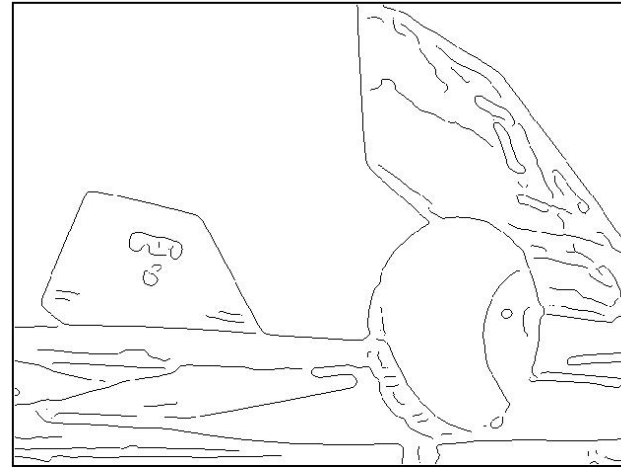


## **Vaje pri predmetu Računalniško zaznavanje 08/09**

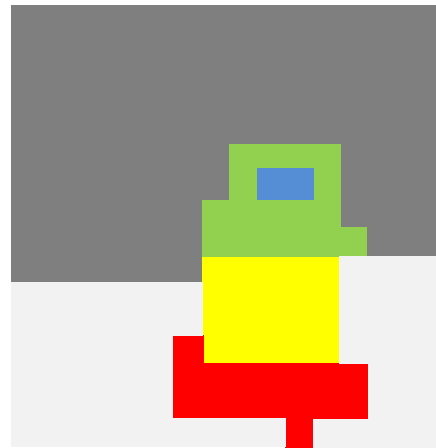
# LOKALNE ZNAČILKE

# Lokalne značilke



Robovi

# Lokalne značilke



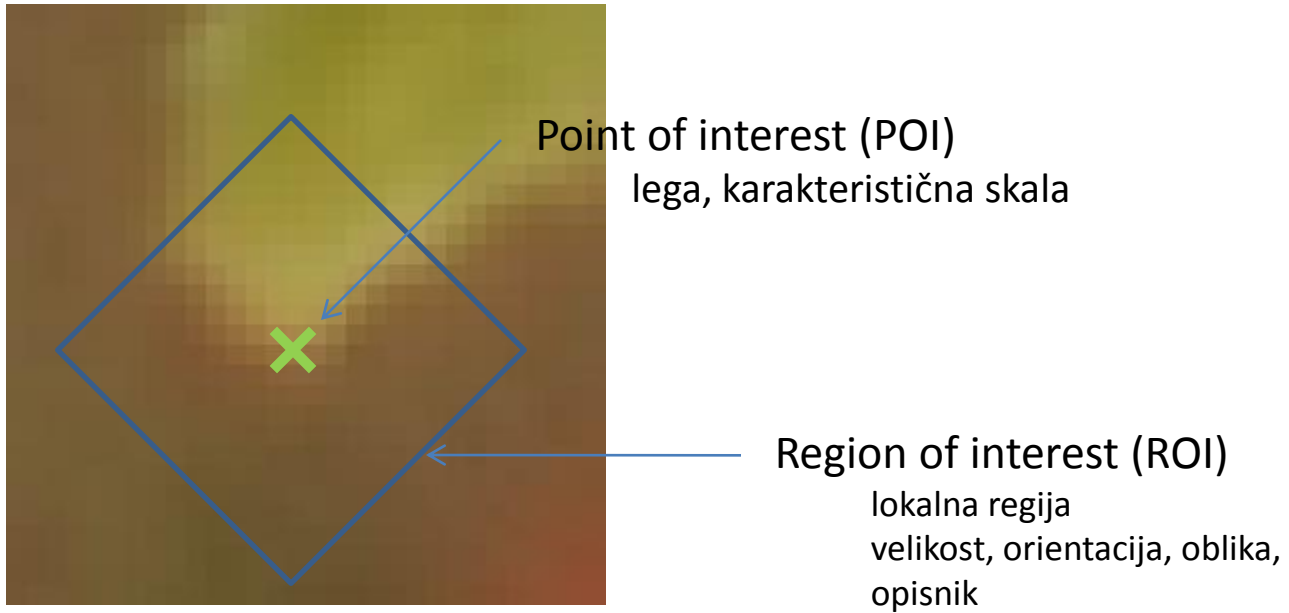
Lokalne regije

# Lokalne značilke



Značilne točke (oglišča, scale space maxima, ...)

