GEOMETRIJSKE PRESLIKAVE

2D preslikave

$$I(x, y) \to I'(x', y')$$

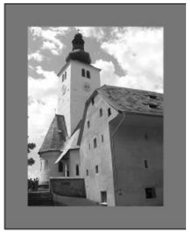
$$T : \mathbb{R}^2 \to \mathbb{R}^2$$

$$\boldsymbol{x} = (x, y) \qquad \boldsymbol{x}' = (x', y')$$

$$\boldsymbol{x}' = T(\boldsymbol{x})$$

$$x' = T_x(x, y) \qquad y' = T_y(x, y)$$

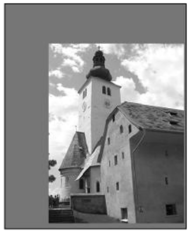
2D preslikave



Premik

$$T_x : x' = x + d_x$$
$$T_y : y' = y + d_y$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} d_x \\ d_y \end{pmatrix}$$



Povečava

$$T_x : x' = s_x \cdot x$$
$$T_y : y' = s_y \cdot y$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

Strižna deformacija

$$T_x : x' = x + b_x \cdot y$$
$$T_y : y' = y + b_y \cdot x$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & b_x \\ b_y & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

Rotacija

$$T_x : x' = x \cdot \cos\alpha - y \cdot \sin\alpha$$
$$T_y : y' = x \cdot \sin\alpha + y \cdot \cos\alpha$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

Homogene koordinate

$$\boldsymbol{x} = \begin{pmatrix} x \\ y \end{pmatrix} \qquad \hat{\boldsymbol{x}} = \begin{pmatrix} \hat{x} \\ \hat{y} \\ h \end{pmatrix} = \begin{pmatrix} h\,x \\ h\,y \\ h \end{pmatrix}$$

$$x = \frac{\hat{x}}{h} \qquad y = \frac{\hat{y}}{h}$$

$$\hat{\boldsymbol{x}}_1 = s \cdot \hat{\boldsymbol{x}}_2 \qquad \boldsymbol{x}_1 = \boldsymbol{x}_2 = \boldsymbol{x}$$

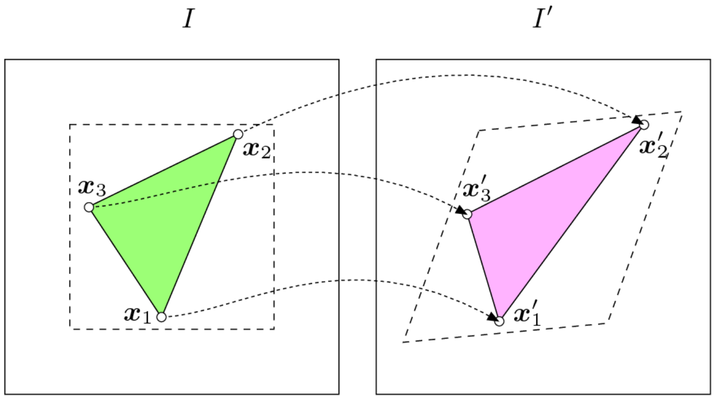
Premik

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} x+d_x \\ y+d_y \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Afina preslikava

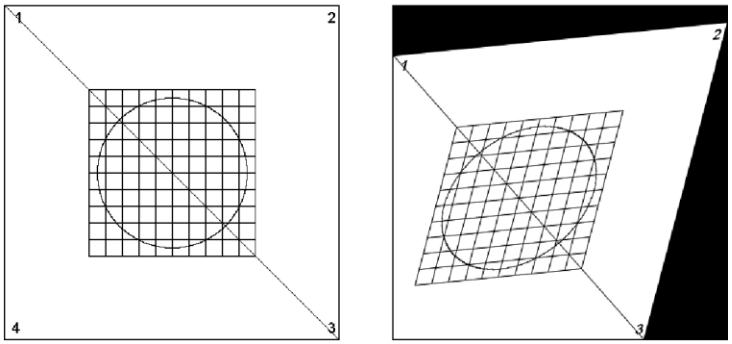
premik + povečava + rotacija

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$



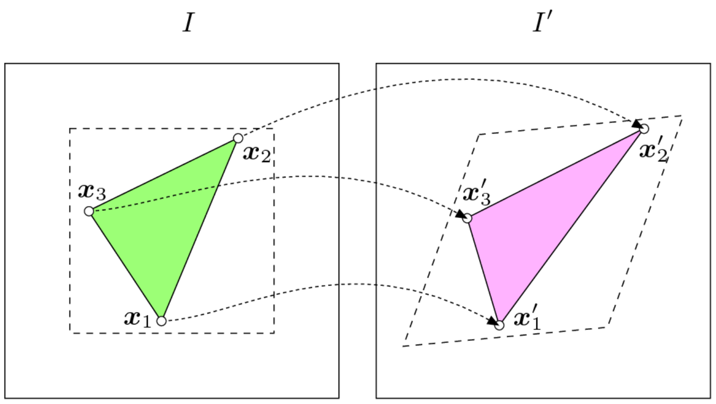
PRESLIKA
premice v premice
pravokotnike v paralelogame

