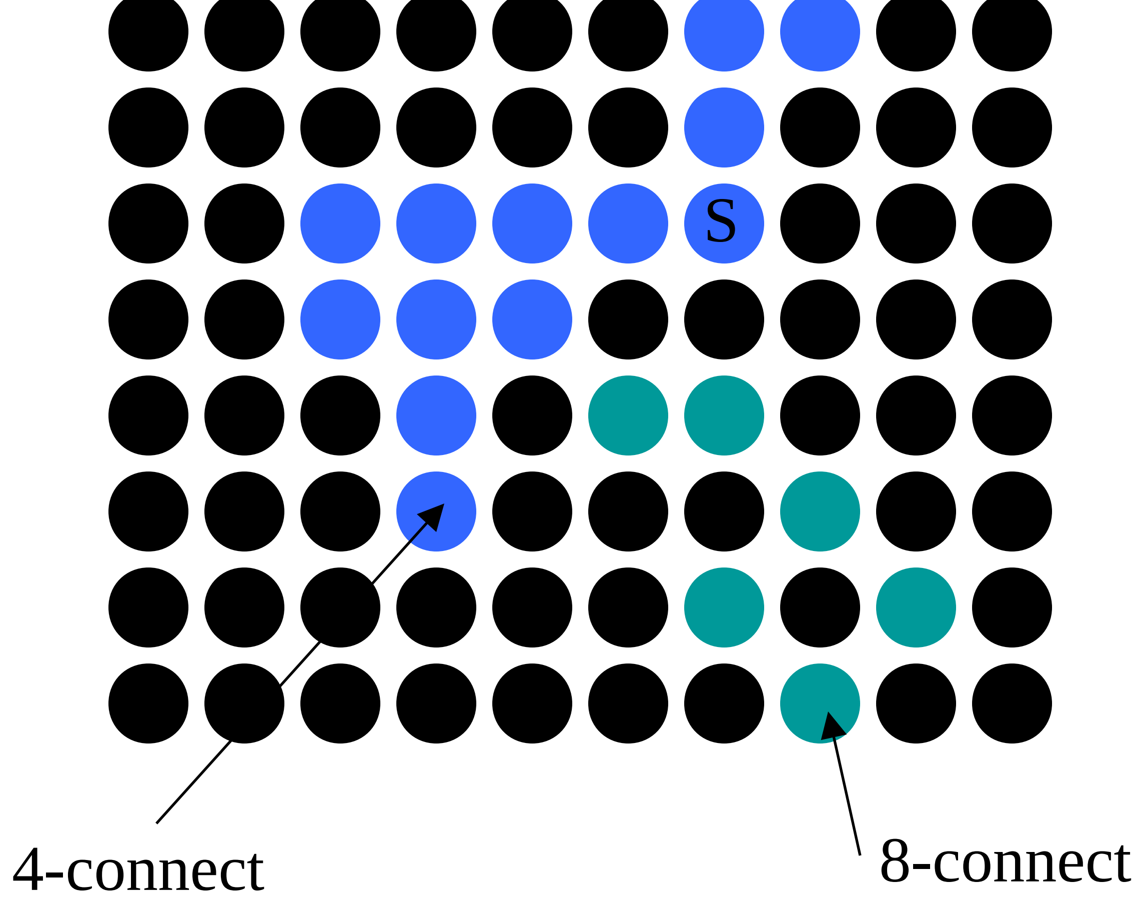
Filling Regions

- Color all pixels in a given region
- Region =
  - All pixels of a certain color (pixel-defined regions)
  - All pixels within a distance of another pixel'
  - All pixels within some given polygon (polygon-defined region)

Filling Pixel-defined regions

- *Region R is the set of all pixels having color C that are “connected” to a given pixel S*
- Connected = there is path of adjacent pixels
- Adjacent
  - 4-adjacent
  - 8-adjacent

Filling Pixel-defined regions



Seed Fill Algorithm

- Basic idea
  - Start at a pixel interior to a polygon

seed

  - Fill the others using connectivity

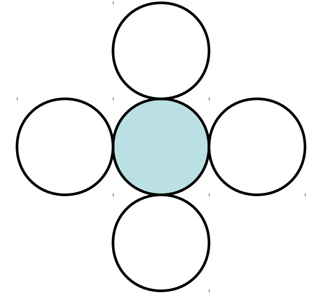
Demo

Seed Fill Algorithms

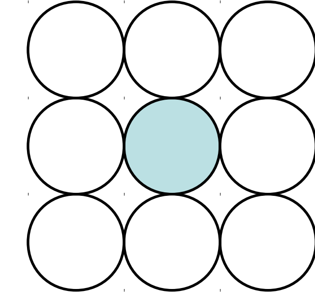
- Assumes that atleast one pixel interior to the polygon is known
- It is a recursive algorithm
- Useful in interactive paint packages

Seed *4-connected* *8 - connected*

Seed Fill Algorithm (Cont')



4-connected

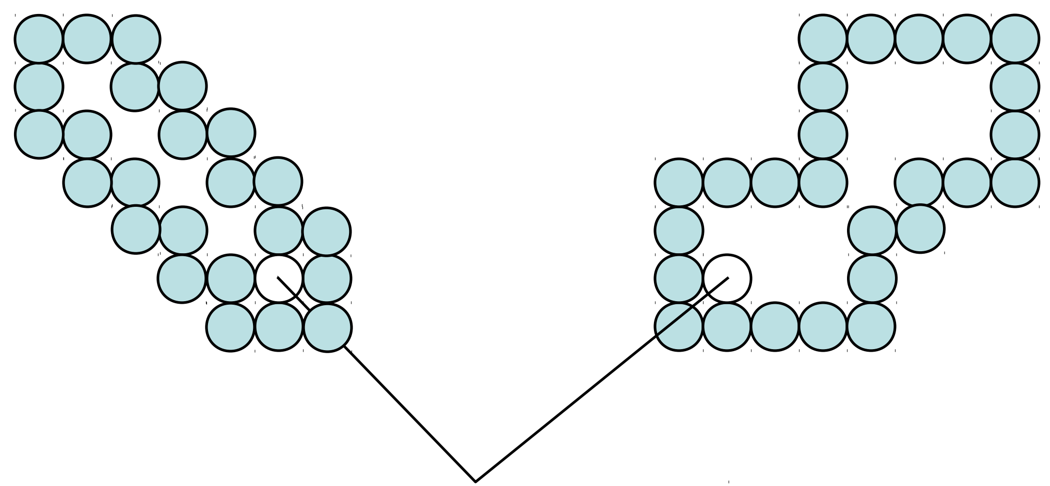


8-connected

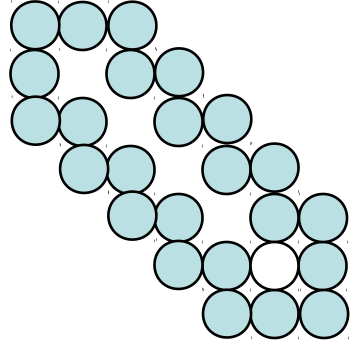
Need a stack.