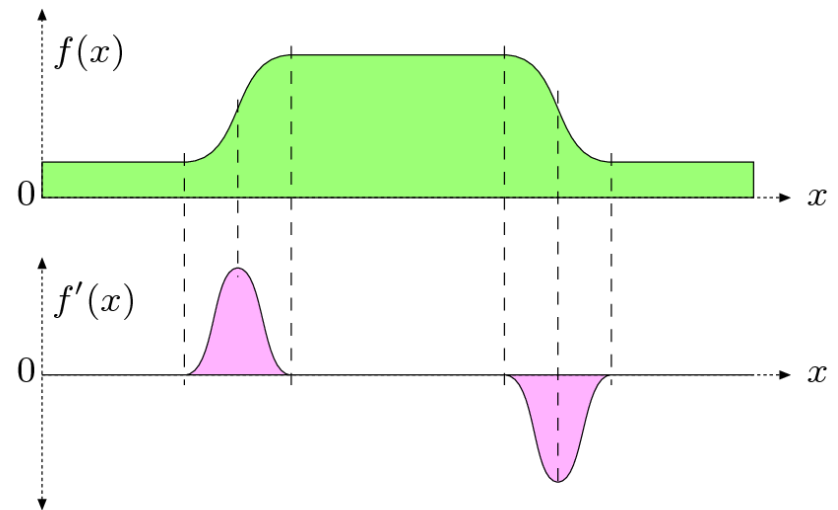
# Lokalne značilke



**ROBOVI**

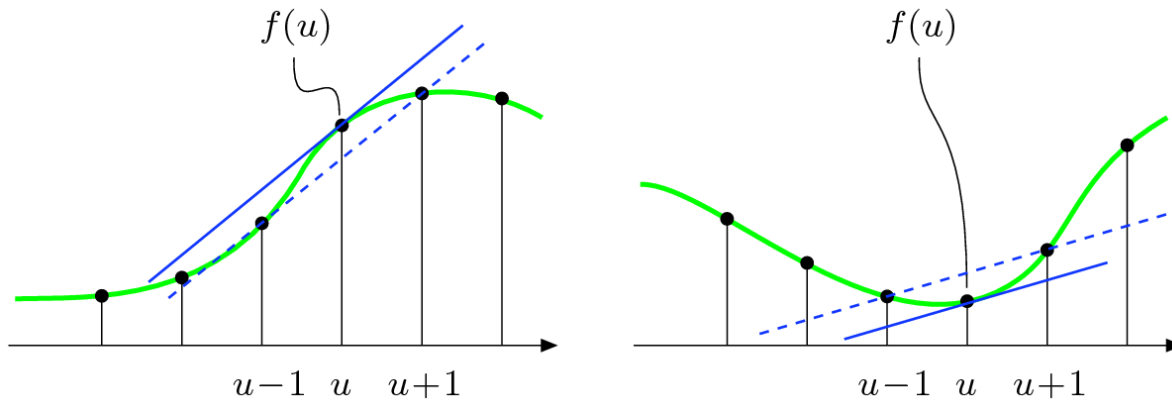
# Odvod funkcije intenzitete

$$f'(x) = \frac{df}{dx}(x)$$





# Diferenca kot diskretna aproksimacija odvoda



$$\frac{df}{du}(u) \approx \frac{f(u+1) - f(u-1)}{2} = 0.5 \cdot (f(u+1) - f(u-1))$$

Realizacija z diferenčnim filtrom:  $H_D = [-1 \ 0 \ 1]$

## Parcialni odvodi funkcije $I(u, v)$

$$\frac{\partial I}{\partial u}(u, v)$$

$$\frac{\partial I}{\partial v}(u, v)$$

$$\nabla I(u, v) = \begin{bmatrix} \frac{\partial I}{\partial u}(u, v) \\ \frac{\partial I}{\partial v}(u, v) \end{bmatrix} \quad \text{Vektor gradienta } \mathbf{v}(u, v)$$

$$\text{Moč gradienta } \mathbf{v}(u, v) \quad |\nabla I|(u, v) = \sqrt{\left(\frac{\partial I}{\partial u}(u, v)\right)^2 + \left(\frac{\partial I}{\partial v}(u, v)\right)^2}$$

# Prewitt in Sobelov operator

Prewitt

$$H_x^P = \begin{bmatrix} -1 & 0 & 1 \\ -1 & \mathbf{0} & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$H_y^P = \begin{bmatrix} -1 & -1 & -1 \\ 0 & \mathbf{0} & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$H_x^P = \begin{bmatrix} 1 \\ \mathbf{1} \\ 1 \end{bmatrix} * \begin{bmatrix} -1 & \mathbf{0} & 1 \end{bmatrix}$$

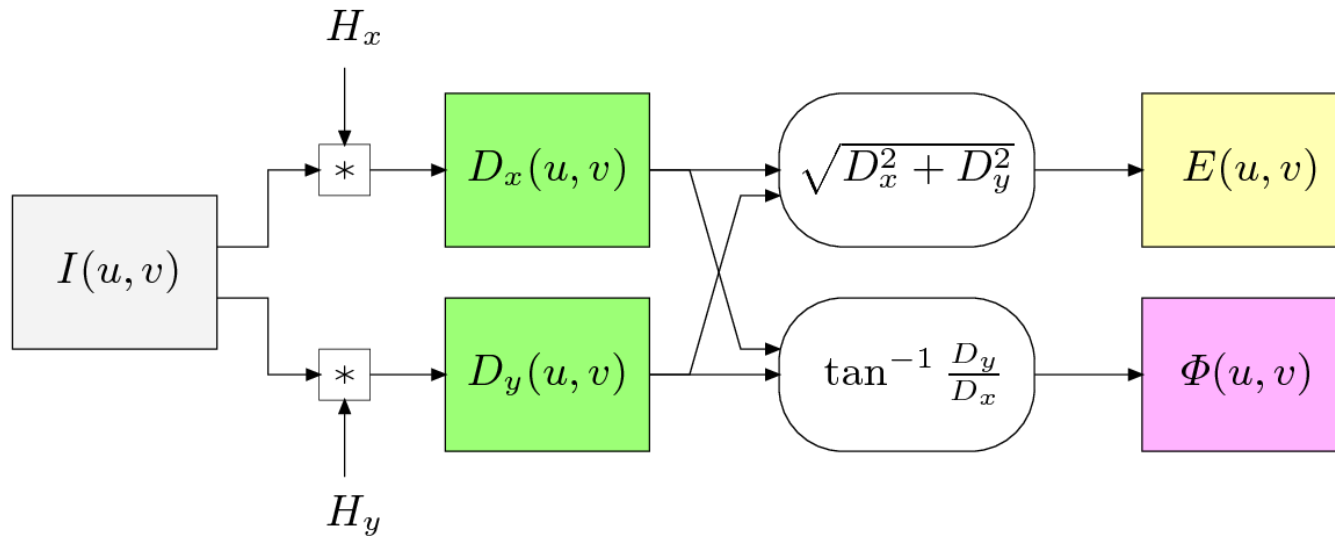
$$H_y^P = \begin{bmatrix} 1 & \mathbf{1} & 1 \end{bmatrix} * \begin{bmatrix} -1 \\ \mathbf{0} \\ 1 \end{bmatrix}$$