OHRANJA
vzporednost
relativne razdalje na premici



Afina preslikava

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$



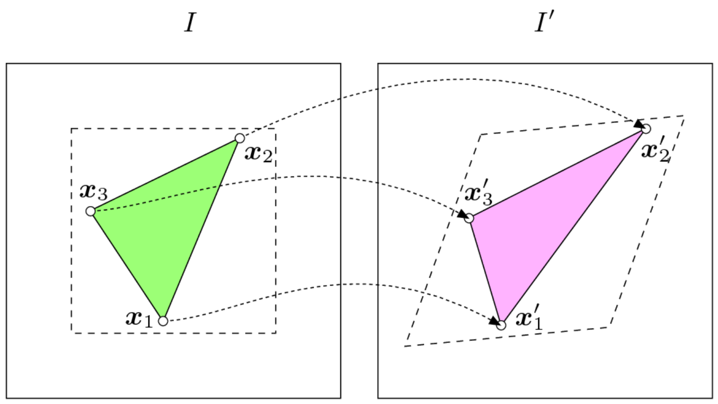
6 parametrov popolnoma definirajo 3 pari točk

$$x'_1 = a_{11} \cdot x_1 + a_{12} \cdot y_1 + a_{13} \qquad y'_1 = a_{21} \cdot x_1 + a_{22} \cdot y_1 + a_{23}$$
$$x'_2 = a_{11} \cdot x_2 + a_{12} \cdot y_2 + a_{13} \qquad y'_2 = a_{21} \cdot x_2 + a_{22} \cdot y_2 + a_{23}$$
$$x'_3 = a_{11} \cdot x_3 + a_{12} \cdot y_3 + a_{13} \qquad y'_3 = a_{21} \cdot x_3 + a_{22} \cdot y_3 + a_{23}$$

$$a_{11} = \frac{1}{d} \cdot \left[y_1(x'_2 - x'_3) + y_2(x'_3 - x'_1) + y_3(x'_1 - x'_2) \right]$$
$$a_{12} = \frac{1}{d} \cdot \left[x_1(x'_3 - x'_2) + x_2(x'_1 - x'_3) + x_3(x'_2 - x'_1) \right]$$
$$a_{21} = \frac{1}{d} \cdot \left[y_1(y'_2 - y'_3) + y_2(y'_3 - y'_1) + y_3(y'_1 - y'_2) \right]$$
$$a_{22} = \frac{1}{d} \cdot \left[x_1(y'_3 - y'_2) + x_2(y'_1 - y'_3) + x_3(y'_2 - y'_1) \right]$$
$$a_{13} = \frac{1}{d} \cdot \left[x_1(y_3 x'_2 - y_2 x'_3) + x_2(y_1 x'_3 - y_3 x'_1) + x_3(y_2 x'_1 - y_1 x'_2) \right]$$
$$a_{23} = \frac{1}{d} \cdot \left[x_1(y_3 y'_2 - y_2 y'_3) + x_2(y_1 y'_3 - y_3 y'_1) + x_3(y_2 y'_1 - y_1 y'_2) \right]$$

$$d = x_1(y_3 - y_2) + x_2(y_1 - y_3) + x_3(y_2 - y_1)$$

Inverzna afina preslikava

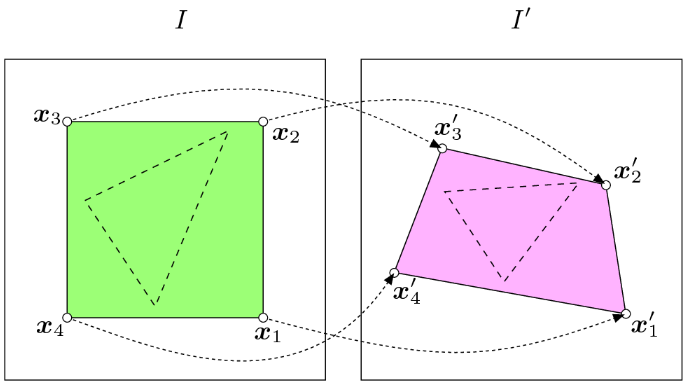


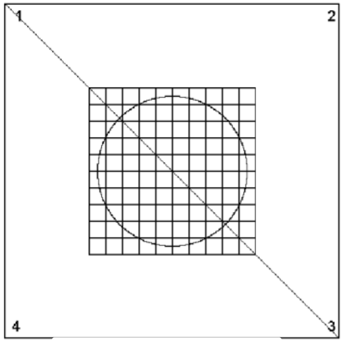
$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}$$

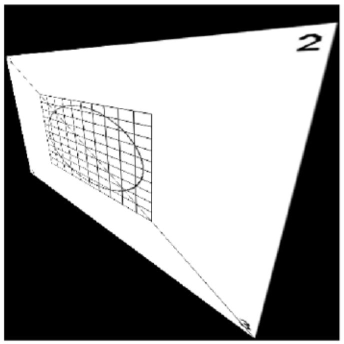
$$= \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{pmatrix} a_{22} & -a_{12} & a_{12}a_{23} - a_{13}a_{22} \\ -a_{21} & a_{11} & a_{13}a_{21} - a_{11}a_{23} \\ 0 & 0 & a_{11}a_{22} - a_{12}a_{21} \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}$$

Projektivna preslikava

$$\begin{pmatrix}\hat{x}'\\ \hat{y}'\\ h'\end{pmatrix} = \begin{pmatrix}h'x'\\ h'y'\\ h'\end{pmatrix} = \begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & 1\end{pmatrix} \cdot \begin{pmatrix}x\\ y\\ 1\end{pmatrix}$$



