1.

Permutacije = bijektivne preslikave na končnih množicah ({1,...,n})

S_n – grupa vseh permutacij
$|S_n| = n!$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} = (1\ 2\ 3) = (2\ 3\ 1) = (3\ 1\ 2)$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 5 & 3 \end{pmatrix} = (1\ 2)(3\ 4\ 5) = (3\ 4\ 5)(1\ 2)$$

$(1\ 2) = (2\ 1)$

definiramo *
$\alpha * \beta := \beta \circ \alpha$

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 1 & 5 & 6 & 8 & 7 & 4 \end{pmatrix} = (1\ 2\ 3)(4\ 5\ 6\ 8)(7) = (1\ 2\ 3)(4\ 5\ 6\ 8)$$

$$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 6 & 4 & 3 & 2 & 1 & 7 & 5 \end{pmatrix} = (1\ 8\ 5\ 2\ 6)(3\ 4)$$

$(1\ 2\ 3)^{-1} = (3\ 2\ 1)$

$\alpha^{-1} = (8\ 6\ 5\ 4)(3\ 2\ 1)$
$\alpha^2 = (1\ 2\ 3)(4\ 5\ 6\ 8)*(1\ 2\ 3)(4\ 5\ 6\ 8) = (1\ 3\ 2)(4\ 6)(5\ 8)$
$\alpha^2 * \beta = (1\ 3\ 2)(4\ 6)(5\ 8)*(1\ 8\ 5\ 2\ 6)(3\ 4) = (1\ 4)(3\ 6)(2\ 8)(5)$

2.

$(1\ 2)(1\ 3)(1\ 4)(1\ 5) = (1\ 2\ 3\ 4\ 5)$

$(5\ 4)(4\ 3)(3\ 2)(2\ 1) = (5\ 1\ 2\ 3\ 4)$
$(i\ j)$ – transpozicija = cikel dolžine 2

$(1\ 8\ 5\ 2\ 6) = (1\ 8)(1\ 5)(1\ 2)(1\ 6)$
$= (6\ 2)(2\ 5)(5\ 8)(8\ 1)$

$(1\ 8\ 5\ 2\ 6)(3\ 4) = (1\ 8)(1\ 5)(1\ 2)(1\ 6)(3\ 4)$ – liha
$(1\ 3\ 2)(4\ 6)(5\ 8) = (1\ 3)(1\ 2)(4\ 6)(5\ 8)$ – soda

3.

$(1\ 2\ 3)*(3\ 4\ 5) = ?$
Ali je to enako $(3\ 4\ 5)*(1\ 2\ 3)$? (oz. ali te dve permutaciji komutirata)

$(1\ 2\ 3)*(3\ 4\ 5) = (1\ 2\ 4\ 5\ 3)$
$(3\ 4\ 5)*(1\ 2\ 3) = (1\ 2\ 3\ 4\ 5)$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 1 & 5 & 3 \end{pmatrix} \neq \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{pmatrix}$$

4.

$\alpha * x * \beta = \gamma \ /*\beta^{-1}$
$\alpha = (1\ 3\ 5\ 7)*(2\ 4\ 6\ 8)$
$\beta = (1\ 5\ 3)*(2\ 6\ 4)*(7\ 8)$
$\gamma = (1\ 5)(2\ 8)$

$\alpha * x * \beta * \beta^{-1} = \gamma * \beta^{-1}$
$\alpha * x = \gamma * \beta^{-1} \ /\alpha^{-1}*$
$\alpha^{-1} * \alpha * x = \alpha^{-1} * \gamma * \beta^{-1}$
$x = \alpha^{-1} * \gamma * \beta^{-1}$

$x = (8\ 6\ 4\ 2)(7\ 5\ 3\ 1)*(1\ 5)(2\ 8)*(8\ 7)(4\ 6\ 2)(3\ 5\ 1)$
$\quad = (1\ 8\ 2\ 4\ 7\ 3)(5)(6)$

$(1\ 8\ 2\ 4\ 7\ 3) = (1\ 8)(1\ 2)(1\ 4)(1\ 7)(1\ 3)$ – liha permutacija

5.

$$\alpha = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

$$\beta = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

1. $\alpha * \pi = \beta \ /\alpha^{-1}*$
$\pi = \alpha^{-1} * \beta$

$\alpha = (1\ 3\ 2)$
$\beta = (1\ 2)(3)$

$\alpha^{-1} = (2\ 3\ 1)$
$\pi = (2\ 3\ 1)*(1\ 2) = (1\ 2\ 3)$

2. $\alpha * \pi^2 = \beta$
$\pi^2 = (2\ 3)$ ... tak π ne obstaja (ker je $(2\ 3)$ liha)

kvadrat permutacije je vedno sodo permutacij

6.