Sobel

$$H_x^S = \begin{bmatrix} -1 & 0 & 1 \\ -2 & \mathbf{0} & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$H_y^S = \begin{bmatrix} -1 & -2 & -1 \\ 0 & \mathbf{0} & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

## Prewitt in Sobelov operator

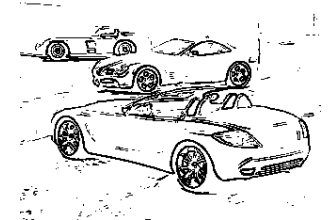
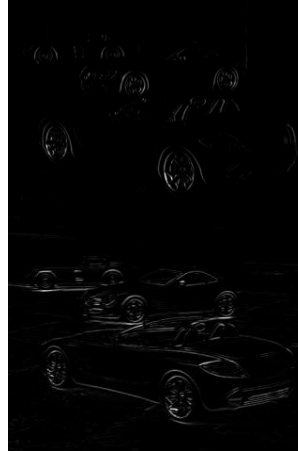


$$D_x(u, v) = H_x * I \quad \text{and} \quad D_y(u, v) = H_y * I$$

$$E(u, v) = \sqrt{(D_x(u, v))^2 + (D_y(u, v))^2}$$

$$\Phi(u, v) = \tan^{-1} \left( \frac{D_y(u, v)}{D_x(u, v)} \right) = \text{ArcTan}(D_x(u, v), D_y(u, v))$$

# Prewittov in Sobelov operator



**OGLIŠČA**

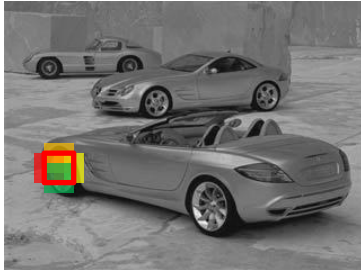
## Harrisov detektor oglišč [Harris & Stephens '88]



$$a(x, y) = \sum_{(x_k, y_k) \in W(x, y)} (I(x_k, y_k) - I(x_k + \Delta x, y_k + \Delta y))^2$$

Avtokorelacija v točki  $(x, y)$  za lokalno soseščino  $W$  in odmik  $(\Delta x, \Delta y)$

# Avtokorelacija



$$a(x, y) = \sum_{(x_k, y_k) \in W(x, y)} (I(x_k, y_k) - I(x_k + \Delta x, y_k + \Delta y))^2$$

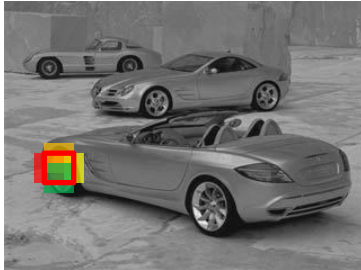
Avtokorelacija v točki  $(x, y)$  za lokalno soseščino  $W$  in odmik  $(\Delta x, \Delta y)$

$a(x, y)$     majhen v vseh smereh – homogeno področje  
                  izrazit v eni smeri – rob  
                  izrazit v vseh smereh - oglišče

Realizacija kot filter - zamudno!



# Aproksimacija avtokorelacije



$$I_x(u, v) = \frac{\partial I}{\partial x}(u, v)$$

$$I_y(u, v) = \frac{\partial I}{\partial y}(u, v)$$

$$a(x, y) = \sum_{(x_k, y_k) \in W(x, y)} (I(x_k, y_k) - I(x_k + \Delta x, y_k + \Delta y))^2$$

Aproksimacija 1. reda

$$I(x_k + \Delta x, y_k + \Delta y) = I(x_k, y_k) + I_x(x_k, y_k) \Delta x + I_y(x_k, y_k) \Delta y$$

# Avtokorelacijska matrika

$$a(x, y) = \sum_{(x_k, y_k) \in W} \left( I_x(x_k, y_k) I_y(x_k, y_k) - \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \right)^2$$
$$= \Delta x \Delta y \begin{bmatrix} \sum_{(x_k, y_k) \in W} (I_x(x_k, y_k))^2 & \sum_{(x_k, y_k) \in W} I_x(x_k, y_k) I_y(x_k, y_k) \\ \sum_{(x_k, y_k) \in W} I_x(x_k, y_k) I_y(x_k, y_k) & \sum_{(x_k, y_k) \in W} (I_y(x_k, y_k))^2 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

# Avtokorelacijska matrika

$$a(x, y) = \Delta x \Delta y \begin{bmatrix} \sum_{(x_k, y_k) \in W} (I_x(x_k, y_k))^2 & \sum_{(x_k, y_k) \in W} I_x(x_k, y_k) I_y(x_k, y_k) \\ \sum_{(x_k, y_k) \in W} I_x(x_k, y_k) I_y(x_k, y_k) & \sum_{(x_k, y_k) \in W} (I_y(x_k, y_k))^2 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$
$$= \Delta x \Delta y \bar{G} \otimes \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

vsoto lahko aproksimiramo z Gaussovimi glajenjem

## Avtokorelacijska matrika

$$M = \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = \begin{bmatrix} A & C \\ C & B \end{bmatrix}$$

$$\bar{M} = G \otimes \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = \begin{bmatrix} I_x^2 \otimes G & I_x I_y \otimes G \\ I_x I_y \otimes G & I_y^2 \otimes G \end{bmatrix} =$$

$$= \begin{bmatrix} A \otimes G & C \otimes G \\ C \otimes G & B \otimes G \end{bmatrix} = \begin{bmatrix} \bar{A} & \bar{C} \\ \bar{C} & \bar{B} \end{bmatrix}$$

$$A(u, v) = I_x^2(u, v)$$

$$B(u, v) = I_y^2(u, v)$$

$$C(u, v) = I_x(u, v) \cdot I_y(u, v)$$

## Avtokorelacijska matrika (matrika lokalne strukture)

$$\bar{M} = \begin{bmatrix} \bar{A} & \bar{C} \\ \bar{C} & \bar{B} \end{bmatrix}$$