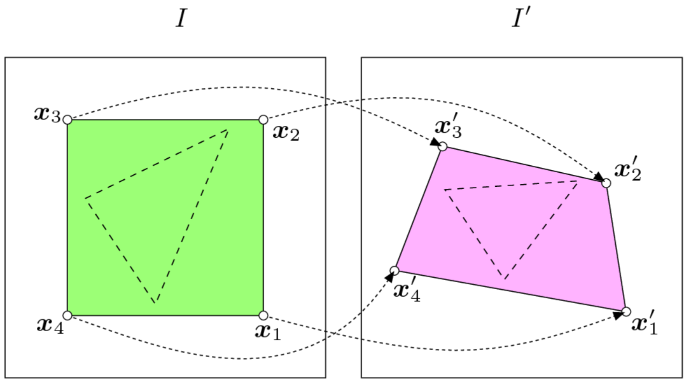




PRESLIKA
premice v premice
funkcije N-tega reda v funkcije N-tega reda

ne ohranja vzporednosti in relativnih razdalj

Iskanje homografije



Štirje pari ujemanj (**x**, **x'**)

$$\mathbf{x}'_i = \mathbf{H}\mathbf{x}_i$$

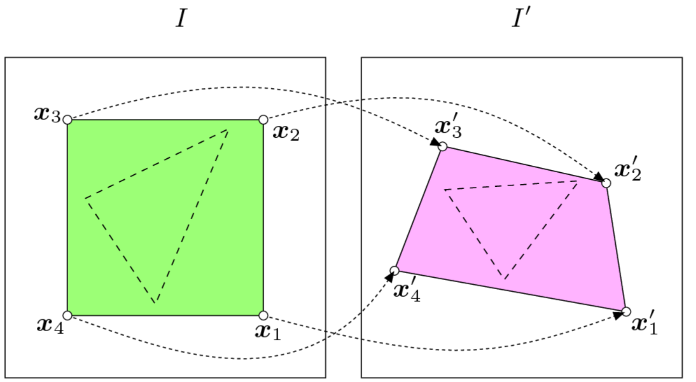
$$\mathbf{x}'_i \times \mathbf{H}\mathbf{x}_i = 0 \qquad \mathbf{H}\mathbf{x_i} = \begin{bmatrix} \mathbf{h}^{1^T}\mathbf{x_i} \\ \mathbf{h}^{2^T}\mathbf{x_i} \\ \mathbf{h}^{3^T}\mathbf{x_i} \end{bmatrix}$$

$$\mathbf{x}'_i \times \mathbf{H}\mathbf{x_i} = \begin{bmatrix} y'_i\mathbf{h}^{3^T}\mathbf{x_i} - w'_i\mathbf{h}^{2^T}\mathbf{x_i} \\ w'_i\mathbf{h}^{1^T}\mathbf{x_i} - x'_i\mathbf{h}^{3^T}\mathbf{x_i} \\ x'_i\mathbf{h}^{2^T}\mathbf{x_i} - y'_i\mathbf{h}^{1^T}\mathbf{x_i} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{0}^T & -w'_i\mathbf{x_i}^T & y'_i\mathbf{x_i}^T \\ w'_i\mathbf{x_i}^T & \mathbf{0}^T & -x'_i\mathbf{x_i}^T \\ -y'_i\mathbf{x_i}^T & x'_i\mathbf{x_i}^T & \mathbf{0}^T \end{bmatrix} \begin{bmatrix} \mathbf{h}^1 \\ \mathbf{h}^2 \\ \mathbf{h}^3 \end{bmatrix} = \mathbf{0} \qquad \mathbf{A_i}\mathbf{h} = \mathbf{0}$$

3x9 9x1

Homografija



Štirje pari ujemanj (**x**, **x’**)

$$\begin{bmatrix} \mathbf{0}^T & -w'_i \mathbf{x_i}^T & y'_i \mathbf{x_i}^T \\ w'_i \mathbf{x_i}^T & \mathbf{0}^T & -x'_i \mathbf{x_i}^T \\ -y'_i \mathbf{x_i}^T & x'_i \mathbf{x_i}^T & \mathbf{0}^T \end{bmatrix} \begin{bmatrix} \mathbf{h}^1 \\ \mathbf{h}^2 \\ \mathbf{h}^3 \end{bmatrix} = \mathbf{0}$$

3x9 9x1

$$\mathbf{A_i h} = \mathbf{0}$$

Štirje pari ujemanj (**x**, **x’**)

