# Naloge

## I. Nedoločeni integral

Izračunaj naslednje nedoločene integrale:

1. $\int x^n dx$
2. $\int (x+a)^n dx$
3. $\int (1+\frac{x}{3})^7 dx$
4. $\int (1+x^2)^2 dx$
5. $\int x(1-x^2)^5 dx$
6. $\int \sin(a+x) dx$
7. $\int \sin(ax+b)\cos(ax+b) dx$

$\star$8. $\int \sin^3 x \cos^2 x dx$

$\star$9. $\int \sin^3 x dx$

10. $\int x\sqrt{1-x^2} dx$
11. $\int \frac{1}{x^2+1} dx$
12. $\int \frac{1}{x^2-1} dx$

$\star$13. $\int \frac{2}{x^2+4x+7} dx$

14. $\int \frac{2x+3}{x^2+1} dx$
15. $\int \frac{dx}{2x+1}$

$\star$16. $\int \frac{\sqrt{x}+1}{\sqrt{x}-1} dx$

17. $\int x\sqrt{x^2+7} dx$
18. $\int \frac{x}{\sqrt{x^2+3}} dx$
19. $\int \frac{\sqrt{x}+\log x}{x} dx$
20. $\int \frac{x}{2x^2+3} dx$

$\star$21. $\int (\cos ax + \sin ax)^2 dx$

22. $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$

$\star$23. $\int \sin^2 x dx$

$\star$24. $\int \frac{x^3}{x^8+5} dx$

⋆25. $\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$

⋆⋆26. $\int \frac{dx}{x\sqrt{x^2-1}}$

⋆27. $\int \frac{dx}{e^x+1}$

28. $\int x(5x^2-3)^7 dx$

29. $\int \frac{x dx}{\sqrt{x+1}}$

⋆30. $\int x^3 e^{-x^2} dx$

31. $\int x e^{-x} dx$

32. $\int \frac{x}{e^x} dx$

⋆33. $\int x^2 e^{3x} dx$

34. $\int \frac{x dx}{\cos^2 x}$

35. $\int \mathrm{tg} x dx$

36. $\int \frac{x dx}{(x-1)(x+1)^2}$

37. $\int \frac{dx}{x^3-2x^2+x}$

38. $\int \frac{x^3+x+1}{x(x^2+1)} dx$

39. $\int \frac{x^2+4x-2}{x^3+x^2-2x} dx$

40. $\int \frac{3x^2-5x-4}{x^3-x^2-x+1} dx$

41. $\int \frac{x^2+6x+7}{(x+2)^3} dx$

42. $\int \frac{1}{x^2+2x+3} dx$

43. $\int \frac{2x^2+6x+6}{(x+1)(x^2+4x+5)} dx$

⋆44. $\int \frac{x^3-x^2-x-2}{(x^2+1)(x^2+2x+2)} dx$

⋆⋆45. $\int \frac{x+1}{3\sqrt[3]{x^2}} dx$

⋆⋆46. $\int \frac{1}{\sqrt{x}(1+\sqrt[5]{x})} dx$

⋆⋆47. $\int \frac{2x \cos x + \sin x}{\sqrt{x}} dx$

48. $\int (x^3 - x^2 + 1) dx$

49. $\int (x-1)^2 (x+1) dx$

50. $\int (\sqrt{x} + \sqrt[3]{x^2}) dx$

51. $\int \left(2x - \frac{1}{x}\right)\sqrt{x}dx$

52. $\int (\sin x + \cos x + x)dx$

53. $\int \sin axdx$

54. $\int (2x+1)^4 dx$

55. $\int \frac{xdx}{1+x^4}$

⋆56. $\int \frac{3x^2+4x-2}{x^3+2x^2-2x-1}dx$

57. $\int \frac{a}{bx+c}dx$

58. $\int x\cos xdx$

59. $\int \log xdx$

60. $\int x\log xdx$

61. $\int x^n \log xdx$

62. $\int e^{\sin x}\cos xdx$

63. $\int x\sin xdx$

64. $\int \frac{x^3-x}{\sqrt{x^2+1}}dx$

65. $\int \frac{dx}{x\log x}$

66. $\int \cos^3 xdx$

67. $\int \frac{4x-2}{x^2-1}dx$

68. $\int \frac{4x-2}{(x-1)^2}dx$

69. $\int \frac{4x-2}{(x^2-1)(x-2)}dx$

70. $\int \frac{3x^2-13x+12}{x^3-6x^2+11x-6}dx$

71. $\int \frac{x^2+x+3}{(x+1)(x^2+2)}dx$

72. $\int \frac{\sqrt{x}}{1+x}dx$

⋆73. $\int x\mathrm{arctg}xdx$

⋆74. $\int \mathrm{arctg}xdx$

⋆75. $\int \arcsin xdx$

76. $\int \sqrt{x+1}dx$

⋆⋆77. $\int \sqrt{1-x^2}dx$

78. $\int x\sqrt{1-x^2}dx$

79. $\int \frac{x}{\sqrt{1-x^2}}dx$

⋆80. $\int \frac{\sqrt{1-x^2}}{x}dx$

81. $\int xe^{-x^2}dx$

82. $\int xe^{-x^2+\log 2}dx$

⋆83. $\int \frac{dx}{2\sin x \mathrm{tg} x+\cos x}$

⋆84. $\int x^2\sin x dx$

85. $\int x\sin x^2 dx$

86. $\int \sin\sqrt{x}dx$

87. $\int \frac{x^2-1}{x-1}dx$

88. $\int \frac{x^2+1}{x-1}dx$