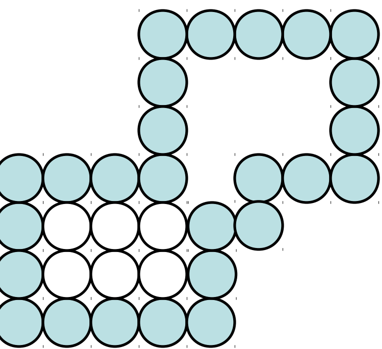
Why?

Seed Fill Algorithm (Cont’)

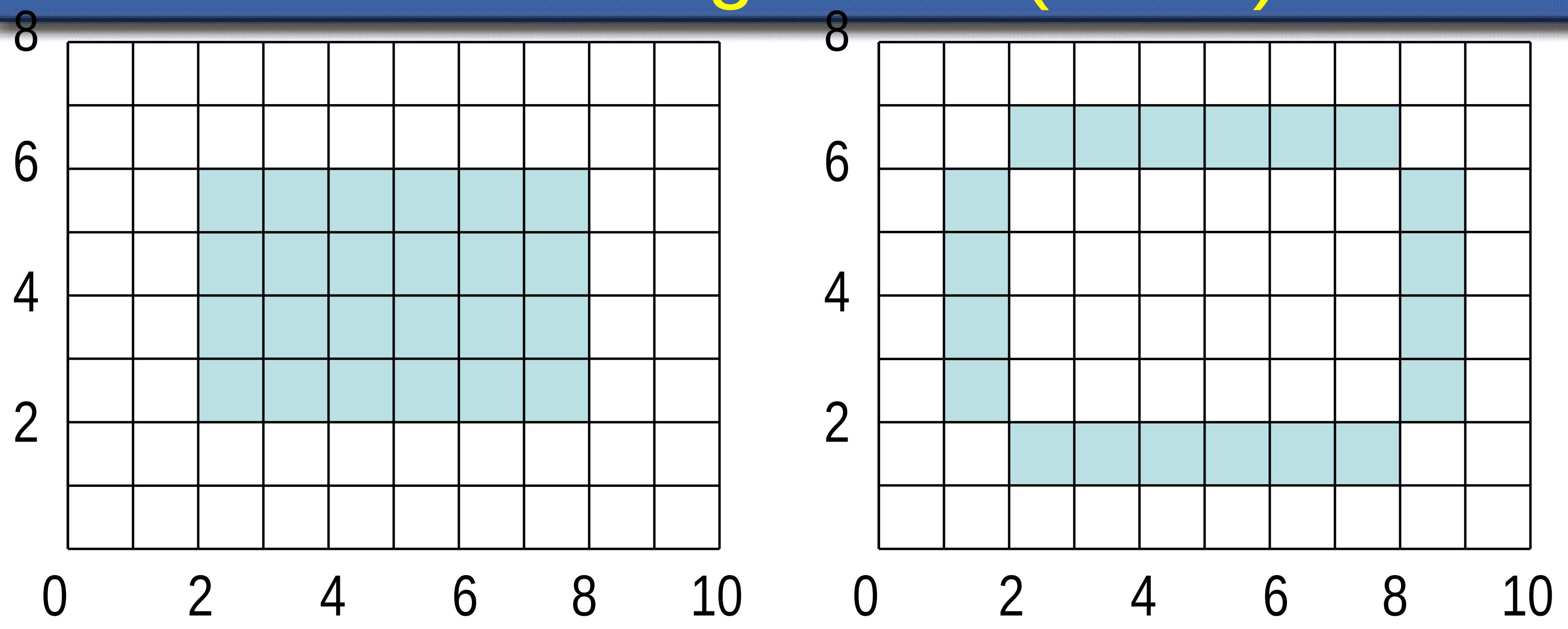


Start Position





Seed Fill Algorithm (Cont’)