$$\mathbf{Ah} = \mathbf{0}$$

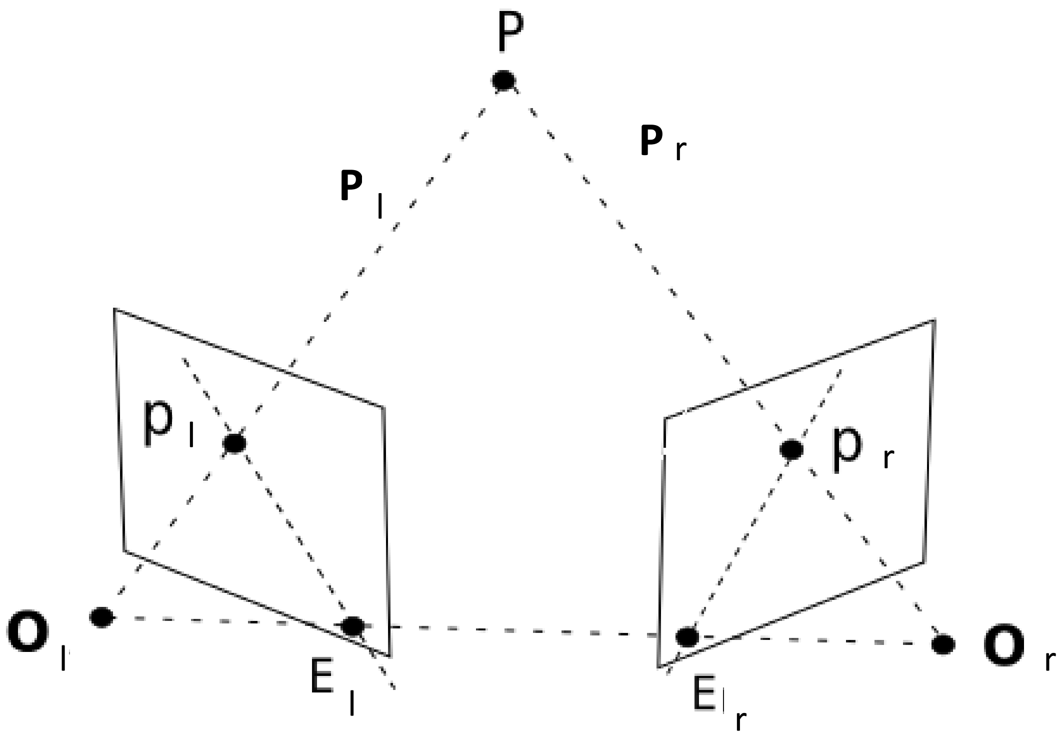
A dimenzije 12x9, rang 8

$$\mathbf{A} = \mathbf{U \Sigma V}^T$$

h je **V** lastni vektor **A** z najmanjšo lastno vrednostjo

KALIBRACIJA

Geometrija dveh pogledov (2-view geometry)



Koordinatna sistema dveh kamer sta povezana s premikom in rotacijo

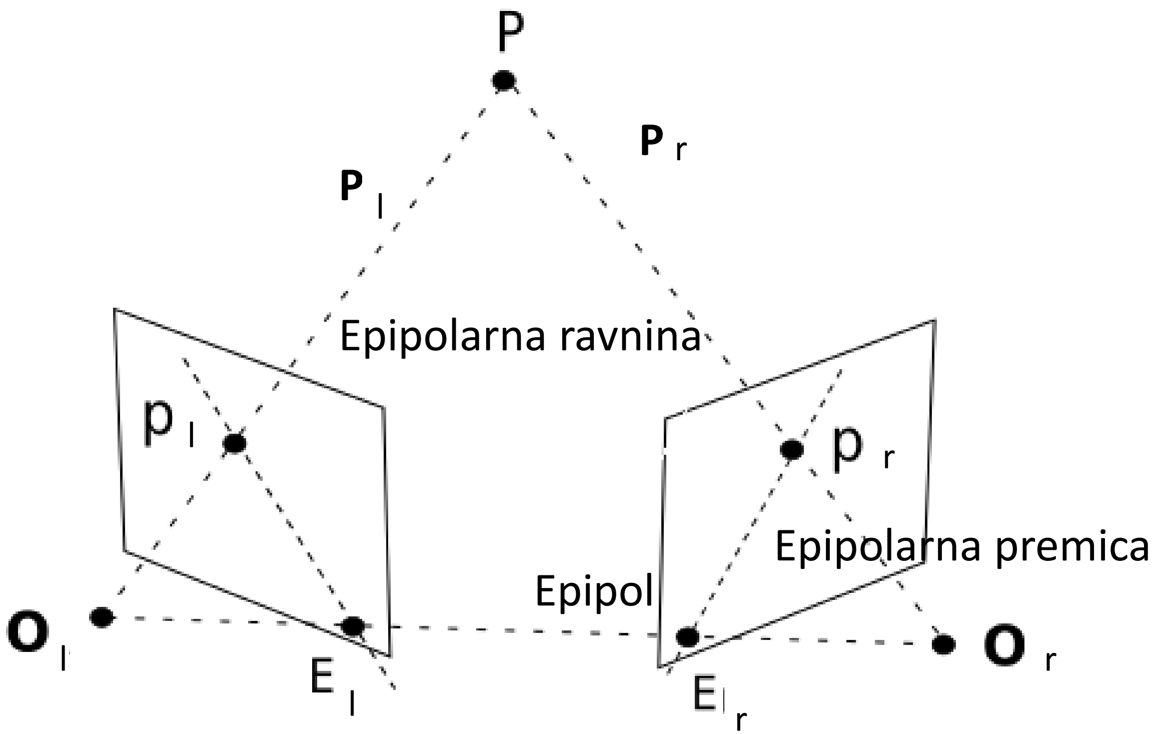
$\mathbf{T} = (O_r - O_l)$

Premik

$\mathbf{P_r} = R(\mathbf{P_l} - \mathbf{T})$

Rotacija

Epipolarna geometrija



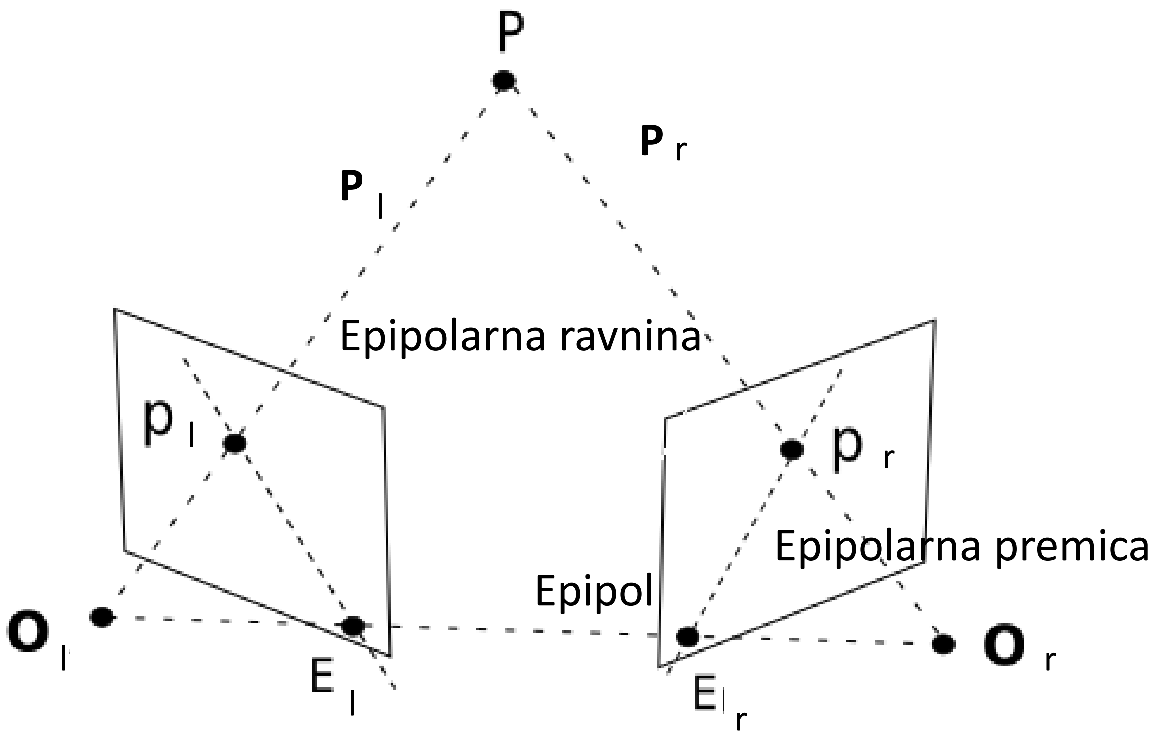
Epipolarna geometrija omejuje možne preslikave točke P

