DETERMINANTE

razvoj po vrstici:

$$\begin{vmatrix}1&2&3\\2&4&6\\3&2&1\end{vmatrix}=1\cdot\begin{vmatrix}4&6\\2&1\end{vmatrix}-2\cdot\begin{vmatrix}2&6\\3&1\end{vmatrix}+3\cdot\begin{vmatrix}2&4\\3&2\end{vmatrix}=1(4\cdot1-2\cdot6)-2(2\cdot1-3\cdot6)+3(2\cdot2-3\cdot4)=0$$

razvoj po stolpcu:

$$\begin{vmatrix}1&2&3\\2&4&6\\3&2&1\end{vmatrix}=-2\cdot\begin{vmatrix}2&6\\3&1\end{vmatrix}+4\cdot\begin{vmatrix}1&3\\3&1\end{vmatrix}-2\cdot\begin{vmatrix}1&3\\2&6\end{vmatrix}=-2(2\cdot1-3\cdot6)+4(1\cdot1-3\cdot3)+2(1\cdot6-2\cdot3)=0$$

1. Razvoj determinante
2. Računamo s pravili
   a. izpostavimo faktor iz vrstice (ali stolpca)

   $$\begin{vmatrix}1&2&3\\2&4&6\\3&2&1\end{vmatrix}=2\cdot\begin{vmatrix}1&2&3\\1&2&3\\3&2&1\end{vmatrix}=2\cdot\begin{vmatrix}1&2&3\\2&2&6\\3&1&1\end{vmatrix}$$

   b. zamenjava 2 vrstic (ali stolpcev), pri tem je treba množiti z -1

   $$\begin{vmatrix}1&2&3\\2&4&6\\3&2&1\end{vmatrix}=2\cdot\begin{vmatrix}1&2&3\\1&2&3\\3&2&1\end{vmatrix}=-\begin{vmatrix}2&4&6\\1&2&3\\3&2&1\end{vmatrix}$$

   c. vrstici (stolpcu) lahko prišteješ večkratnik druge vrstice (stolpca)

   $$\begin{vmatrix}1&2&3\\2&4&6\\3&2&1\end{vmatrix}=2\cdot\begin{vmatrix}1&2&3\\1&2&3\\3&2&1\end{vmatrix}=-\begin{vmatrix}2&4&6\\1&2&3\\3&2&1\end{vmatrix}=2\cdot\begin{vmatrix}1&2&3\\0&0&0\\3&2&1\end{vmatrix}=2\cdot\begin{vmatrix}1&2&3\\0&0&0\\0&-4&-8\end{vmatrix}$$

1.

$$\begin{vmatrix}2&1&1\\0&5&-2\\1&-3&4\end{vmatrix}=2\cdot\begin{vmatrix}5&-2\\-3&4\end{vmatrix}-0\cdot\begin{vmatrix}1&1\\-3&4\end{vmatrix}+1\cdot\begin{vmatrix}1&1\\5&-2\end{vmatrix}=$$

$$2(5\cdot4-(-3)\cdot(-2))-0+(1\cdot(-2)-5\cdot1)=2\cdot(20-6)-7=21$$

2.

$$\begin{vmatrix}-6&21&-30\\1&-3&5\\2&7&-4\end{vmatrix}=3\cdot\begin{vmatrix}-2&7&-10\\1&-3&5\\2&7&-4\end{vmatrix}=3\cdot\begin{vmatrix}-2&7&-10\\1&-3&5\\0&14&-14\end{vmatrix}=-3\cdot\begin{vmatrix}1&-3&5\\0&1&0\\0&14&-14\end{vmatrix}=$$

$$-3\cdot\begin{vmatrix}1&-3&5\\0&1&0\\0&0&-14\end{vmatrix}=-3\cdot(-14)=42$$

3.

$$\begin{vmatrix} 1 & 2 & 0 & -1 \\ 2 & 3 & -1 & 0 \\ 0 & -1 & 2 & 4 \\ -1 & 0 & 4 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 0 & -1 \\ 0 & -1 & -1 & 2 \\ 0 & -1 & 2 & 4 \\ 0 & 2 & 4 & -2 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 0 & -1 \\ 0 & -1 & -1 & 2 \\ 0 & -1 & 2 & 4 \\ 0 & 0 & 8 & 6 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 2 & 0 & -1 \\ 0 & -1 & -1 & 2 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 8 & 6 \end{vmatrix} = 1 \cdot \begin{vmatrix} -1 & -1 & 2 \\ 0 & 3 & 2 \\ 0 & 8 & 6 \end{vmatrix} - 0 + 0 - 0 = -1 \cdot \begin{vmatrix} 3 & 2 \\ 8 & 6 \end{vmatrix} =$$

$$= -1((3 \cdot 6) - (2 \cdot 8)) = -(18 - 16) = -2$$

4.