Interior-defined

↓

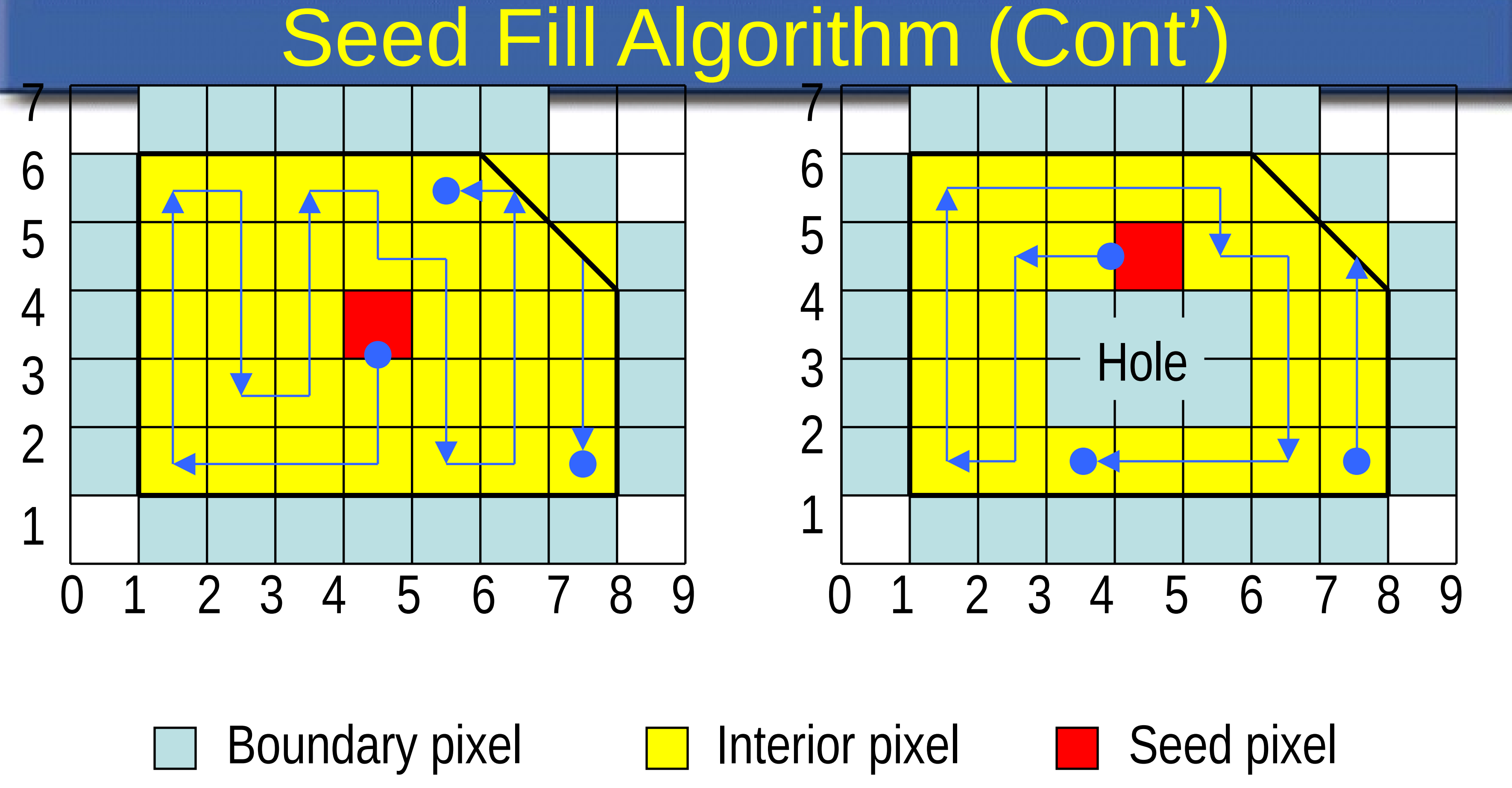
flood fill algorithm

boundary-defined

↓

boundary fill algorithm

Seed Fill Algorithm (Cont’)



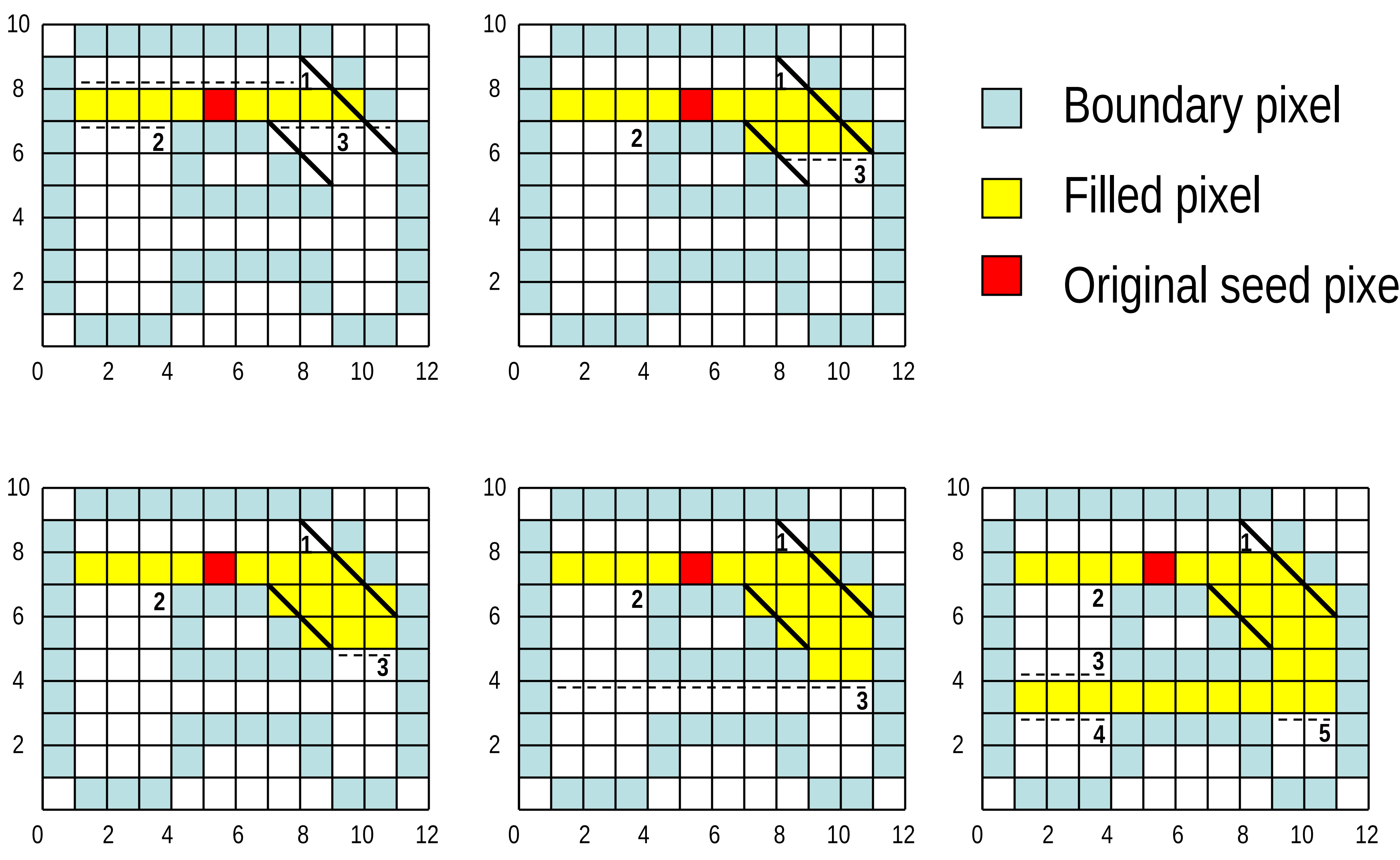
The stack may contain duplicated or unnecessary information !!!