Avtokorelacijska matrika opisuje strukturne lastnosti soseščine

Matrika je simetrična – lahko jo diagonaliziramo

$$\bar{M} = U \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} U^T$$

$\lambda_1$  in  $\lambda_2$  sta lastni vrednosti:

$\lambda_1 = \lambda_2 = 0$  : homogena soseščina

$\lambda_1 > 0, \lambda_2 = 0$  : rob

$\lambda_1 > 0, \lambda_2 > 0$  : oglišče

# Lastne vrednosti

$$\lambda_1 \approx \lambda_2 \approx 0$$

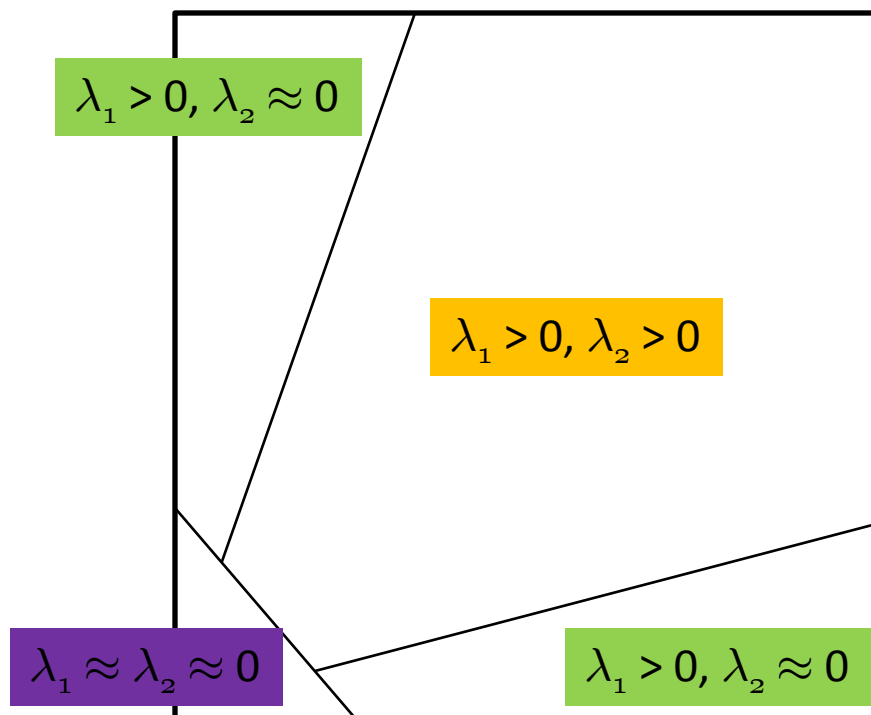
$$\lambda_1 > 0, \lambda_2 \approx 0$$

$$\lambda_1 > 0, \lambda_2 > 0$$

: homogena soseščina

: rob

: oglišče



## Ocena oglatosti (Corner Response Function, CRF)

$$\begin{aligned}\lambda_{1,2} &= \frac{\text{trace}(\bar{M})}{2} \pm \sqrt{\left(\frac{\text{trace}(\bar{M})}{2}\right)^2 - \det(\bar{M})} \\ &= \frac{1}{2} \left( \bar{A} + \bar{B} \pm \sqrt{\bar{A}^2 - 2\bar{A}\bar{B} + \bar{B}^2 + 4\bar{C}^2} \right)\end{aligned}$$

$$\lambda_1 - \lambda_2 = 2 \cdot \sqrt{\frac{1}{4} \cdot (\text{trace}(\bar{M}))^2 - \det(\bar{M})}$$

$$\begin{aligned}Q(u, v) &= \det(\bar{M}) - \alpha \cdot (\text{trace}(\bar{M}))^2 \\ &= (\bar{A}\bar{B} - \bar{C}^2) - \alpha \cdot (\bar{A} + \bar{B})^2\end{aligned} \quad \text{Ocena oglatosti CRF}$$