$$\mathbf{T} = (O_r - O_l) \qquad \mathbf{P_r} = R(\mathbf{P_l} - \mathbf{T})$$

Epipolarna ravnina

$$(\mathbf{P_l} - \mathbf{T})^T . \mathbf{T} \times \mathbf{P_l} = 0$$

Kako lahko opišemo medsebojno lego kamer?

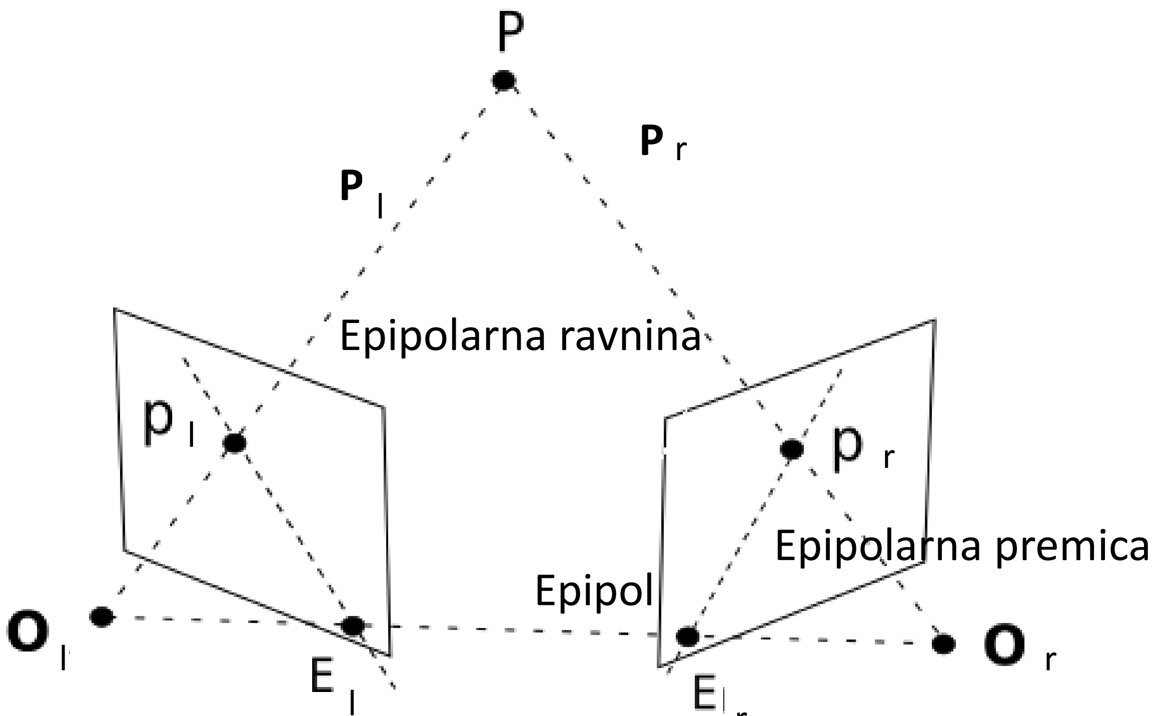


$$(\mathbf{P_l} - \mathbf{T})^T . \mathbf{T} \times \mathbf{P_l} = 0$$

$$\mathbf{P_r} = R(\mathbf{P_l} - \mathbf{T})$$

$$(R^T \mathbf{P_r})^T \mathbf{T} \times \mathbf{P_l} = 0$$

Osnovna matrika (essential matrix) E



$$(R^T \mathbf{P_r})^T \mathbf{T} \times \mathbf{P_l} = 0$$

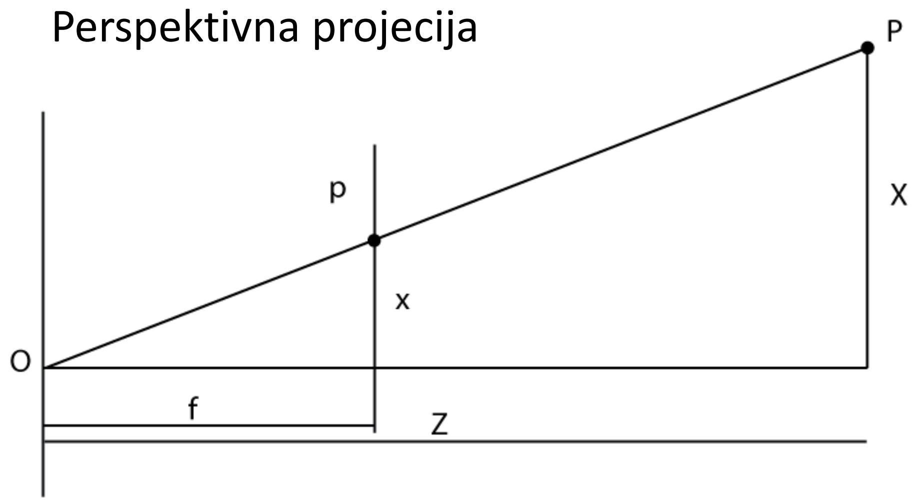
$$\mathbf{T} \times \mathbf{P_l} = S\,\mathbf{P_l}$$