Scan Line Seed Fill

Scan Line conversion + Seed filling

Shani, U., “Filling Regions in Binary Raster Images: A Graph-Theoretic Approach”, Computer Graphics, 14, (1981), 321-327

Scan Line Seed Fill (Cont’)



Filling Pixel-defined regions

- Recursive flood-fill
  - If a pixel is part of the region, switch its color
  - Apply the same procedure to each neighbor
- Neighbor = 4-connect or 8-connect

Filling Pixel-defined regions

```
floodFill (x, y, color) {
    If (getColor(x,y) = color) {
        setColor(x,y, color)
        floodFill(x-1, y, color)
        floodFill(x+1, y, color)
        floodFill(x, y+1, color)
        floodFill(x, y-1, color)
        }
    }
```

Filling Symbolic Regions

- If we have a description of the region (e.g. polygon), filling might be more efficient!
- Scan-line fill of polygon
  - For each scan line, find intersections with the polygon
  - Fill in the spans
  - Go to next scanline

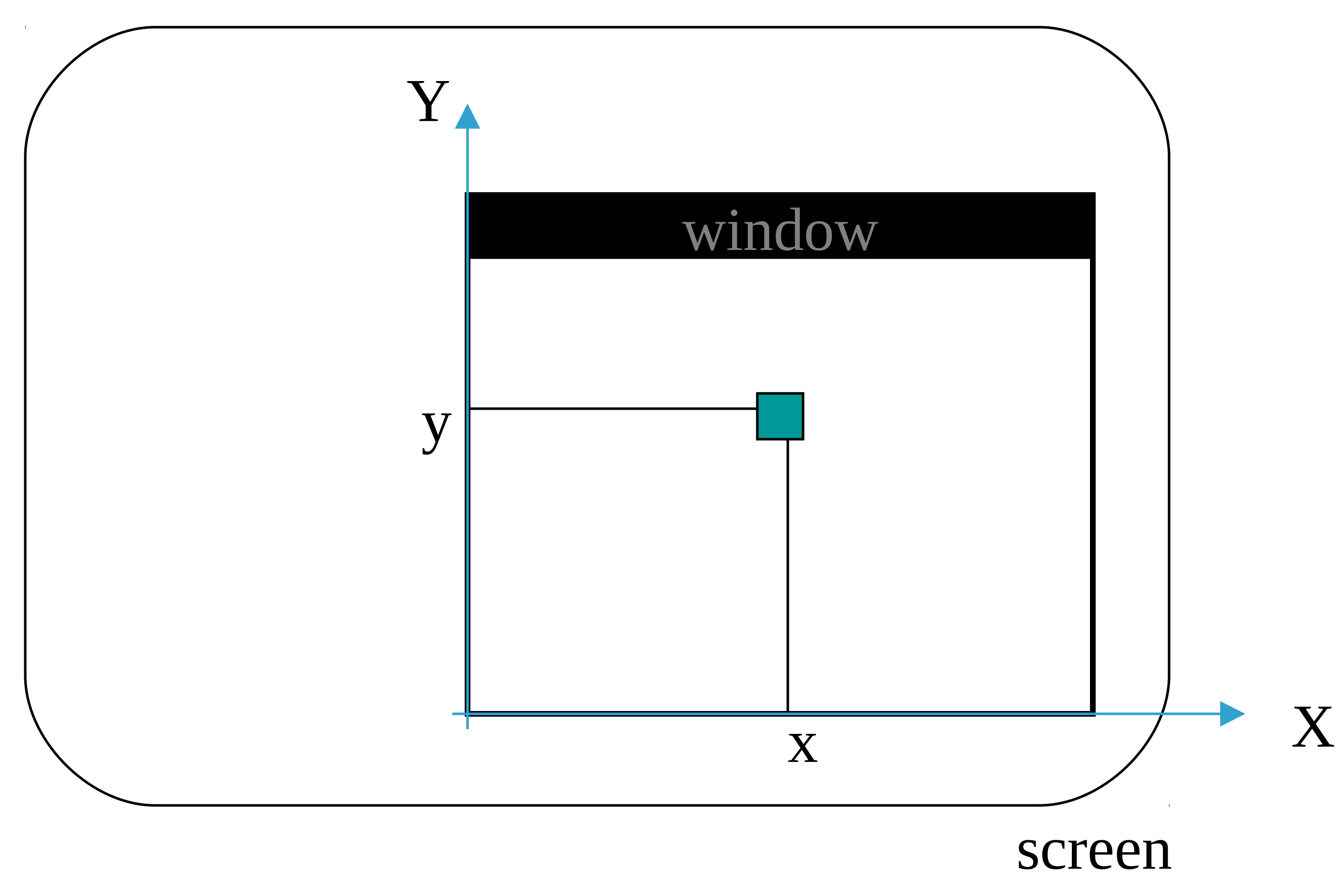
How to draw things?

- Given: window on the screen
- Graphics API (e.g. OpenGL) has something of the form:

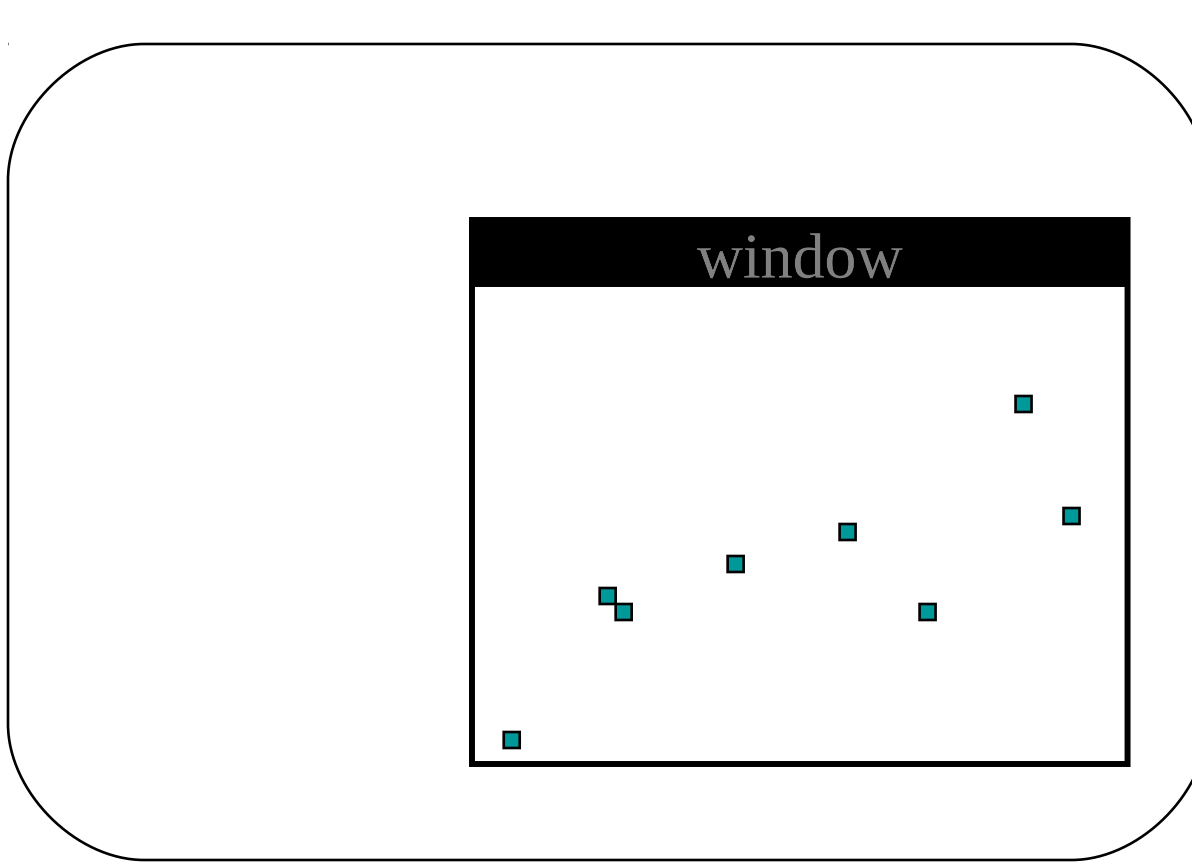
```
plotPixel(int x, int y)
```

How to draw things?

plotPixel(x,y)



How to draw things?



- plotPixel(289,190)
- plotPixel(320,128)
- plotPixel(239,67)
- plotPixel(194,101)
- plotPixel(129,83)
- plotPixel(75,73)
- plotPixel(74,74)
- plotPixel(20,10)

Why is this impractical?

- Coordinates are expressed in *screen space*, but objects live in (3D) world space
- Resizing window implies we have to change coordinates of objects to be drawn

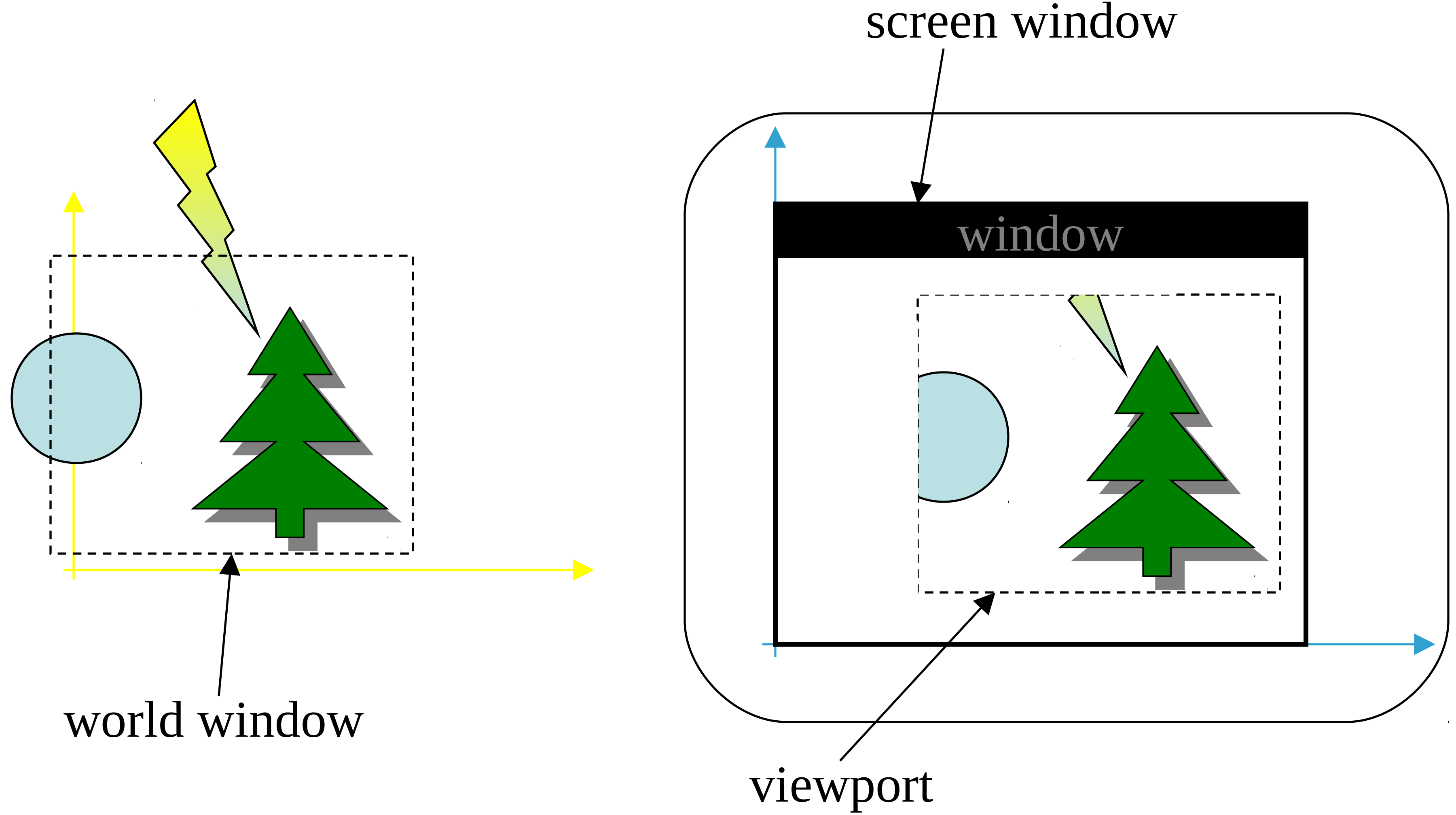
- We want to make a separation between:
  - values to describe geometrical objects
  - values needed to draw these objects on the screen