**občutljivost Harrisovega detektorja**

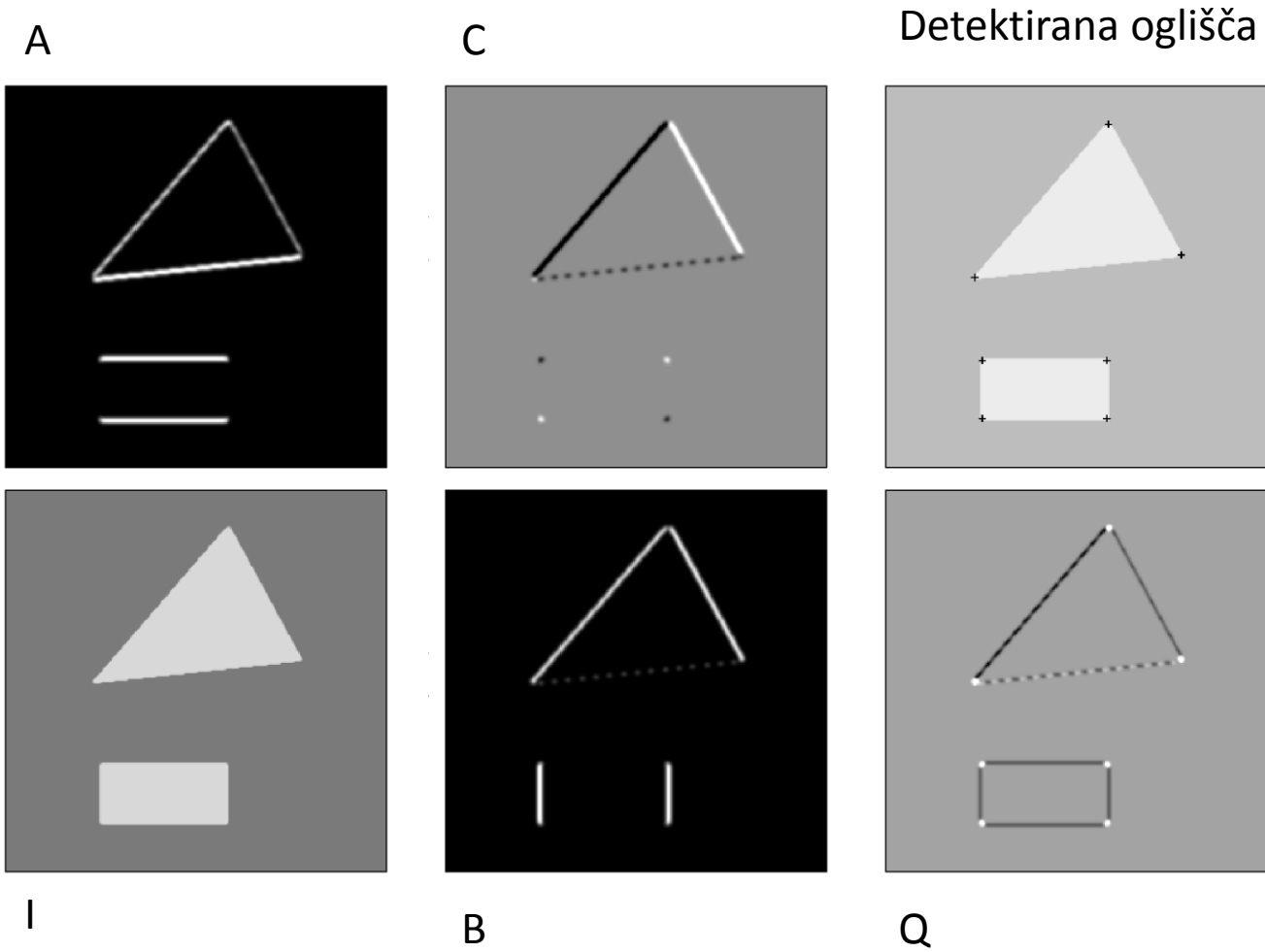
# Harrisov detektor oglišč

- Detekcija oglišč
  - Za vsak slikovni element izračunaj oceno oglatosti
  - Upragovi (absoluten ali relativen prag, lahko glede na število oglišč)
  - Poišči lokalne maksimume

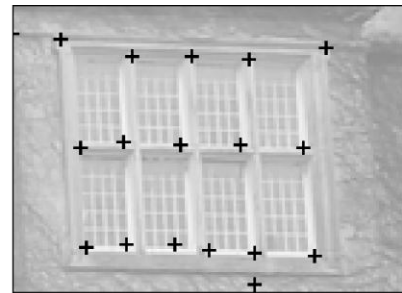
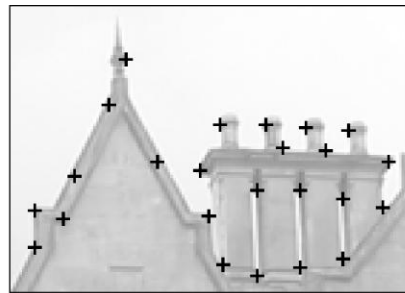
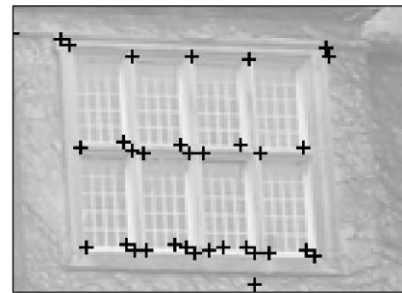
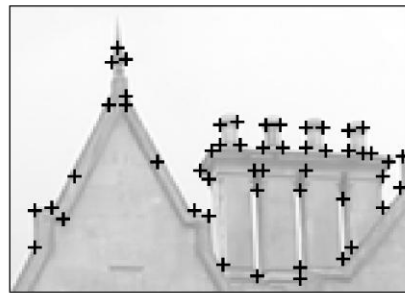
$$Q(x, y) > \text{prag} \wedge \forall x', y' \in \mathcal{N} - \text{sosescini: } Q(x, y) \geq Q(x', y')$$