89. $\int \frac{x^2+1}{x^2-6x+10}dx$

90. $\int \sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)dx$

91. $\int (19x+10)^3\,dx$

⋆⋆92. $\int \frac{x^4+8}{x^3+1}dx$

93. $\int \frac{1+x+x^2+x^3}{x^2+1}dx$

94. $\int \frac{x^3}{x^2+1}dx$

95. $\int \sqrt{\sin x}\cos x dx$

96. $\int \sqrt{1+\sin x}\cos x dx$

⋆⋆97. $\int \frac{dx}{\sin x\cos x}$

98. $\int \frac{\sin x}{\cos^2 x}dx$

99. $\int \frac{\sin x}{\cos^3 x}dx$

100. $\int \mathrm{ctg} x dx$

101. $\int x^2(1-x^3)^2dx$

## II. Določeni integral

Po definiciji izračunaj naslednje določene integrale:

102. $\int_0^1 x dx$

103. $\int_0^1 x^2 dx$

104. $\int_0^1 e^x dx$

Izračunaj naslednje določene integrale:

105. $\int_1^3 \sqrt{4x-3} dx$

106. $\int_0^{2\pi} \sin x dx$

107. $\int_0^{2\pi} \sin^2 x dx$

## III. Uporabne naloge iz integriranja

108. Odvajaj funkcijo $\sin^2 x$, nato izraz poenostavi in ga integriraj. Katero znano formulo dobiš?

109. Po definiciji izračunaj dolžino daljice od točke $(0,0)$ do točke $(1,0)$.

110. Po definiciji izračunaj dolžino daljice od točke $(0,0)$ do točke $(a,b)$.

111. Izračunaj dolžino krivulje $y=\sqrt{x}^3$ od točke $(0,0)$ do točke $(1,1)$.

⋆⋆112. Izračunaj dolžino krivulje $y=\sqrt{x}$ od točke $(0,0)$ do točke $(1,1)$.

⋆⋆113. Izračunaj dolžino krivulje $y=x^2$ od točke $(0,0)$ do točke $(1,1)$.

⋆114. Izračunaj obseg kroga z radijem 1.

⋆⋆115. Izračunaj ploščino kroga z radijem 1.

116. Izračunaj ploščino lika, ki ga omejujeta krivulji $y=x^2$, $y=\frac{1}{2}x+\frac{1}{2}$.

117. Izračunaj ploščino lika, ki ga omejujeta krivulji $y=x^2$, $y=1-x^2$.

118. Izračunaj ploščino lika, ki ga omejujeta krivulji $y=x^4$, $y=1-x^4$.

⋆⋆119. Izračunaj ploščino lika, ki ga omejujeta krivulji $y=x^3-x$, $y=x-x^3$.

⋆120. Izračunaj ploščino lika, ki ga omejujejo krivulje $x=0$, $y=x^2$, $y=3-2x^3$.

121. Izračunaj ploščino lika, ki ga omejujejo krivulje $x=0$, $y=1+x^2$, $y=\frac{13}{6}x$.

122. Izračunaj ploščino lika, ki ga omejujeta krivulji $y = 1 + x^2$, $y = \frac{13}{6}x$.

123. Izračunaj ploščino trikotnika z oglišči $(0,0)$, $(2,3)$ in $(4,7)$.

⋆124. Izračunaj ploščino trikotnika z oglišči $(0,0)$, $(a,b)$ in $(c,d)$.

⋆⋆125. Imamo valjasto 3000 litrsko cisterno za olje, ki je postavljena ležeče. Nivo olja v cisterni je polovico polmera pod vrhom. Ali lahko v cisterno dotočimo 600 litrov olja?

126. Izračunaj ploščino lika, ki ga omejujejo krivulje $y = x + 2$, $y = x - 2$, $y = 0$ in $y = 2$.

127. Izračunaj ploščino lika, ki ga omejujejo krivulje $y = x^2 - 2x$, $y = x^2 + 2$ in $y = x^2 + 2x$.

128. Izračunaj ploščino lika, ki ga omejujejo krivulje $y = x^2 - 2x - 2$, $y = x^2 + 2x + 2$ in $y = x^2 + x + 4$.

129. Izračunaj ploščino lika, ki ga omejujeta krivulji $y = x^4 + x^3 + x^2 + x + 1$ in $y = 3x^2 + 7x + 5$.

130. Izračunaj ploščino lika, ki ga omejujeta krivulji $y = \frac{1}{x}$ in $y = -x + \frac{5}{2}$.

131. Izračunaj ploščino lika, ki ga omejujeta krivulji $y = x + \frac{1}{x}$ in $y = \frac{10}{3}$.

# Rešitve

## I. Nedoločeni integral

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$; $\int \frac{dx}{x} = \log x + C$.

