

1. $123^{203} - 45^{62}$ ali je to deljivo s 43?

$$123^{203} \equiv x(43)$$

$$123 \equiv ?(43)$$

$$123 \equiv 37(43)$$

$$123 \equiv -6(43)$$

$$123^{203} \equiv (-6)^{203}(43)$$

$$(-6)^0 \equiv 1(43)$$

$$(-6)^1 \equiv -6(43)$$

$$(-6)^2 \equiv 36(43) \equiv -7(43)$$

$$(-6)^3 \equiv -6 \cdot -7 \equiv 42(43) \equiv -1(43) \rightarrow (-6)^6 \equiv (-1)^2(43) \equiv 1(43)$$

$$(-6)^4 \equiv -6 \cdot -1 \equiv 6(43)$$

$$(-6)^5 \equiv -6 \cdot 6 \equiv -36(43) \equiv 7(43)$$

$$(-6)^6 \equiv -6 \cdot 7 \equiv -42(43) \equiv 1(43)$$

$$203 \equiv ?(6)$$

$$203 \equiv 5(6)$$

$$(-6)^{203} \equiv (-6)^5(43) \equiv 7(43)$$

$$123^{203} \equiv (-6)^{203} \equiv (-6)^5 \equiv 7(43)$$

$$45^{62} \equiv x(43)$$

$$45 \equiv 2(43)$$

$$2^0 \equiv 1(43)$$

$$2^1 \equiv 2(43)$$

$$2^2 \equiv 4(43)$$

$$2^3 \equiv 8(43)$$

$$2^4 \equiv 16(43)$$

$$2^5 \equiv 32(43)$$

$$2^6 \equiv 64(43) \equiv 21(43) \text{ /*2...}$$

$$2^7 \equiv 42(43) \equiv -1(43) \text{ /*}^2$$

$$2^{14} \equiv (-1)^2 \equiv 1(43)$$

$$62 \equiv ?(14)$$

$$62 \equiv 6(14) \Leftrightarrow 2^{62} \equiv 2^6 \equiv 21(43)$$

$$45^{62} \equiv 2^{62} \equiv 2^6 \equiv 21(43)$$

$$123^{203} - 45^{62} \equiv 7 - 21 \equiv -14 \equiv 29(43)$$

$$[-14 + 43 = 29]$$

2. $2001^{2001} \equiv x(11)$

$$2001 \equiv ?(11)$$

$$2001 \equiv 10(11)$$

$$2001^{2001} \equiv 10^{2001}(11)$$

$$10^0 \equiv 1(11)$$

$$10^1 \equiv -1(11)$$

$$10^2 \equiv -10(11) \equiv 1(11)$$

$$2001 \equiv 1(2)$$

$$10^{2001} \equiv 10^1 \equiv -1(2)$$

$$2001^{2001} \equiv 10^{2001} \equiv -1(2)$$

3. DN: $15^{100}(17)$

4. $150^{17} + 14^{136} \cdot 5^{19} \equiv x(6)$

$$150 \equiv 25 \cdot 6 \equiv 0(6)$$

$$150^{17} \equiv 0(6)$$

$$14^{136} \equiv 14(6)$$

$$2^0 \equiv 1(6)$$

$$2^1 \equiv 2(6)$$

$$2^2 \equiv 4(6)$$

$$2^3 \equiv 8(6) \equiv 2^6$$

$$2^4 \equiv 4(6)$$

$$2^5 \equiv 8(6) \equiv 2(6)$$

$$2^{136} \equiv 2^2 \equiv 4(6)$$

$$14^{136} \equiv 2^{136}(6) \equiv 4(6)$$

$$5^{19}(6)$$

$$5 \equiv -1(6)$$

$$5^2 \equiv -5 \equiv 1(6)$$

...

$$5^{19} \equiv 5^1 \equiv -1(6)$$

$$150^{17} + 14^{136} \cdot 5^{19} \equiv 0 + 4 \cdot (-1) \equiv -4(6) \equiv 2(6)$$