# Harrisov detektor oglišč



# Harrisov detektor oglišč



# Harrisov detektor oĝliĉ

```
1: HARRISCORNERS( $I$ )
   Returns a list of the strongest corners found in the image  $I$ .

2: STEP 1—COMPUTE THE CORNER RESPONSE FUNCTION:
3:   Prefilter (smooth) the original image:  $I' \leftarrow I * H_p$ 
4:   Compute the horizontal and vertical image derivatives:
        $I_x \leftarrow I' * H_{dx}$ 
        $I_y \leftarrow I' * H_{dy}$ 
5:   Compute the local structure matrix  $M(u, v) = \begin{pmatrix} A & C \\ C & B \end{pmatrix}$ :
        $A(u, v) \leftarrow I_x^2(u, v)$ 
        $B(u, v) \leftarrow I_y^2(u, v)$ 
        $C(u, v) \leftarrow I_x(u, v) \cdot I_y(u, v)$ 
6:   Blur each component of the structure matrix:  $\bar{M} = \begin{pmatrix} \bar{A} & \bar{C} \\ \bar{C} & \bar{B} \end{pmatrix}$ :
        $\bar{A} \leftarrow A * H_b$ 
        $\bar{B} \leftarrow B * H_b$ 
        $\bar{C} \leftarrow C * H_b$ 
7:   Compute the corner response function:
        $Q \leftarrow (\bar{A} \cdot \bar{B} - \bar{C}^2) - \alpha \cdot (\bar{A} + \bar{B})^2$ 

8: STEP 2—COLLECT CORNER POINTS:
9:   Create an empty list:
        $Corners \leftarrow []$ 
10:  for all image coordinates  $(u, v)$  do
11:    if  $Q(u, v) > t_H$  and ISLOCALMAX( $Q, u, v$ ) then
12:      Create a new corner:
          $c_i \leftarrow \langle u_i, v_i, q_i \rangle = \langle u, v, Q(u, v) \rangle$ 
13:      Add  $c_i$  to  $Corners$ 
14:  Sort  $Corners$  by  $q_i$  in descending order (strongest corners first)
15:   $GoodCorners \leftarrow \text{CLEANUPNEIGHBORS}(Corners)$ 
16:  return  $GoodCorners$ .

17: ISLOCALMAX( $Q, u, v$ )    ▷ determine if  $Q(u, v)$  is a local maximum
18:   Let  $q_c \leftarrow Q(u, v)$  (center pixel)
19:   Let  $\mathcal{N} \leftarrow \text{Neighbors}(Q, u, v)$     ▷ values of all neighboring pixels
20:   if  $q_c \geq q_i$  for all  $q_i \in \mathcal{N}$  then
21:     return true
22:   else
23:     return false.

24: CLEANUPNEIGHBORS( $Corners$ )    ▷  $Corners$  is sorted by descending  $q$ 
25:   Create an empty list:
      $GoodCorners \leftarrow []$ 
26:   while  $Corners$  is not empty do
27:      $c_i \leftarrow \text{REMOVEFIRST}(Corners)$ 
28:     Add  $c_i$  to  $GoodCorners$ 
29:     for all  $c_j$  in  $Corners$  do
30:       if  $\text{Dist}(c_i, c_j) < d_{\min}$  then
31:         Delete  $c_j$  from  $Corners$ 
32:   return  $GoodCorners$ .
```

**Prefilter** (line 3): Smoothing with a small  $xy$ -separable filter

$H_p = H_{px} * H_{py}$ , where

$$H_{px} = \frac{1}{9} [2 \ 5 \ 2] \quad \text{and} \quad H_{py} = H_{px}^T = \frac{1}{9} \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}.$$

**Gradient filter** (line 4): Computing the first partial derivative in the  $x$  and  $y$  directions with

$$H_{dx} = [-0.453014 \ 0 \ 0.453014] \quad \text{and} \quad H_{dy} = H_{dx}^T = \begin{bmatrix} -0.453014 \\ 0 \\ 0.453014 \end{bmatrix}.$$

**Blurfilter** (line 6): Smoothing the individual components of the structure matrix  $M$  with separable Gaussian filters  $H_b = H_{bx} * H_{by}$  with

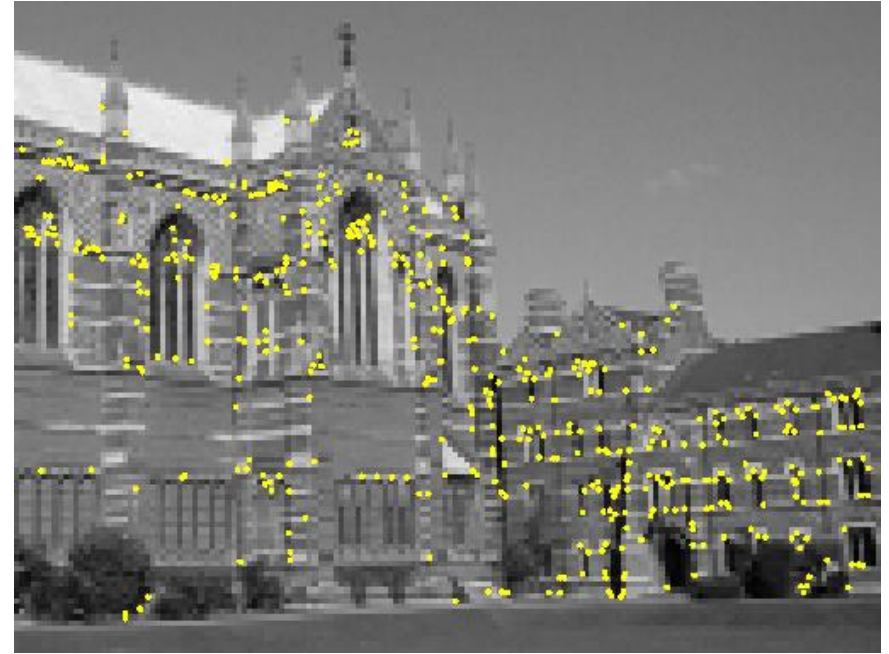
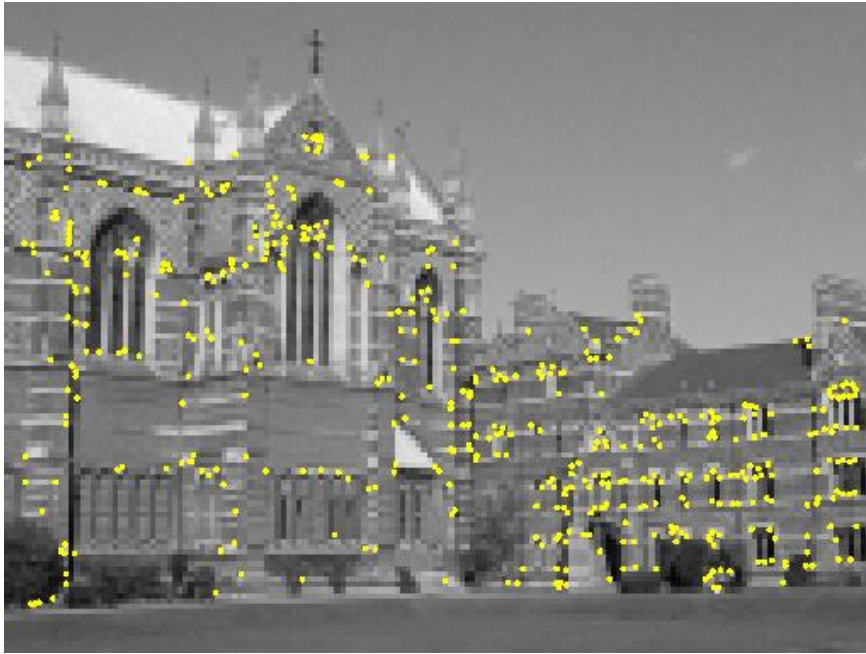
$$H_{bx} = \frac{1}{64} [1 \ 6 \ 15 \ 20 \ 15 \ 6 \ 1], \quad H_{by} = H_{bx}^T = \frac{1}{64} \begin{bmatrix} 1 \\ 6 \\ 15 \\ 20 \\ 15 \\ 6 \\ 1 \end{bmatrix}.$$