2. $t = x + a$: $\int (x+a)^n dx = \int t^n dt = \frac{t^{n+1}}{n+1} + C = \frac{(x+a)^{n+1}}{n+1} + C$.

3. $t = 1 + \frac{x}{3}$: $\int (1 + \frac{x}{3})^7 dx = \int t^7 3dt = \frac{3t^8}{8} + C = \frac{3(1+\frac{x}{3})^8}{8} + C$.

4. $\int (1 + x^2)^2 dx = \int (1 + 2x^2 + x^4) dx = x + \frac{2}{3}x^3 + \frac{1}{5}x^5 + C$.

5. $t = 1 - x^2$: $\int x(1 - x^2)^5 dx = \int t^5 (-\frac{1}{2}) dt = -\frac{1}{12}t^6 + C = -\frac{1}{12}(1 - x^2)^6 + C$.

6. $t = a + x$: $\int \sin(a + x) dx = \int \sin t dt = -\cos t + C = -\cos(a + x) + C$.

7. $t = \sin(ax + b)$: $\int \sin(ax+b)\cos(ax+b) dx = \int t \frac{dt}{a} = \frac{t^2}{2a} + C = \frac{\sin^2(ax+b)}{2a} + C$.

8. $t = \cos x$: $\int \sin^3 x \cos^2 x dx = -\int (1 - t^2) t^2 dt = \int (t^4 - t^2) dt = \frac{t^5}{5} - \frac{t^3}{3} + C =$
$= \frac{\cos^3 x}{15}(3\cos^2 x - 5) + C$.

9. $t = \cos x$: $\int \sin^3 x dx = -\int (1 - t^2) dt = \int (t^2 - 1) dt = \frac{t^3}{3} - t + C = \frac{\cos x}{3}(\cos^2 x - 3) + C$.

10. $t = \sqrt{1 - x^2}$: $\int x\sqrt{1 - x^2} dx = \int \sqrt{1 - t^2}\, t \frac{-tdt}{\sqrt{1-t^2}} = -\int t^2 dt = -\frac{t^3}{3} + C = -\frac{1}{3}(1 - x^2)^{\frac{3}{2}} + C$.

11. $\int \frac{1}{x^2+1} dx = \mathrm{arctg} x + C$.

12. $\int \frac{1}{x^2-1} dx = \frac{1}{2}\int (\frac{1}{x-1} - \frac{1}{x+1}) dx = \frac{1}{2}\log|x - 1| - \frac{1}{2}\log|x + 1| + C = \log\sqrt{\frac{x-1}{x+1}} + C$.

13. $t=\frac{x+2}{\sqrt{3}}$: $\int \frac{2}{x^2+4x+7}dx = \int \frac{2}{(x+2)^2+3}dx = \int \frac{2}{3t^2+3}\sqrt{3}dt = \frac{2}{\sqrt{3}}\int \frac{1}{t^2+1}dt = \frac{2}{\sqrt{3}}\mathrm{arctg}t + C =$
$= \frac{2}{\sqrt{3}}\mathrm{arctg}\frac{x+2}{\sqrt{3}} + C.$

14. $\int \frac{2x+3}{x^2+1}dx = \int \frac{2x}{x^2+1}dx + 3\int \frac{1}{x^2+1}dx = \log(x^2+1) + 3\mathrm{arctg}x + C.$

15. $\int \frac{dx}{2x+1} = \frac{1}{2}\log|2x+1| + C.$

16. $t^2 = x$: $\int \frac{\sqrt{x}+1}{\sqrt{x}-1}dx = \int \frac{t+1}{t-1}2tdt = \int(2t+4+\frac{4}{t-1})dt = t^2+4t+4\log|t-1| + C =$
$= x + 4\sqrt{x} + 4\log|\sqrt{x}-1| + C.$

17. $t = x^2+7$: $\int x\sqrt{x^2+7}dx = \int \frac{1}{2}\sqrt{t}dt = \frac{1}{3}t^{\frac{3}{2}} + C = \frac{1}{3}(x^2+7)^{\frac{3}{2}} + C.$

18. $t = \sqrt{x^2+3}$: $\int \frac{x}{\sqrt{x^2+3}}dx = \int dt = t + C = \sqrt{x^2+3} + C.$

19. $\int \frac{\sqrt{x}+\log x}{x}dx = \int x^{-\frac{1}{2}}dx + \int \frac{\log x}{x}dx = 2\sqrt{x} + \frac{1}{2}\log^2 x + C.$