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 2 & 3 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 5 \end{vmatrix} = 1 \begin{vmatrix} 2 & 2 & 3 \\ 0 & 3 & 4 \\ 0 & 0 & 5 \end{vmatrix} = 1 \cdot 2 \cdot \begin{vmatrix} 3 & 4 \\ 0 & 5 \end{vmatrix} = 1 \cdot 2 \cdot 3 \cdot 5 = 30$$

5.

$$\begin{vmatrix} 1 & 0 & 2 \\ 3 & 0 & 4 \\ 2 & -5 & 1 \end{vmatrix} = 0 + 0 - (-5) \cdot \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -(-5) \cdot (1 \cdot 4 - 3 \cdot 2) = 5 \cdot (-2) = -10$$

- transponiranje

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{vmatrix}$$

6.

$$Kdaj\ je \begin{vmatrix} x+y & x-y \\ x-y & x+y \end{vmatrix} = 0?$$

$(x+y)(x+y)-(x-y)(x-y)=0$

$x^2 + 2xy + y^2 - x^2 + 2xy - y^2 = 0$

$4xy = 0$

$xy = 0$

1. $x = 0$
2. $y = 0$

7.

$$\begin{vmatrix} 0 & 0 & 5 & 1 \\ 0 & 0 & 6 & 2 \\ 5 & 6 & 7 & 3 \\ 1 & 2 & 3 & 4 \end{vmatrix} = \begin{vmatrix} 5 & 6 & 7 & 3 \\ 1 & 2 & 3 & 4 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 6 & 2 \end{vmatrix} = \frac{1}{5}\begin{vmatrix} 5 & 6 & 7 & 3 \\ 0 & 4 & 8 & 17 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 6 & 2 \end{vmatrix} = \frac{1}{5}\begin{vmatrix} 5 & 6 & 7 & 3 \\ 0 & 4 & 8 & 17 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} =$$

$$-\frac{1}{5}\begin{vmatrix} 5 & 6 & 7 & 3 \\ 0 & 4 & 8 & 17 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 5 & 1 \end{vmatrix} = -\frac{1}{5}\begin{vmatrix} 5 & 6 & 7 & 3 \\ 0 & 4 & 8 & 17 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -4 \end{vmatrix} = -\frac{1}{5}\cdot 5\cdot 4\cdot(-4) = 16$$

8.

$$\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 2 & 3 & 4 \\ 0 & 2 & 1 & 2 & 3 \\ 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -3 & -4 & -5 & -6 \\ 0 & 2 & 1 & 2 & 3 \\ 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 2 & 1 \end{vmatrix} = \frac{1}{6}\begin{vmatrix} -6 & -8 & -10 & -12 \\ 6 & 3 & 6 & 9 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{vmatrix} =$$

$$\frac{1}{6}\begin{vmatrix} -6 & -8 & -10 & -12 \\ 0 & -5 & -4 & -3 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{vmatrix} = \frac{1}{60}\begin{vmatrix} -6 & -8 & -10 & -12 \\ 0 & -10 & -8 & -6 \\ 0 & 0 & -3 & 4 \\ 0 & 0 & 2 & 1 \end{vmatrix} = -3-8 = -11$$

9.

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = \begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 0 & 0 & -c & d \end{vmatrix} =$$

$$-(-c)\cdot\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1 \end{vmatrix} + d\cdot\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} =$$

$$c\cdot\begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 1 & 1 & 1 \end{vmatrix} + d\cdot\begin{vmatrix} a & 0 & -c \\ 0 & b & -c \\ 1 & 1 & 1+c \end{vmatrix} = a\cdot b\cdot c + d\cdot\left(a\cdot\begin{vmatrix} b & -c \\ 1 & 1+c \end{vmatrix} + \begin{vmatrix} 0 & -c \\ b & -c \end{vmatrix}\right) =$$

$$a\cdot b\cdot c + d(a(b(1+c)+c) - b\cdot c) = a\cdot b\cdot c + d(a(b+b\cdot c+c)+c) =$$

$$a\cdot b\cdot c + a\cdot b\cdot d + a\cdot b\cdot c\cdot d + a\cdot c\cdot d + b\cdot c\cdot d$$

10.

$$\begin{vmatrix} a & b & c & d \\ a & -b & -c & -d \\ a & b & -c & -d \\ a & b & c & -d \end{vmatrix} = \begin{vmatrix} a & b & c & d \\ 0 & -2b & -2c & -2d \\ 0 & 0 & -2c & -2d \\ 0 & 0 & 0 & -2d \end{vmatrix} = a\cdot(-2b)\cdot(-2c)\cdot(-2d) =$$

$$-8\cdot a\cdot b\cdot c\cdot d$$

11.