$$\mathbf{P_r}^T E \mathbf{P_l} = 0$$

$$E = RS$$

$$S = \begin{pmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{pmatrix}$$

Osnovna matrika in koordinate slikovnih elementov



$x = f.X/Z$

$y = f.Y/Z$

$$\mathbf{p_l} = (f_l.\ \mathbf{P_l})/Z_l$$

$$\mathbf{p_r} = (f_r.\ \mathbf{P_r})/Z_r$$

$$\mathbf{p_r}^T E \mathbf{p_l} = 0$$

Notranji parametri kamere

$$\mathbf{p_r}^T E \mathbf{p_l} = 0$$

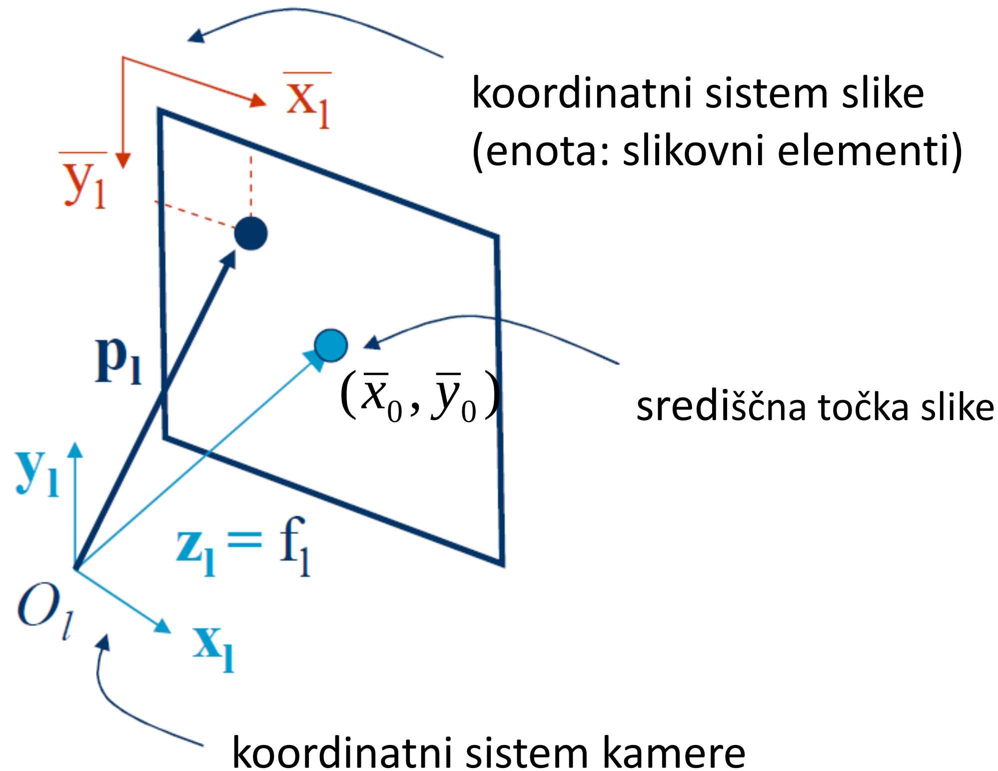
E določa zunanja razmerja dveh kamer

Notranji parametri:

m_x
s.e. na enoto
m_y

$$\bar{x}_0 = m_x p_x$$

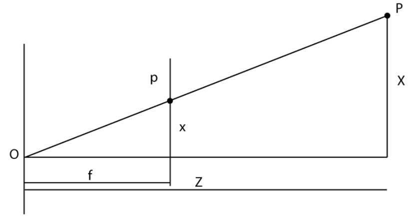
$$\bar{y}_0 = m_y p_y$$



$$M = \begin{bmatrix} f_l m_x & s & \bar{x}_0 \\ 0 & f_l m_y & \bar{y}_0 \\ 0 & 0 & 1 \end{bmatrix}$$

Končni projektivni model kamere (camera matrix)

$$M = \begin{bmatrix} f_l m_x & s & \bar{x}_0 \\ 0 & f_l m_y & \bar{y}_0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$MR[I \mid -\mathbf{C}]$$

$$\mathbf{p} = MR[I \mid -\mathbf{C}]\mathbf{P}$$

Zunanji parametri

11 prostostnih stopenj

Fundamentalna matrika

$$F = M_r^{-T} E M_l^{-1}$$

$$\mathbf{p_l} = M_l^{-1} \, \bar{\mathbf{p}}_\mathbf{l} \qquad \mathbf{p_r} = M_r^{-1} \, \bar{\mathbf{p}}_\mathbf{r}$$

$$\bar{\mathbf{p}}_\mathbf{r}^{\mathrm{T}} F \, \bar{\mathbf{p}}_\mathbf{l} = 0$$

Fundamentalna matrika povezuje **slikovne koordinate** dveh kamer
9 prostostnijh stopenj