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 5 & 2 & 6 & 3 & 1 \end{pmatrix}$$

$\alpha^{2005} = ?$
$\alpha = (1\ 4\ 6)(2\ 5\ 3)$
$\alpha^{2005} = ((1\ 4\ 6)(2\ 5\ 3))^{2005} = (1\ 4\ 6)^{2005} * (2\ 5\ 3)^{2005} = (1\ 4\ 6)(2\ 5\ 3)$

$(1\ 2\ 3)^1 = (1\ 2\ 3)$
$(1\ 2\ 3)^2 = (1\ 2\ 3)(1\ 2\ 3) = (1\ 3\ 2)$
$(1\ 2\ 3)^3 = (1\ 3\ 2)(1\ 2\ 3) = (1)(2)(3) = id$

$(\alpha*\beta)^2 = \alpha\beta\alpha\beta \neq \alpha^2\beta^2$

7.

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 1 & 5 & 8 & 2 & 7 & 6 & 9 & 4 \end{pmatrix}$$

$\alpha^{2005} = ?$

$\alpha = (1\ 3\ 5\ 2)(4\ 8\ 9)(6\ 7)$
$\alpha^{2005} = (1\ 3\ 5\ 2)^{2005}*(4\ 8\ 9)^{2005}*(6\ 7)^{2005} = (1\ 3\ 5\ 2)(4\ 8\ 9)(6\ 7)$

8.

Red permutacije π je najmanjši n, tako da je $\pi^n = id$
red $\alpha = 3*4 = 12$
red π = najmanjši skupni večkratnik dolžin ciklov

9.

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 3 & 5 & 2 & 4 & 1 & 7 \end{pmatrix}$$

$$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 7 & 2 & 1 & 3 & 4 & 5 \end{pmatrix}$$

Določi parnost, red, in ali komutirata?

$\alpha = (1\ 6)(2\ 3\ 5\ 4)(7) = (1\ 6)(2\ 3)(2\ 5)(2\ 4)$ soda, red = 4
$\beta = (1\ 6\ 4)(2\ 7\ 5\ 3) = (1\ 6)(1\ 4)(2\ 7)(2\ 5)(2\ 3)$ liha, red = 12

$\beta*\alpha = (1\ 6\ 4)(2\ 7\ 5\ 3)*(1\ 6)(2\ 3\ 5\ 4) = (1)(2\ 7\ 4\ 6)(3)(5)$ torej ne komutirata

10.

$y*(1\ 2\ 3)*(3\ 1\ 2)*y = (7\ 8\ 9)*y\ /y^{-1}$
$y*(1\ 2\ 3)*(3\ 1\ 2)*y*y^{-1} = (7\ 8\ 9)*y*y^{-1}$
$y*(1\ 2\ 3)(3\ 1\ 2) = (7\ 8\ 9)\ /*(3\ 1\ 2)^{-1}$
$y = (7\ 8\ 9)(2\ 1\ 3)(3\ 2\ 1) = (1\ 2\ 3)(7\ 8\ 9)$

11.

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 6 & 9 & 8 & 5 & 4 & 1 & 2 & 3 \end{pmatrix}$$

$$\alpha * x = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 4 & 9 & 7 & 6 & 5 & 2 & 3 & 1 \end{pmatrix} = \beta$$

Določi parnost, red α, β, x

$\alpha = (1\ 7)(2\ 6\ 4\ 8)(3\ 9)(5)$
$\beta = (1\ 8\ 3\ 9)(2\ 4\ 7)(5\ 6)$

$x = \alpha^{-1} * \beta$

$x = (9\ 3)(8\ 4\ 6\ 2)(7\ 1) * (1\ 8\ 3\ 9)(2\ 4\ 7)(5\ 6)$
$\quad = (1\ 2\ 3)(4\ 5\ 6)(7\ 8)(9)$

$\alpha = (1\ 7)(2\ 6)(2\ 4)(2\ 8)(3\ 9)$ – liha
$\beta = (1\ 2)(1\ 3)(4\ 5)(4\ 6)(7\ 8)$ – liha

red $\alpha = 4$
red $\beta = 12$
red $x = 6$

12.

$\alpha = (2\ 5)(3\ 1\ 4\ 6)$
$\pi(i) \equiv 2i+3\ (7) + 1$
$i = 1,..,7$
Izračunaj $(\pi * \alpha^{-1})^{2004}$

$\alpha^{-1} = (6\ 4\ 1\ 3)(5\ 2)$

$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 1 & 3 & 5 & 7 & 2 & 4 \end{pmatrix}$$

$\pi = (1\ 6\ 2)(3)(4\ 5\ 7)$

$((1\ 6\ 2)(4\ 5\ 7) * (6\ 4\ 1\ 3)(5\ 2))^{2005} =$
$= (1\ 4\ 2\ 3\ 6\ 5\ 7)^{2005} = (1\ 4\ 2\ 3\ 6\ 5\ 7)^{3} = (1\ 3\ 7\ 2\ 5\ 4\ 6)$

$2005 \equiv 3\ (7)$

13.