20. $t = 2x^2+3$: $\int \frac{x}{2x^2+3}dx = \int \frac{\frac{1}{4}dt}{t} = \frac{1}{4}\log t + C = \frac{1}{4}\log(2x^2+3) + C.$

21. $\int(\cos ax + \sin ax)^2 dx = \int(\cos^2 ax + 2\cos ax \sin ax + \sin^2 ax)dx = \int(1+\sin 2ax)dx =$
$= x - \frac{1}{2a}\cos 2ax + C.$

22. $t = \sqrt{x}$: $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx = \int 2\cos t dt = 2\sin t + C = 2\sin\sqrt{x} + C.$

23. Integriramo po delih ($\sin x = u$, $\sin x dx = dv$): $\int \sin^2 x dx = -\sin x\cos x + \int \cos^2 x dx$. Poleg tega velja $\int \sin^2 x dx + \int \cos^2 x dx = \int 1 dx = x + C$. Imamo torej dve enačbi in dva neznana integrala. Rešitev je $\int \sin^2 x dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$, $\int \cos^2 x dx = \frac{x}{2} + \frac{\sin 2x}{4} + C$.

24. $t = x^4$: $\int \frac{x^3}{x^8+5}dx = \frac{1}{4}\int \frac{dt}{t^2+5} = \frac{1}{4\sqrt{5}}\mathrm{arctg}\frac{t}{\sqrt{5}} + C = \frac{1}{4\sqrt{5}}\mathrm{arctg}\frac{x^4}{\sqrt{5}} + C.$

25. $t = \sin x + \cos x$: $\int \frac{\sin x - \cos x}{\sin x + \cos x}dx = -\int \frac{dt}{t} = -\log|t| + C = -\log|\sin x + \cos x| + C.$

26. $t = \frac{1}{x}$: $\int \frac{dx}{x\sqrt{x^2-1}} = \int \frac{-dt}{\sqrt{1-t^2}} = -\arcsin(t) + C = -\arcsin(\frac{1}{x}) + C.$

27. $t = e^{-x}$: $\int \frac{dx}{e^x+1} = -\int \frac{dt}{1+t} = -\log|1+t| + C = -\log(1+e^{-x}) + C.$

28. $t = 5x^2-3$: $\int x(5x^2-3)^7 dx = \frac{1}{10}\int t^7 dt = \frac{1}{10}\frac{t^8}{8} + C = \frac{1}{80}(5x^2-3)^8 + C.$

29. $t = \sqrt{x+1}$: $\int \frac{xdx}{\sqrt{x+1}} = \int 2(t^2-1)dt = \frac{2}{3}t^3 - 2t + C = \frac{2}{3}\sqrt{x+1}^3 - 2\sqrt{x+1} + C.$

30. $t = x^2$: $\int x^3 e^{-x^2}dx = \frac{1}{2}\int te^{-t}dt = -\frac{1}{2}(t+1)e^{-t} + C = -\frac{1}{2}(x^2+1)e^{-x^2} + C.$

31. $u = x$, $dv = e^{-x}dx$: $\int xe^{-x}dx = x(-e^{-x}) - \int(-e^{-x})dx = -xe^{-x} - e^{-x} + C =$
$= -(x+1)e^{-x} + C.$

32. $\int \frac{x}{e^x}dx = \int xe^{-x}dx = -(x+1)e^{-x} + C.$

33. Dvakrat integriramo po delih (prvič $u = x^2$, $dv = e^{3x}dx$, drugič $u = x$, $dv = e^{3x}dx$):
$\int x^2 e^{3x}dx = x^2\frac{1}{3}e^{3x} - \int 2x\frac{1}{3}e^{3x}dx = x^2\frac{1}{3}e^{3x} - \frac{2}{3}\left[x\frac{1}{3}e^{3x} - \int \frac{1}{3}e^{3x}dx\right] = \frac{e^{3x}}{27}(9x^2-6x+2) + C.$

34. $u = x$, $dv = \frac{1}{\cos^2}dx$: $\int \frac{xdx}{\cos^2 x} = x\mathrm{tg}x - \int \mathrm{tg}x dx = x\mathrm{tg}x + \log|\cos x| + C.$

35. $t=\cos x$: $\int \mathrm{tg} x dx = \int \frac{-dt}{t} = -\log|t| + C = -\log|\cos x| + C.$

36. $\int \frac{xdx}{(x-1)(x+1)^2} = \int (\frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{\frac{1}{2}}{(x+1)^2})dx = \frac{1}{4}\log|x-1| - \frac{1}{4}\log|x+1| - \frac{1}{2}\frac{1}{(x+1)} + C.$

37. $\int \frac{dx}{x^3-2x^2+x} = \int (\frac{1}{x} - \frac{1}{x-1} + \frac{1}{(x-1)^2})dx = \log|x| - \log|x-1| + \frac{-1}{(x-1)} + C.$