5. $5^{6^7 \cdot 8^9} \equiv x(44)$

$$5^0 \equiv 1(44)$$

$$5 \equiv 5(44)$$

$$5^2 \equiv 25(44)$$

$$5^3 \equiv 125(44) \equiv -7(44)$$

$$5^4 \equiv -35(44) \equiv 9(44)$$

$$5^5 \equiv 45(44) \equiv 1(44)$$

$$6^{7^8 \cdot 9} \equiv ?(5) \text{ [vsakih 5 je ponovitev]}$$

$$6^0 \equiv 1(5)$$

$$6 \equiv 1(5)$$

$$6^x \equiv 1(5)$$

$$6^{7^8 \cdot 9} \equiv 1(5)$$

$$5^{6^7^8^9} \equiv 5^1 \equiv 5(6)$$

FUNKCIJE

$$xRy \Leftrightarrow y=f(x)$$

isti x se sme pojaviti samo enkrat v tej relaciji

A = {(1,2),(2,3),(3,1)} je funkcija

B = {(1,1),(1,2),(2,3)} ni funkcija

se ne sme na prvi lokaciji cifra ponoviti

$$D_f = \text{vsi } x\text{-i}$$

$$Z_f = \text{vsi } y\text{-i}$$

$$xRy \Leftrightarrow y^2 = x$$

y ni funkcija x-a (ker se x ponovi)

$$1R1 \wedge (-1)R1$$

$$(-1)^2 = 1$$

$$y = \sqrt{x}$$

y = -sqrt(x) sta funkciji

$y^2=x$, x je funkcija y-a

injektivnost: f je injektivna, če iz $f(x_1) = f(x_2) \Rightarrow x_1=x_2$

ali $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

surjektivnost: če f zavzame vse vrednosti

$$\forall y \exists x: f(x)=y$$

6. f,g,j,k: $\mathbb{R} \rightarrow \mathbb{R}$

$$f(x)=x^2$$

$$g(x)=x^3$$

$$h(x)=x(x-1)(x-2)$$

$$k(x)=e^x$$

$$l(x)=\log x$$

Določí D_f , Z_f , surj., inj., bij.

$$D_f: \mathbb{R}$$

$$Z_f: \mathbb{R}^+ \cup \{0\}$$

ni inj., ni surj.

$$D_g: \mathbb{R}$$

$$Z_g: \mathbb{R}$$

inj., surj., bij.

$$D_h: \mathbb{R}$$

$$Z_h: \mathbb{R}$$

ni inj., surj.

$$D_k: \mathbb{R}$$

$Z_k: \mathbb{R}^+$
inj., ni surj.

$D_l: \mathbb{R}^+$
 $Z_l: \mathbb{R}$
inj., surj., bij.

$$7. \quad \begin{aligned} f &= \begin{pmatrix} a & b & c & d & e & f \\ e & a & d & d & c & b \end{pmatrix} \\ g &= \begin{pmatrix} a & b & c & d & e & f \\ b & c & d & c & b & a \end{pmatrix} \end{aligned}$$

Določite $D_f, Z_f, D_g, Z_g, \text{surj.}, \text{inj.}, f \circ g, g \circ f, g \circ g, f \circ f$

$$Z_f = \{a, b, c, d, e\}$$

$$Z_g = \{a, b, c, d\}$$

$$D_f = D_g = \{a, b, c, d, e, f\}$$

f ni inj. (ker se d ponovi)

f ni surj. (ker manjka f)

g je inj., je surj. \rightarrow je bij.

$$\begin{aligned} f \circ g &= \begin{pmatrix} a & b & c & d & e & f \\ b & c & d & d & a & e \end{pmatrix} \\ g \circ f &= \begin{pmatrix} a & b & c & d & e & f \\ b & b & c & c & d & c \end{pmatrix} \\ g \circ g &= \begin{pmatrix} a & b & c & d & e & f \\ a & b & c & d & e & f \end{pmatrix} = \text{id} \\ f \circ f &= \begin{pmatrix} a & b & c & d & e & f \\ c & e & d & d & d & a \end{pmatrix} \end{aligned}$$

$$8. \quad \begin{aligned} f: \mathbb{N} \rightarrow \mathbb{N} \\ f(n+1) &= \begin{cases} 1/2 * f(n), & \text{če je } f(n) \text{ sodo} \\ 5 * f(n) + 1, & \text{če je } f(n) \text{ liho} \end{cases} \end{aligned}$$

$$f(1) = 1$$

$$f(2) = 5 * f(1) + 1 = 6$$

$$f(3) = 1/2 * f(2) = 3$$

$$f(4) = 5 * f(3) + 1 = 16$$

$$f(5) = 1/2 * f(4) = 8$$

$$f(6) = 1/2 * f(5) = 4$$

$$f(7) = 1/2 * f(6) = 2$$

$$f(8) = 1/2 * f(7) = 1 = f(1)$$

f je periodična

ni inj.

ni surj.

$$Z_f = \{1, 6, 3, 16, 8, 4, 2\}$$

9. $f(x) = 3/(2x-5)$
 $g(x) = |x-2|$
 $D_f = \mathbb{R} \setminus \{5/2\}$
 $D_g = \mathbb{R}$