**Steering parameter** (line 7):  $\alpha = 0.04$  to  $0.06$  (default  $0.05$ )

**Response threshold** (line 13):  $t_H = 10,000$  to  $1,000,000$  (default  $25,000$ )

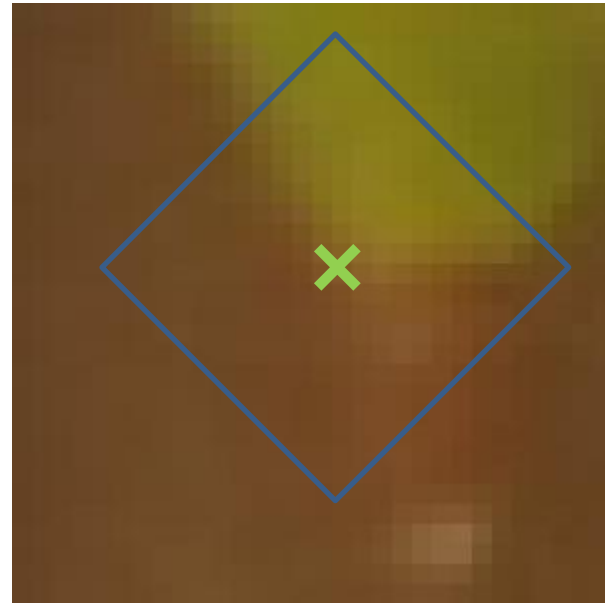
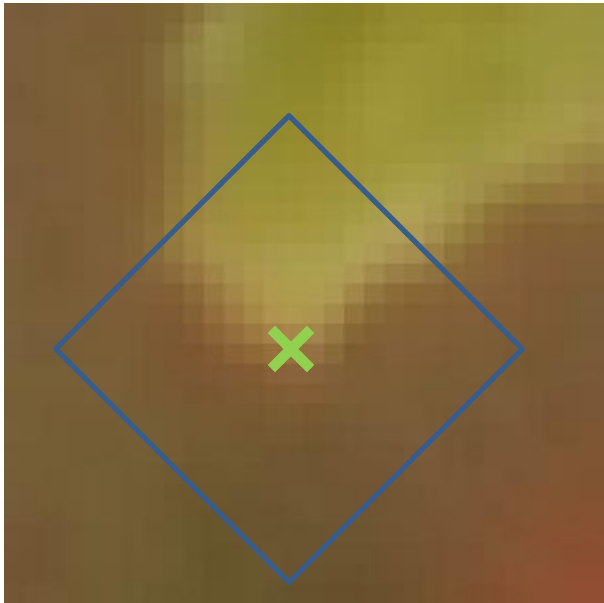
**Neighborhood radius** (line 31):  $d_{\min} = 10$  pixels

# Lokalna ujemanja



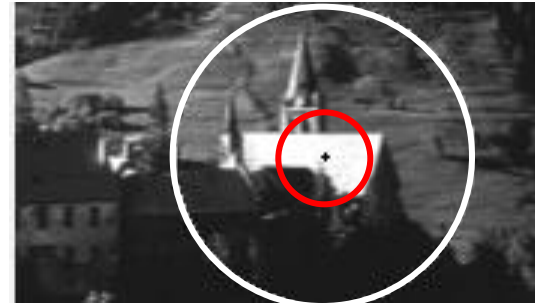
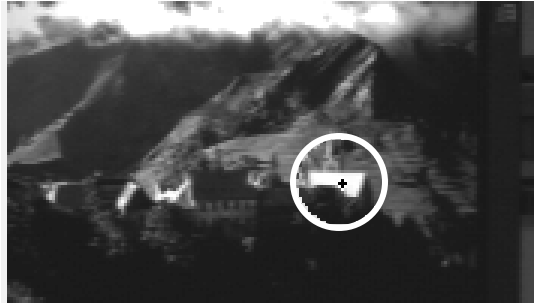
- Sledenje lokalnih regij
- Ujemanje slik (razpoznavanje objektov, stereo, zleпки, panorame...)
- Oglišča so primer značilnih točk. Potrebujemo še opisnike.