38. $\int \frac{x^3+x+1}{x(x^2+1)}dx = \int (1 + \frac{1}{x} - \frac{x}{x^2+1})dx = x + \log|x| - \frac{1}{2}\log(x^2+1) + C.$

39. $\int \frac{x^2+4x-2}{x^3+x^2-2x}dx = \int (\frac{1}{x} + \frac{1}{x-1} + \frac{-1}{x+2})dx = \log|x| + \log|x-1| - \log|x+2| + C.$

40. $\int \frac{3x^2-5x-4}{x^3-x^2-x+1}dx = \int (\frac{2}{x-1} + \frac{-3}{(x-1)^2} + \frac{1}{x+1})dx = 2\log|x-1| + \frac{3}{x-1} + \log|x+1| + C.$

41. $t = x+2$: $\int \frac{x^2+6x+7}{(x+2)^3}dx = \int \frac{(t-2)^2+6(t-2)+7}{t^3}dt = \int \left(\frac{1}{t} + \frac{2}{t^2} - \frac{1}{t^3}\right)dt = \log|t| - \frac{2}{t} + \frac{1}{2t^2} + C =$
$= \log|x+2| - \frac{2}{x+2} + \frac{1}{2(x+2)^2} + C.$

42. $t = \frac{x+1}{\sqrt{2}}$: $\int \frac{1}{x^2+2x+3}dx = \int \frac{1}{(x+1)^2+2}dx = \sqrt{2}\int \frac{1}{2t^2+2}dx = \frac{1}{\sqrt{2}}\mathrm{arctg} t + C = \frac{1}{\sqrt{2}}\mathrm{arctg}\frac{x+1}{\sqrt{2}} + C.$

43. $\int \frac{2x^2+6x+6}{(x+1)(x^2+4x+5)}dx = \int \left(\frac{1}{x+1} + \frac{x+1}{(x+2)^2+1}\right)dx =$
$= \log|x+1| + \frac{1}{2}\log\left((x+2)^2+1\right) - \mathrm{arctg}(x+2) + C.$

44. $\int \frac{x^3-x^2-x-2}{(x^2+1)(x^2+2x+2)}dx = \int \left(\frac{x}{x^2+2x+2} - \frac{1}{x^2+1}\right)dx = \frac{1}{2}\log(x^2+2x+2) - \mathrm{arctg}(x+1) - \mathrm{arctg} x + C.$

45. $t^3 = x$: $\int \frac{x+1}{3\sqrt[3]{x^2}}dx = \int \frac{(t^3+1)}{3t^2}3t^2 dt = \int (t^3+1)dt = \frac{1}{4}t^4 + t + C = \frac{1}{4}x^{\frac{4}{3}} + \sqrt[3]{x} + C.$

46. $t^{10} = x$: $\int \frac{1}{\sqrt{x}(1+\sqrt[5]{x})}dx = \int \frac{10t^9}{t^5(1+t^2)}dt = \int \left(10t^2 - 10 + \frac{10}{1+t^2}\right)dt = \frac{10}{3}t^3 - 10t + 10\mathrm{arctg} t + C =$
$= \frac{10}{3}x^{\frac{3}{10}} - 10\sqrt[10]{x} + 10\mathrm{arctg}\sqrt[10]{x} + C.$

47. $t = \sqrt{x}\sin x$: $\int \frac{2x\cos x + \sin x}{\sqrt{x}}dx = \int 2dt = 2t + C = 2\sqrt{x}\sin x + C.$

48. $\int (x^3 - x^2 + 1)dx = \frac{1}{4}x^4 - \frac{1}{3}x^3 + x + C.$

49. $\int (x-1)^2(x+1)dx = \int (x^3 - x^2 - x + 1)dx = \frac{1}{4}x^4 - \frac{1}{3}x^3 - \frac{1}{2}x^2 + x + C.$

50. $\int (\sqrt{x} + \sqrt[3]{x^2})dx = \int (x^{\frac{1}{2}} + x^{\frac{2}{3}})dx = \frac{2}{3}x^{\frac{3}{2}} + \frac{3}{5}x^{\frac{5}{3}} + C.$

51. $\int \left(2x - \frac{1}{x}\right)\sqrt{x}dx = \int \left(2x^{\frac{3}{2}} - x^{-\frac{1}{2}}\right)dx = \frac{4}{5}x^{\frac{5}{2}} - 2x^{\frac{1}{2}} + C.$

52. $\int (\sin x + \cos x + x)dx = -\cos x + \sin x + \frac{1}{2}x^2 + C.$

53. $\int \sin ax dx = -\frac{\cos ax}{a} + C.$

54. $t = 2x+1$: $\int (2x+1)^4 dx = \frac{1}{2}\int t^4 dt = \frac{1}{10}t^5 + C = \frac{1}{10}(2x+1)^5 + C.$

55. $t = x^2$: $\int \frac{xdx}{1+x^4} = \int \frac{\frac{1}{2}dt}{1+t^2} = \frac{1}{2}\mathrm{arctg} t + C = \frac{1}{2}\mathrm{arctg} x^2 + C.$

56. $t = x^3 + 2x^2 - 2x - 1$: $\int \frac{3x^2+4x-2}{x^3+2x^2-2x-1}dx = \int \frac{dt}{t} = \log|t| + C = \log|x^3 + 2x^2 - 2x - 1| + C.$

57. $\int \frac{a}{bx+c}dx = \frac{a}{b}\log|bx+c| + D.$

58. Integriramo po delih ($x = u$, $\cos x dx = dv$): $\int x \cos x dx = x \sin x - \int \sin x dx =$
$= x \sin x + \cos x + C$.