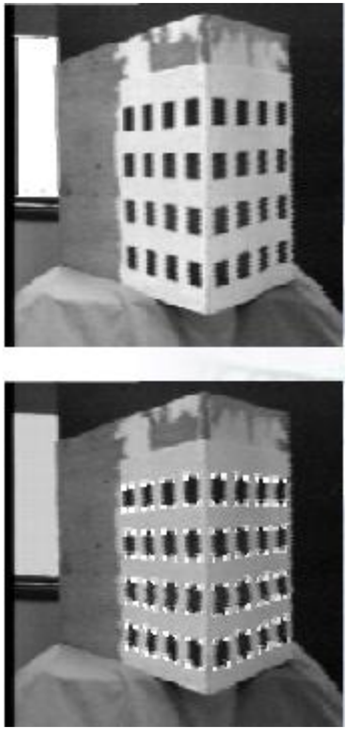
E **opisuje zunanje (extrinsic) parametre kamer**
F **opisuje zunanje in notranje (intrinsic) parametre**

Kalibracija

a) izračunamo M_l, M_r in E

ali

b) izračunamo F (neposredna kalibracija)



Neposredna kalibracija

$$\bar{\mathbf{p}}_{\mathbf{r}}^{T} F \, \bar{\mathbf{p}}_{\mathbf{l}} = 0$$

Potrebujemo vsaj 8 (linearno neodvisnih) ujemanj (natančnost do merila)

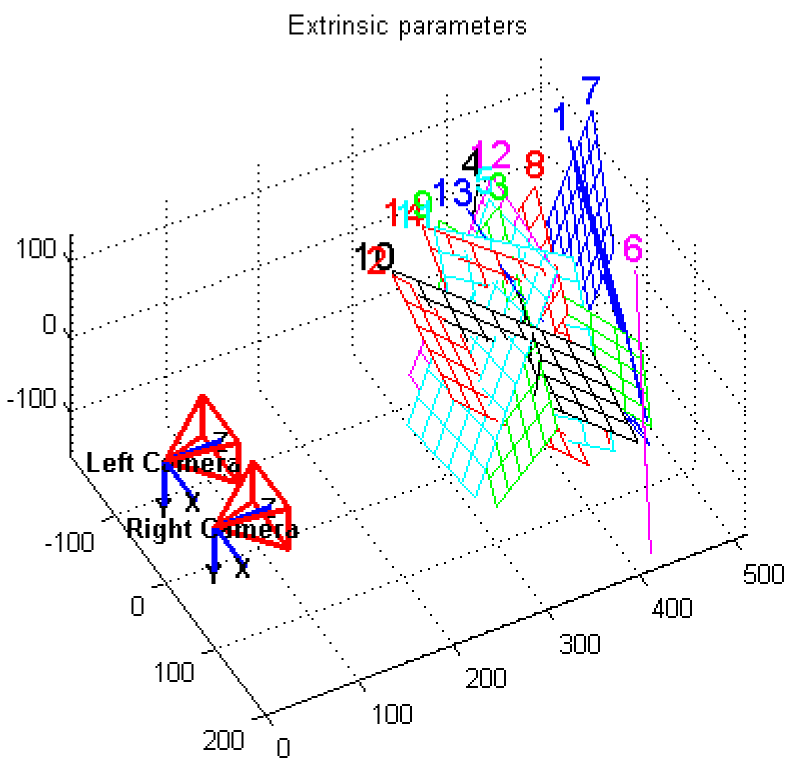
Radialna in tangentna deformacija

$$x_d = \begin{bmatrix} x_d(1) \\ x_d(2) \end{bmatrix} = \left(1 + kc(1)\,r^2 + kc(2)\,r^4 + kc(5)\,r^6\right) x_n + dx$$

Tangentna

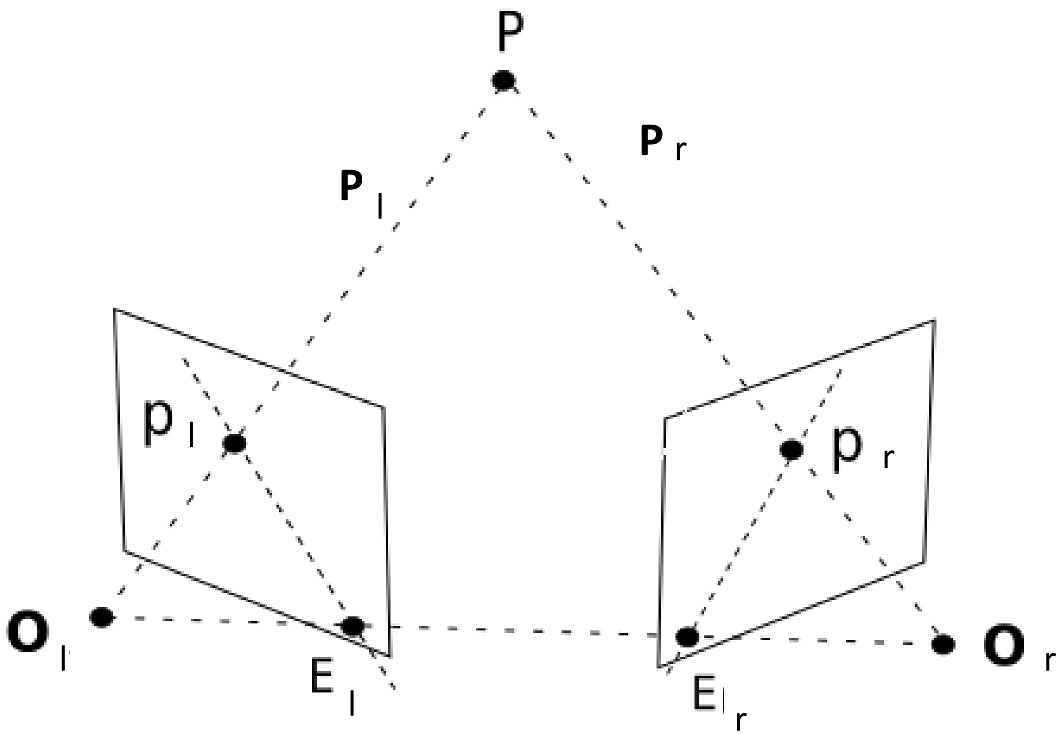
$$dx = \begin{bmatrix} 2\,kc(3)\,x\,y + kc(4)\left(r^2 + 2x^2\right) \\ kc(3)\left(r^2 + 2y^2\right) + 2\,kc(4)\,x\,y \end{bmatrix}$$