World window & viewport

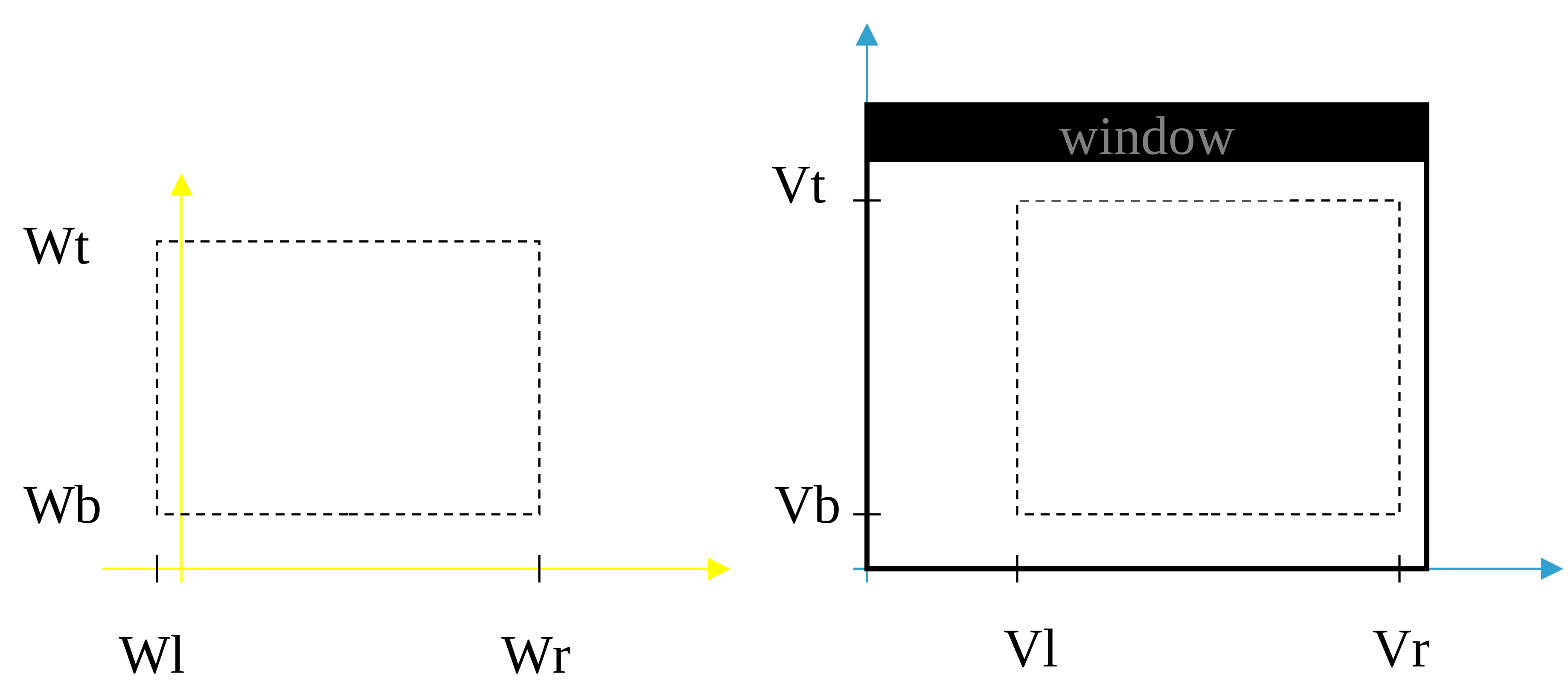
- World window:
  specifies what part of the world should be drawn