59. Po delih ($\log x = u$, $dx = dv$): $\int \log x dx = x \log x - \int x \frac{1}{x} dx = x \log x - x + C =$
$= x(\log x - 1) + C$.

60. Po delih ($\log x = u$, $x dx = dv$): $\int x \log x dx = \frac{x^2}{2} \log x - \int \frac{x^2}{2} \frac{1}{x} dx = \frac{x^2}{2} \log x - \frac{x^2}{4} + C$.

61. Po delih ($\log x = u$, $x^n dx = dv$): $\int x^n \log x dx = \frac{x^{n+1}}{n+1} \log x - \int \frac{x^{n+1}}{n+1} \frac{1}{x} dx =$
$= \frac{x^{n+1}}{n+1} \log x - \frac{x^{n+1}}{(n+1)^2} + C$.

62. $t = \sin x$: $\int e^{\sin x} \cos x dx = \int e^t dt = e^t + C = e^{\sin x} + C$.

63. Po delih ($x = u$, $\sin x dx = dv$): $\int x \sin x dx = -x \cos x + \int \cos x dx = -x \cos x + \sin x + C$.

64. $t = \sqrt{x^2+1}$: $\int \frac{x^3 - x}{\sqrt{x^2+1}} dx = \int (t^2 - 2) dt = \frac{1}{3} t^3 - 2t + C = \frac{1}{3} \sqrt{x^2+1}^3 - 2\sqrt{x^2+1} + C$.

65. $t = \log x$: $\int \frac{dx}{x \log x} = \int \frac{dt}{t} = \log |t| + C = \log |\log x| + C$.

66. $t = \sin x$: $\int \cos^3 x dx = \int (1 - t^2) dt = t - \frac{1}{3} t^3 + C = \sin x - \frac{1}{3} \sin x^3 + C$.

67. $\int \frac{4x-2}{x^2-1} dx = \int \left( \frac{1}{x-1} + \frac{3}{x+1} \right) dx = \log |x-1| + 3 \log |x+1| + C$.

68. $\int \frac{4x-2}{(x-1)^2} dx = \int \left( \frac{4}{x-1} + \frac{2}{(x-1)^2} \right) dx = 4 \log |x-1| - \frac{2}{x-1} + C$.

69. $\int \frac{4x-2}{(x^2-1)(x-2)} dx = \int \left( \frac{-1}{x-1} + \frac{-1}{x+1} + \frac{2}{x-2} \right) dx = -\log |x-1| - \log |x+1| + 2 \log |x-2| + C$.

70. $\int \frac{3x^2-13x+12}{x^3-6x^2+11x-6} dx = \int \left( \frac{1}{x-1} + \frac{2}{x-2} + \frac{0}{x-3} \right) dx = \log |x-1| + 2 \log |x-2| + C$.

71. $\int \frac{x^2+x+3}{(x+1)(x^2+2)} dx = \int \left( \frac{1}{x+1} + \frac{1}{x^2+2} \right) dx = \log |x+1| + \frac{1}{\sqrt{2}} \mathrm{arctg} \frac{x}{\sqrt{2}} + C$.

72. $t = \sqrt{x}$: $\int \frac{\sqrt{x}}{1+x} dx = \int \frac{t}{1+t^2} 2t dt = 2 \int \left( 1 - \frac{1}{1+t^2} \right) dt = 2t - 2\mathrm{arctg} t + C =$
$= 2\sqrt{x} - 2\mathrm{arctg}\sqrt{x} + C$.

73. Po delih ($\mathrm{arctg} x = u$, $x dx = dv$): $\int x \mathrm{arctg} x dx = \frac{x^2}{2} \mathrm{arctg} x - \int \frac{x^2}{2} \frac{1}{1+x^2} dx =$
$= \frac{x^2}{2} \mathrm{arctg} x - \frac{1}{2} \int \left( 1 - \frac{1}{1+x^2} \right) dx = \frac{x^2}{2} \mathrm{arctg} x - \frac{1}{2} x + \frac{1}{2} \mathrm{arctg} x + C$.

74. Po delih ($\mathrm{arctg} x = u$, $dx = dv$): $\int \mathrm{arctg} x dx = x \mathrm{arctg} x - \int \frac{x}{1+x^2} dx =$
$= x \mathrm{arctg} x - \frac{1}{2} \log(1 + x^2) + C$.