$\alpha = (2\ 4)(3\ 2\ 1)(7\ 4\ 1\ 5)$
α^{-2005}

$\alpha^{-1} = (5\ 1\ 4\ 7)(1\ 2\ 3)(4\ 2)$
če cikli niso disjunktni, potem ne moreš potencirat vsakega posebej, ker ne komutirajo (imata vsaj eno skupno število. v tem primeru "1")

če pa komutirajo, potem lahko potenciramo vsakega posebej

$(5\ 1\ 4\ 7)(1\ 2\ 3)(4\ 2) = (1\ 2\ 3)(4\ 7\ 5)$
$\alpha^{-2005} = (1\ 2\ 3)^{2005} * (4\ 7\ 5)^{2005}$

$2005 \equiv 1\ (3)$

$\alpha^{-2005} = \alpha^{-1}$

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Poišči β:

$\alpha^{2}*\beta = (2\ \ 3)(7\ \ 5\ \ 1)\ /\ \alpha^{-2}*$

$\alpha^{2} = (1\ \ 2\ \ 3)(4\ \ 7\ \ 5)$

$\beta = (5\ \ 7\ \ 4)(3\ \ 2\ \ 1)*(2\ \ 3)(7\ \ 5\ \ 1) = (1\ \ 2\ \ 7\ \ 4)(3)(5)$

14.

Ali obstaja rešitev enačbe $\pi^{1986} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 7 & 6 & 1 & 3 & 5 & 2 \end{pmatrix}$?

$(1\ \ 4)(2\ \ 7)(3\ \ 6\ \ 5)$

Preveri parnost

$(1\ \ 4)(2\ \ 7)(3\ \ 6)(3\ \ 5)$ – soda, če bi bla liha bi takoj vedeli da rešitev ne bi obstajala (zaradi ^{1986})

Kakšna mora biti dolžina ciklov?

- recimo, da je π cikel dolžine 7.
  $1986 = 6\ \ (7)$
  $(\ \dots\ )^{6} = (\ \dots\ )$
  $\ \ 7 \qquad\quad 7$
  $(1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7)^{6} = (1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7)^{-1}$
  →← protislovje
- recimo, da je 6
  potem π ima eno fiksno točko
  $\Rightarrow \pi^{1986}$ mora tudi imeti fiksno točko, pa je nima
  →← protislovje
- recimo $\pi = (\ \dots\ )(\ \dots\ )$
  $\qquad\qquad\quad 5 \qquad 2$ (← dolžina cikla)
  $\pi^{1986} = (\ \dots\ )^{1986}()()\ (ij)^{1986} = id$
  $\qquad\qquad\quad 5$
  →←
- recimo $\pi = (\ \dots\ )(\ \dots\ )$
  $\qquad\qquad\quad 4 \qquad 3$
  $\pi^{1986} = (\ \dots\ )^{1986}(\ \dots\ )^{1986}$
  $\qquad\qquad\quad 4 \qquad\quad 3 = id$

$\Rightarrow$ tak π ne obstaja

15.

$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 5 & 7 & 6 & 3 & 4 \end{pmatrix}$

$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 7 & 6 & 4 & 5 & 1 \end{pmatrix}$

Določi red α, β, $\alpha^{2}*\beta^{-3}$ in sodost.

$\alpha = (1\ \ 2)(3\ \ 5\ \ 6)(4\ \ 7) = (1\ \ 2)(3\ \ 5)(3\ \ 6)(4\ \ 7)$, red $\alpha = 6$, soda

$\beta = (1\ \ 2\ \ 3\ \ 7)(4\ \ 6\ \ 5) = (1\ \ 2)(1\ \ 3)(1\ \ 7)(4\ \ 6)(4\ \ 5)$, red $\beta = 12$, liha

$\alpha^{2}*\beta^{-3} = (1\ \ 2)(3\ \ 5\ \ 6)(4\ \ 7)*(1\ \ 2)(3\ \ 5\ \ 6)(4\ \ 7)*(5\ \ 6\ \ 4)(7\ \ 3\ \ 2\ \ 1)*(5\ \ 6\ \ 4)(7\ \ 3\ \ 2\ \ 1)*(5\ \ 6\ \ 4)(7\ \ 3\ \ 2\ \ 1) = (1\ \ 2\ \ 3\ \ 6\ \ 5\ \ 7)(4) = (1\ \ 2)(1\ \ 3)(1\ \ 6)(1\ \ 5)(1\ \ 7)$, red $\alpha^{2}*\beta^{-3} = 6$, liha