- Viewport:
  rectangular area in the screen window in which we will draw

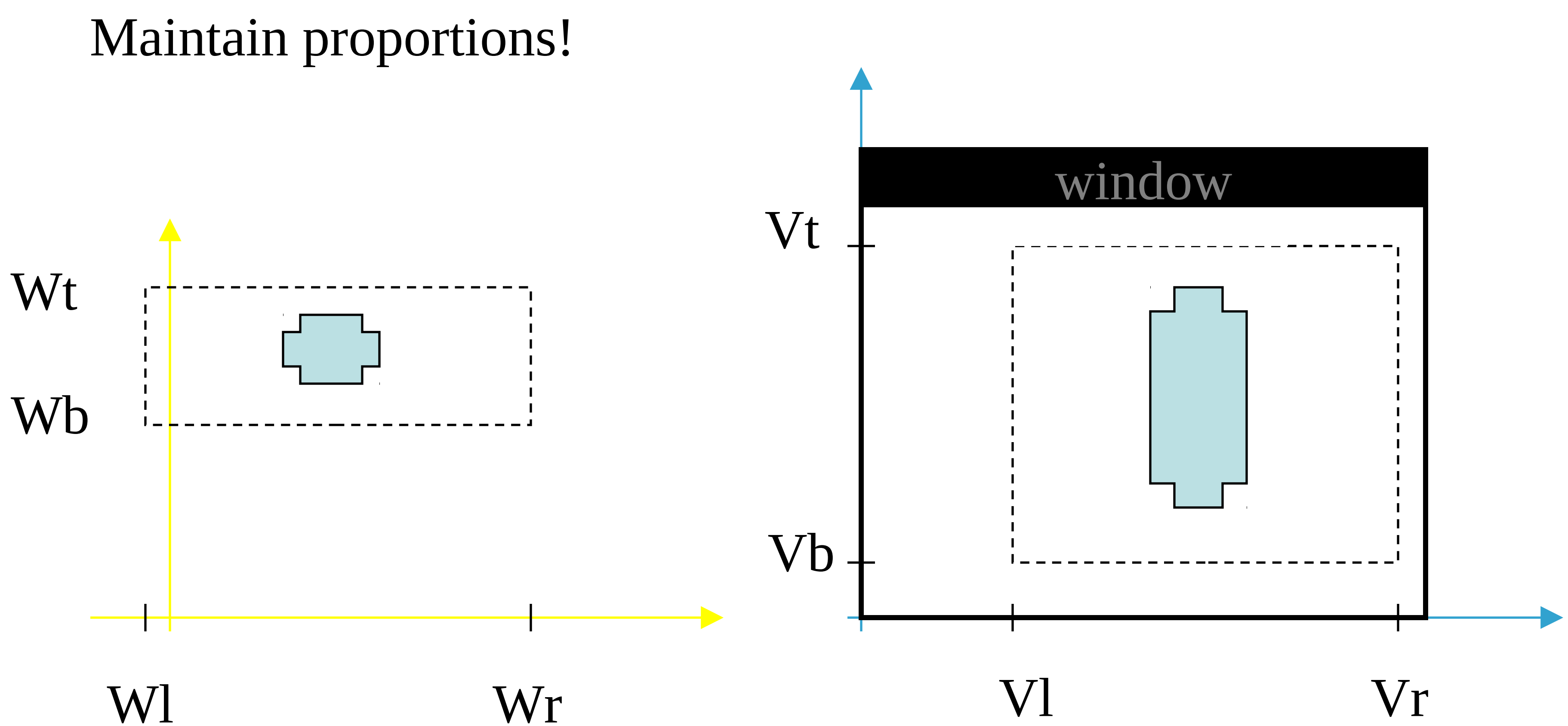
World window & viewport



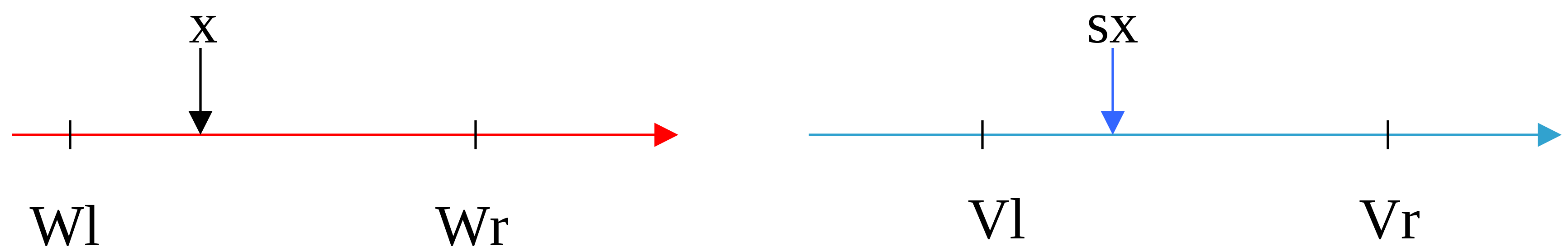
Mapping: world window to viewport



Mapping: world window to viewport



Mapping: world window to viewport



$$sx = \frac{Vr - Vl}{Wr - Wl} x + (Vl + \frac{Vr - Vl}{Wr - Wl} Wl)$$

$$sy = \frac{Vt - Vb}{Wt - Wb} y + (Vb + \frac{Vt - Vb}{Wt - Wb} Wb)$$

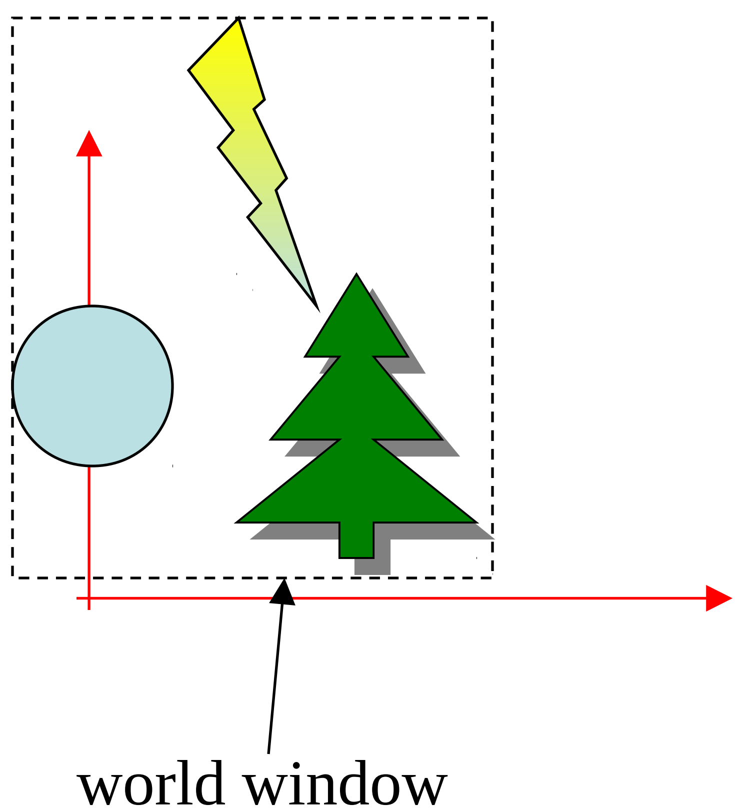
Mapping: world window to viewport

- If x = Wl, then sx = Vl
- If x = Wr, then sx = Vr
- If x = f*(Wr-Wl), then sx = f*(Vr-Vl)
- If x < Wl, then sx < Vl
- If x > Wr, then sx > Vr