$Z_f = \mathbb{R} \setminus \{0\}$ ni surj.

$$x = (3+5y)/2y \rightarrow y \neq 0 \rightarrow f \text{ ni surj.}$$

$Z_g = \mathbb{R}^+ \cup \{0\}$ ni surj.

$$f(x_1) = f(x_2) \Rightarrow x_2 = x_1 \text{ je inj.}$$

$$g(x_1) = g(x_2)$$

$$|x_1 - 2| = |x_2 - 2|$$

$$x_1 = 0, x_2 = 4 \Rightarrow |0 - 2| = 2 = |4 - 2| \neq \Rightarrow f(x_1) \neq f(x_2), g \text{ ni inj.}$$

10. $f, g: \mathbb{N} \cup \{0\} \rightarrow \mathbb{N} \cup \{0\}$

$$f(n) = 2n$$

$$g(n) = \lfloor n/2 \rfloor$$

$\lfloor x \rfloor$ je celi del od x

$$\lfloor 1.5 \rfloor = 1$$

$$\lfloor 2.97 \rfloor = 2$$

$$\lfloor 0.331 \rfloor = 0$$

$$f, g, f \circ g, g \circ$$

f inj.

$$\text{injektivnost: } f(n_1) = f(n_2) \sim \sim \sim \sim \rightarrow n_1 = n_2$$

$$2n_1 = 2n_2, n_1 = n_2$$

$$2n = m$$

$$m = 3$$

$2n = 3$: ni rešljivo v \mathbb{N}

ni surj (manjkajo liha števila)

f

$$0 \mapsto 0$$

$$1 \mapsto 2$$

$$2 \mapsto 4$$

$$3 \mapsto 6$$

$$4 \mapsto 8$$

$$5 \mapsto 10$$

g

$$0 \mapsto 0$$

$$1 \mapsto 0$$

$$2 \mapsto 1$$

$$3 \mapsto 1$$

$$4 \mapsto 2$$

$$5 \mapsto 2$$

$$6 \mapsto 3$$

$7 \mapsto 3$
 je surj.
 ni inj. (ker se števila na desni ponavljajo)

$f \circ g$, ni inj. (ker se 0,2,4...ponovijo na desni), ni surj. (ker manjkajo liha števila)

$0 \mapsto 0$
 $1 \mapsto 0$
 $2 \mapsto 2$
 $3 \mapsto 2$
 $4 \mapsto 4$
 $5 \mapsto 4$
 $6 \mapsto 6$
 $7 \mapsto 6$

$g \circ f = \text{id}$, surj.

$0 \mapsto 0$
 $1 \mapsto 1$
 $2 \mapsto 2$
 $3 \mapsto 3$
 $4 \mapsto 4$
 $5 \mapsto 5$
 $6 \mapsto 6$
 $7 \mapsto 7$

ALGEBRA

množice + operacija \in na množici

$(\mathbb{N}, +)$, $(\mathbb{R}, +, *)$

$(\mathbb{N}, *)$

$(\mathbb{N}, +, *)$

$(G, *)$

1. Asociativnost

$$(a*b)*c = a*(b*c)$$

2. Kommutativnost

$$a*b = b*a$$

3. Enota

poseben element e

$$e*a = a \quad \forall a$$

$$a*e = a \quad \forall a$$

npr.: $(\mathbb{N}, *)$: $e=1$

$(\mathbb{N}, +)$: ni e , (ker $0 \notin \mathbb{N}$)

$(\mathbb{Z}, +)$: $e=0$, (ker $0+n=n$)

4. Inverz a

a^{-1} je inverz od a , če

$$a*a^{-1} = e \wedge a^{-1}*a = e$$

$(\mathbb{N}, +)$: ni enote, ni inverzov

$(\mathbb{Z}, +)$: $e=0$, $n^{-1}=-n$, ker $n+(-n)=0$

$(\mathbb{Z}, *)$: $e=1, n^*n^{-1}=1; 2^*n^{-1}=1, n=1/2 \notin \mathbb{Z}$. razen za

$(\mathbb{Q}, *)$: $n^*n^{-1}=1; n^{-1}=1/n$, za $n \neq 0$

5. Absorpcijski el. w

$$w^*a=w \quad \forall a$$

$$a^*w=w$$

$(\mathbb{N}, +)$: ni abs.

$(\mathbb{Z}, *)$: $w=0$, ker $0^*n=0 \quad \forall n$

$(\mathbb{Z}, +)$: ni abs. el.