75. Po delih ($\arcsin x = u$, $dx = dv$): $\int \arcsin x dx = x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx =$
$= x \arcsin x + \sqrt{1-x^2} + C$.

76. $\int \sqrt{x+1} dx = \frac{2}{3} (x+1)^{\frac{3}{2}} + C$.

77. Najhitreje gre s substitucijo $t = \sin x$, lepa rešitev pa gre tako:
$\int \sqrt{1-x^2} dx = \int \frac{1-x^2}{\sqrt{1-x^2}} dx = \int \frac{1}{\sqrt{1-x^2}} dx - \int \frac{x^2}{\sqrt{1-x^2}} dx$. Prvi integral znamo izračunati, drugega pa izračunamo po delih ($x = u$, $\frac{x}{\sqrt{1-x^2}} dx = dv$): $\int \frac{1}{\sqrt{1-x^2}} dx - \int \frac{x^2}{\sqrt{1-x^2}} dx =$
$= \arcsin x - \left[ -x\sqrt{1-x^2} + \int \sqrt{1-x^2} dx \right] = \arcsin x + x\sqrt{1-x^2} - \int \sqrt{1-x^2} dx$. Sledi
$\int \sqrt{1-x^2} dx = \frac{1}{2} \arcsin x + \frac{1}{2} x \sqrt{1-x^2} + C$.

78. $t=1-x^2$: $\int x\sqrt{1-x^2}dx=-\frac{1}{2}\int\sqrt{t}dt=-\frac{1}{3}t^{\frac{3}{2}}+C=-\frac{1}{3}\sqrt{1-x^2}^3+C$.

79. $\int\frac{x}{\sqrt{1-x^2}}dx=-\sqrt{1-x^2}+C$.

80. $t=\sqrt{1-x^2}$: $\int\frac{\sqrt{1-x^2}}{x}dx=\int\frac{t}{\sqrt{1-t^2}}\frac{-tdt}{\sqrt{1-t^2}}=\int\frac{-t^2}{1-t^2}dt=\int\left(1+\frac{1}{t^2-1}\right)dt=$
$=t+\frac{1}{2}\log|t-1|-\frac{1}{2}\log|t+1|+C=$
$=\sqrt{1-x^2}+\frac{1}{2}\log|\sqrt{1-x^2}-1|-\frac{1}{2}\log|\sqrt{1-x^2}+1|+C$.

81. $t=x^2$: $\int xe^{-x^2}dx=\frac{1}{2}\int e^{-t}dt=-\frac{1}{2}e^{-t}+C=-\frac{1}{2}e^{-x^2}+C$.

82. $\int xe^{-x^2+\log 2}dx=\int xe^{-x^2}e^{\log 2}dx=2\int xe^{-x^2}dx=-e^{-x^2}+C$.

83. $t=\sin x$: $\int\frac{dx}{2\sin x\,\mathrm{tg}x+\cos x}=\int\frac{\cos x dx}{2\sin^2 x+\cos^2 x}=\int\frac{dt}{1+t^2}=\mathrm{arctg}t+C=\mathrm{arctg}(\sin x)+C$.

84. Dvakrat po delih: $\int x^2\sin xdx=-x^2\cos x+2\int x\cos xdx=$
$=-x^2\cos x+2x\sin x-2\int\sin xdx=-x^2\cos x+2x\sin x+2\cos x+C$.

85. $t=x^2$: $\int x\sin x^2dx=\int\sin t\frac{dt}{2}=-\frac{1}{2}\cos t+C=-\frac{1}{2}\cos x^2+C$.

86. $t^2=x$: $\int\sin\sqrt{x}dx=\int 2t\sin tdt=2\left[-t\cos t+\sin t\right]+C=-2\sqrt{x}\cos\sqrt{x}+2\sin\sqrt{x}+C$.

87. $\int\frac{x^2-1}{x-1}dx=\int(x+1)dx=\frac{1}{2}x^2+x+C$.

88. $\int\frac{x^2+1}{x-1}dx=\int\left(x+1+\frac{2}{x-1}\right)dx=\frac{1}{2}x^2+x+2\log|x-1|+C$.

89. $\int\frac{x^2+1}{x^2-6x+10}dx=\int\left(1+3\frac{2x-6}{x^2-6x+10}+\frac{9}{(x-3)^2+1}\right)dx=x+3\log(x^2-6x+10)+9\mathrm{arctg}(x-3)+C$.

90. $\int\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)dx=\int\left(x^{\frac{3}{2}}+3x+2x^{\frac{1}{2}}\right)dx=\frac{2}{5}x^{\frac{5}{2}}+\frac{3}{2}x^2+\frac{4}{3}x^{\frac{3}{2}}+C$.

91. $t=19x+10$: $\int(19x+10)^3dx=\int t^3\frac{dt}{19}=\frac{1}{4\cdot 19}t^4+C=\frac{1}{4\cdot 19}(19x+10)^4+C$.