$$\begin{vmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{vmatrix} = \begin{vmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & b-a & c-a & c-a \\ 0 & b-a & c-a & d-a \end{vmatrix} = \begin{vmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & b-a & c-a & d-a \end{vmatrix} =$$

$$\begin{vmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & c-b & d-b \end{vmatrix} = \begin{vmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{vmatrix} = a(b-a)(c-b)(d-c)$$

12.

$$\begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 2-x^2 & 2 & 3 \\ 2 & 3 & 1 & 5 \\ 2 & 3 & 1 & 9-x^2 \end{vmatrix} = 0?$$

$$\begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & 1-x^2 & 0 & 0 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & 3-x^2 \end{vmatrix} = \begin{vmatrix} 1-x^2 & 0 & 0 \\ 1 & -3 & -1 \\ 1 & -3 & 3-x^2 \end{vmatrix} = \left(1-x^2\right)\begin{vmatrix} -3 & -1 \\ -3 & 3-x^2 \end{vmatrix} =$$

$$\left(1-x^2\right)\left(-3(3-x^2)-3\right) = (1-x^2)(-12+3x^2) = -12+15x^2-3x^4 = 0$$

13.

$$\begin{vmatrix} t^2 & 0 & 0 & t+1 \\ 0 & t+2 & 0 & 0 \\ t-1 & 0 & t-3 & 0 \\ 0 & t^{10} & -2t & 2 \end{vmatrix} = 0$$

$$t^2\cdot\begin{vmatrix} t+2 & 0 & 0 \\ 0 & t-3 & 0 \\ t^{10} & -2t & 2 \end{vmatrix} - (t+1)\cdot\begin{vmatrix} 0 & t+2 & 0 \\ t-1 & 0 & t-3 \\ 0 & t^{10} & -2t \end{vmatrix} =$$

$$t^2\cdot(t+2)\cdot(t-3)\cdot 2-(t+1)(-(t+2))\begin{vmatrix} t-1 & t-3 \\ 0 & -2t \end{vmatrix} =$$ a

$$2t^2\cdot(t+2)(t-3)+(t+1)(t+2)(t-1)(-2t) = 2t(t+2)(t\cdot(t-3)-(t+1)(t-1)) =$$

$$2t(t+2)(t^2-3t-t^2+1) = 2t(t+2)(-3t+1) = 0$$

$t_1=1/3$
$t_2=-2$
$t_3=0$

14.

$$\begin{vmatrix} a^2 & (1+a)^2 & (a+2)^2 \\ b^2 & (1+b)^2 & (b+2)^2 \\ c^2 & (1+c)^2 & (c+2)^2 \end{vmatrix} = \begin{vmatrix} a^2 & 1+a^2+2a & 4+a^2+4a \\ b^2 & 1+b^2+2b & 4+b^2+4b \\ c^2 & 1+c^2+2c & 4+c^2+4c \end{vmatrix} = \begin{vmatrix} a^2 & 1+2a & 4+4a \\ b^2 & 1+2b & 4+4b \\ c^2 & 1+2c & 4+4c \end{vmatrix} =$$

$$2\cdot\begin{vmatrix} a^2 & 1+2a & 1 \\ b^2 & 1+2b & 1 \\ c^2 & 1+2c & 1 \end{vmatrix} = 2\cdot\begin{vmatrix} a^2 & 2a & 1 \\ b^2 & 2b & 1 \\ c^2 & 2c & 1 \end{vmatrix} = 2\cdot\begin{vmatrix} a^2 & 2a & 1 \\ b^2-a^2 & 2b-2a & 0 \\ c^2-a^2 & 2c-2a & 0 \end{vmatrix} = 4\cdot\begin{vmatrix} b^2-a^2 & b-a \\ c^2-a^2 & c-a \end{vmatrix} =$$

$$4((b^2-a^2)(c-a)-(c^2-a^2)(b-a)) = 4((b-a)(b+a)(c-a)-(c-a)(c+a)(b-a)) =$$
$$=4(b-a)(c-a)(b-c)$$

15.

$$\begin{vmatrix} a & 0 & c & 0 \\ 0 & a & 0 & c \\ b & 0 & d & 0 \\ 0 & b & 0 & d \end{vmatrix} = a\cdot\begin{vmatrix} a & 0 & c \\ 0 & d & 0 \\ b & 0 & d \end{vmatrix} + c\cdot\begin{vmatrix} 0 & a & c \\ b & 0 & 0 \\ 0 & 0 & d \end{vmatrix} = a\cdot d\begin{vmatrix} a & c \\ b & d \end{vmatrix} - c\cdot b\begin{vmatrix} a & c \\ b & d \end{vmatrix} =$$

$$(a\cdot d - c\cdot b)^2$$