MATLAB Calibration Toolbox



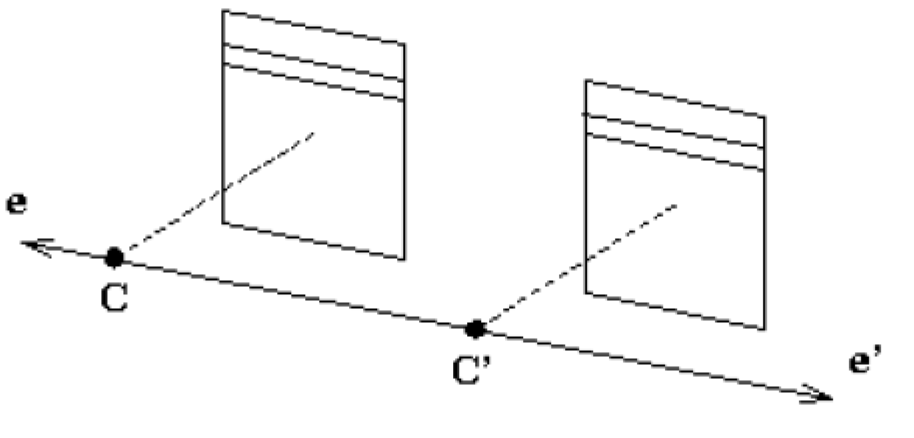
STEREO VID

Epipolarna geometrija stereo sistema

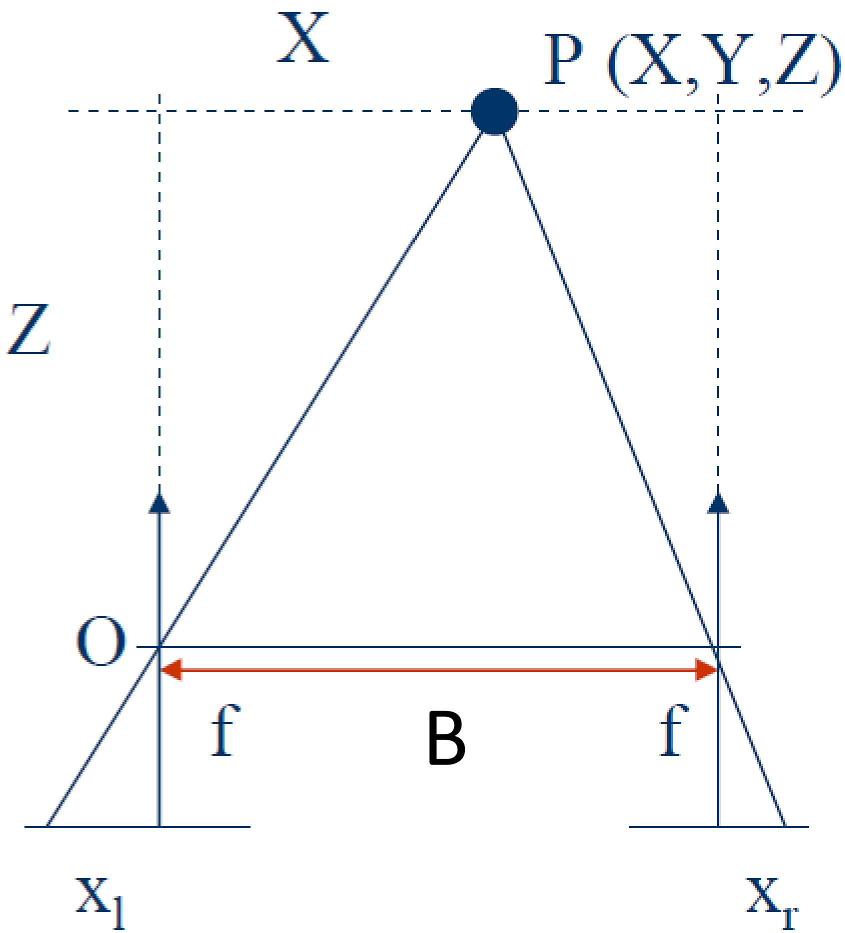


http://www.ai.sri.com/~luong/research/Meta3DViewer/EpipolarGeo.html

Epipolarna geometrija stereo sistema – vzporedni ravnini, fl = fr, vodoraven raster



Epipolarna geometrija stereo sistema – vzporedni ravnini, fl = fr, vodoraven raster



$$\frac{x_l}{f} = \frac{X}{Z} \Rightarrow x_l = \frac{fX}{Z}$$

$$\frac{x_r}{f} = \frac{X - B}{Z} \Rightarrow x_r = \frac{f(X - B)}{Z}$$

$$x_r - x_l = \frac{f}{Z}\left(X - (X - B)\right)$$

$$Z = \frac{fB}{x_r - x_l}$$