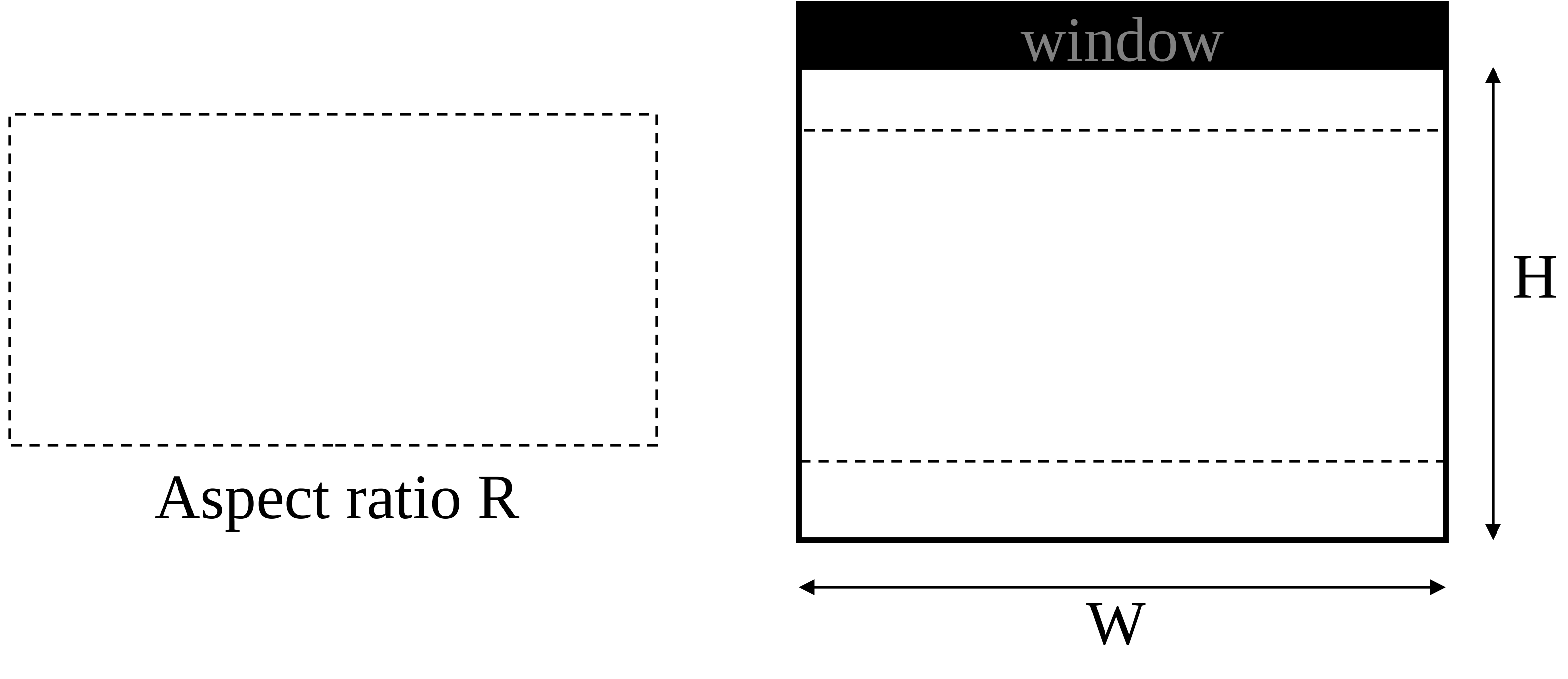
- … also for y and sy

World window

- Pick size automatically

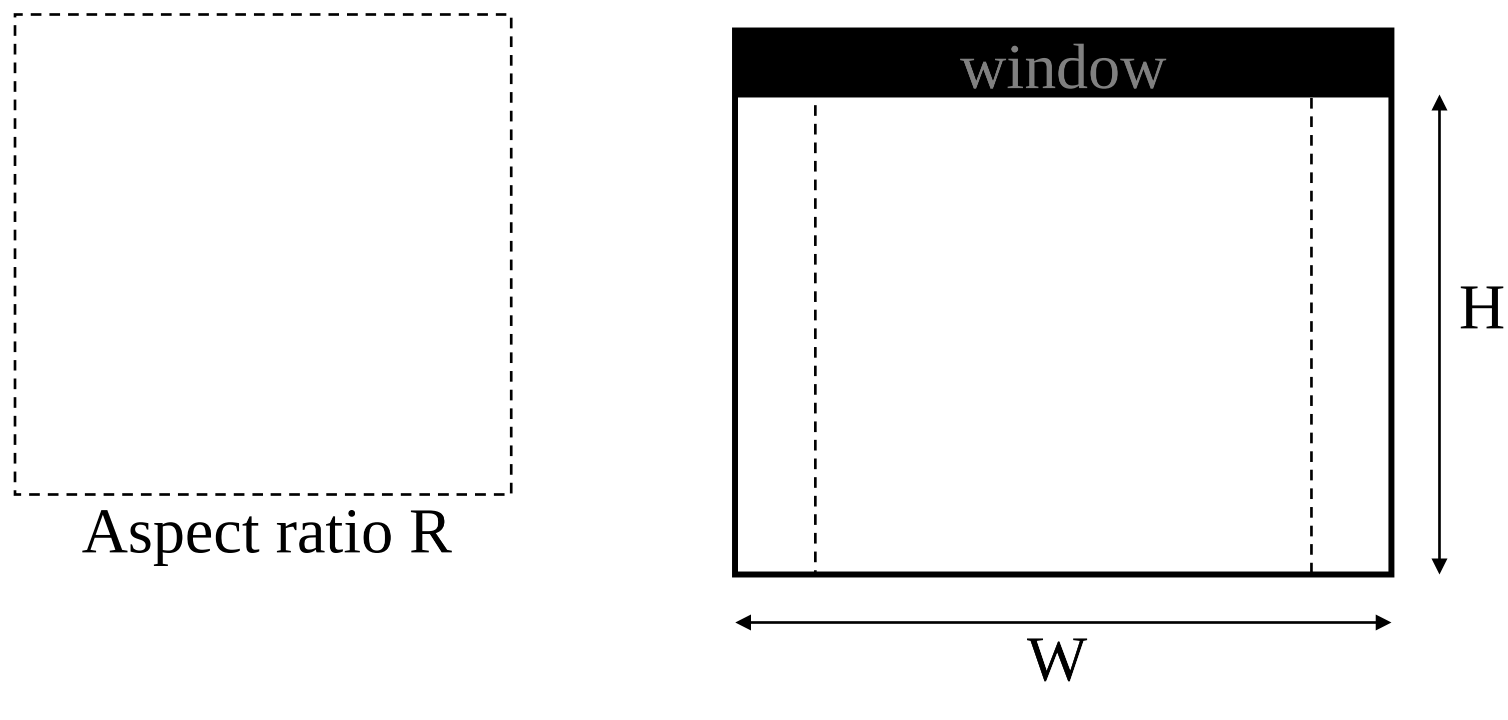


Automatic setting to preserve aspect ratio & center



R > W/H

Automatic setting to preserve aspect ratio & center



R < W/H