# Lokalna ujemanja



Za izračun opisnikov potrebujemo podporno regijo  
Spremembe v osvetlitvi  
Spremembe v legi, orientaciji, velikosti

# Detektor oglišč neodvisen od merila (scale invariant)



Sprememba velikosti:

$$I_1 \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = I_2 \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = I_2 \begin{pmatrix} sx_1 \\ sy_1 \end{pmatrix}$$

# Prostor merila (scale space)

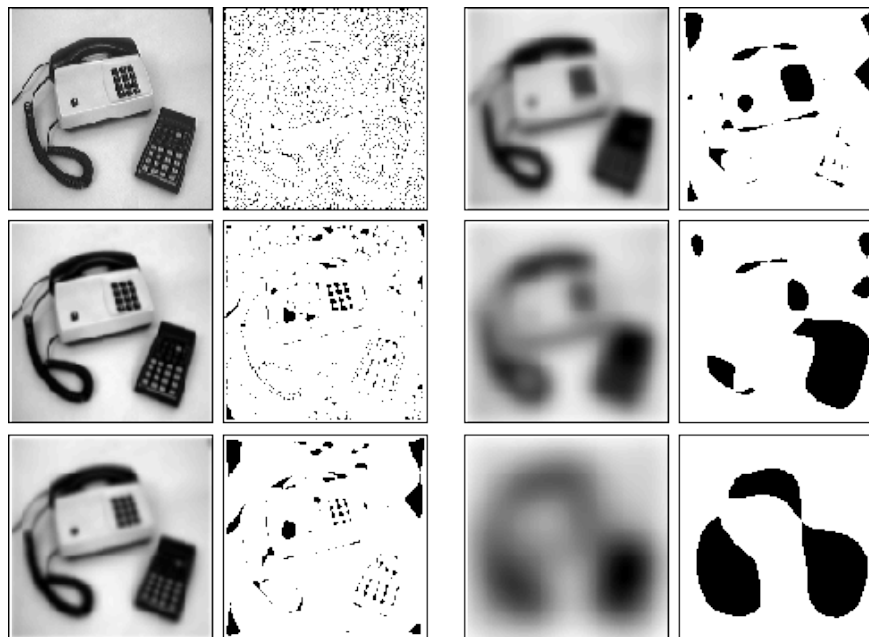
**Scale-space: A framework for handling image structures at multiple scales**

**T. Lindenberg**

Prostor merila je predstavitev slikovne informacije pri različnih nivojih podrobnosti

Pogoj: sprememba merila ne dodaja nove strukturne informacije (robovi...)

Pokazati se da, da je ustrezen prostor merila natanko tista predstavitev, ki jo dobimo s konvolucijo z Gaussovimi jedrom.





## Prostor merila (scale space)

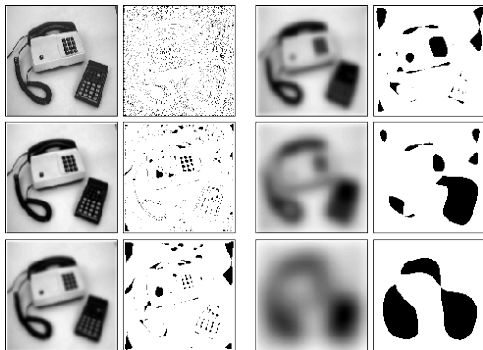
**Scale-space: A framework for handling image structures at multiple scales**

**T. Lindenberg**

Prostor merila je predstavitev slikovne informacije pri različnih nivojih podrobnosti

Pogoj: sprememba merila ne dodaja nove strukturne informacije (robovi...)

Pokazati se da, da je ustrezen prostor merila natanko tista predstavitev, ki jo dobimo s konvolucijo z Gaussovim jedrom.



$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y),$$

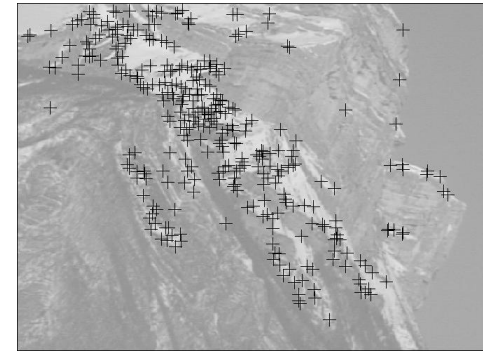
$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}.$$

Witkin (1983), Koenderink(1984), Lindeberg(1994)

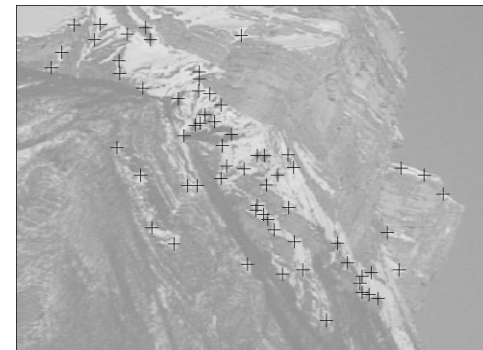
$$L(x, y, s\sigma); \quad s = 1, 2, \dots, k$$

# Harrisov detektor na več merilih

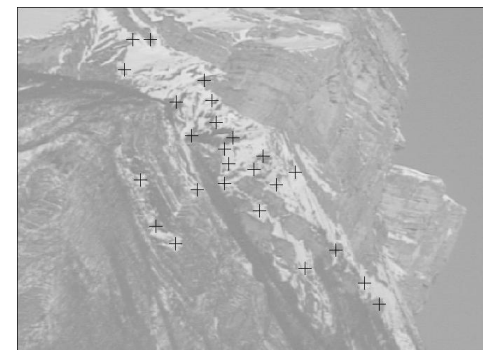
$$M_s = s^2 G(s\tilde{\sigma}) \otimes \begin{bmatrix} L_x^2(s\sigma) & L_x L_y(s\sigma) \\ L_x L_y(s\sigma) & L_y^2(s\sigma) \end{bmatrix}$$



$s = 1$



$s = 3$



$s = 5$

# Harrisov detektor na več merilih

$$M_s = s^2 G(s\tilde{\sigma}) \otimes \begin{bmatrix} L_x^2(s\sigma) & L_x L_y(s\sigma) \\ L_x L_y(s\sigma) & L_y^2(s\sigma) \end{bmatrix}$$