92. $\int\frac{x^4+8}{x^3+1}dx=\int\left(x+\frac{-x+8}{(x+1)(x^2-x+1)}\right)dx=\int\left(x+\frac{3}{x+1}+\frac{-3x+5}{x^2-x+1}\right)dx=$
$=\int\left(x+3\frac{1}{x+1}-\frac{3}{2}\frac{2x-1}{x^2-x+1}+\frac{7}{2}\frac{1}{(x-\frac{1}{2})^2+\frac{3}{4}}\right)dx=$
$=\frac{1}{2}x^2+3\log|x+1|-\frac{3}{2}\log(x^2-x+1)+\frac{7}{2}\frac{1}{\frac{\sqrt{3}}{2}}\mathrm{arctg}\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}+C$.

93. $\int\frac{1+x+x^2+x^3}{x^2+1}dx=\int(x+1)dx=\frac{1}{2}x^2+x+C$.

94. $\int\frac{x^3}{x^2+1}dx=\int\left(x-\frac{x}{x^2+1}\right)dx=\frac{1}{2}x^2-\frac{1}{2}\log(x^2+1)+C$

95. $t=\sin x$: $\int\sqrt{\sin x}\cos xdx=\int\sqrt{t}dt=\frac{2}{3}t^{\frac{3}{2}}+C=\frac{2}{3}\sin^{\frac{3}{2}}x+C$.

96. $t=1+\sin x$: $\int\sqrt{1+\sin x}\cos xdx=\int\sqrt{t}dt=\frac{2}{3}t^{\frac{3}{2}}+C=\frac{2}{3}(1+\sin x)^{\frac{3}{2}}+C$.

97. $t=\mathrm{tg}x$: $\int\frac{dx}{\sin x\cos x}=\int\frac{dx}{\mathrm{tg}x\cos^2 x}=\int\frac{dt}{t}=\log|t|+C=\log|\mathrm{tg}x|+C$.

98. $t=\cos x$: $\int\frac{\sin x}{\cos^2 x}dx=\int\frac{-dt}{t^2}=\frac{1}{t}+C=\frac{1}{\cos x}+C$.

99. $t=\cos x$: $\int\frac{\sin x}{\cos^3 x}dx=\int\frac{-dt}{t^3}=\frac{1}{2t^2}+C=\frac{1}{2\cos^2 x}+C$.

100. $t=\sin x$: $\int\mathrm{ctg}xdx=\int\frac{\cos x}{\sin x}dx=\int\frac{dt}{t}=\log|t|+C=\log|\sin x|+C$.

101. $t=1-x^3$: $\int x^2(1-x^3)^2dx=-\int t^2\frac{dt}{3}=-\frac{1}{9}t^3+C=-\frac{1}{9}(1-x^3)^3+C$.

# II. Določeni integral

102. Uporabimo definicijo z zgornjimi vsotami: $\int\limits_0^1 x dx = \lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \frac{1}{n} \cdot \frac{k}{n} = \lim\limits_{n\to\infty} \frac{1}{n^2} \sum\limits_{k=1}^{n} k =$
$= \lim\limits_{n\to\infty} \frac{1}{n^2} \cdot \frac{n(n+1)}{2} = \lim\limits_{n\to\infty} \frac{n^2+n}{2n^2} = \frac{1}{2}$.

103. Uporabimo definicijo s spodnjimi vsotami: $\int\limits_0^1 x^2 dx = \lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \frac{1}{n} \cdot \left(\frac{k-1}{n}\right)^2 =$
$= \lim\limits_{n\to\infty} \frac{1}{n^3} \sum\limits_{k=1}^{n} (k-1)^2 = \lim\limits_{n\to\infty} \frac{1}{n^3} \cdot \frac{(n-1)n(2n-1)}{6} = \lim\limits_{n\to\infty} \frac{2n^3-3n+1}{6n^3} = \frac{1}{3}$.

104. Uporabimo definicijo z zgornjimi vsotami: $\int\limits_0^1 e^x dx = \lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \frac{1}{n} \cdot e^{\frac{k}{n}} = \lim\limits_{n\to\infty} \frac{1}{n} \sum\limits_{k=1}^{n} e^{\frac{k}{n}} =$
$= \lim\limits_{n\to\infty} \frac{1}{n} e^{\frac{1}{n}} \frac{e^{\frac{n}{n}}-1}{e^{\frac{1}{n}}-1} = \lim\limits_{n\to\infty} \frac{e^{\frac{1}{n}}(e-1)}{n(e^{\frac{1}{n}}-1)}$. To limito lahko izračunamo na mnogo načinov. Eden od njih je s pomočjo Taylorjeve vrste za $e^x$: $\lim\limits_{n\to\infty} \frac{e^{\frac{1}{n}}(e-1)}{n(e^{\frac{1}{n}}-1)} = (e-1) \lim\limits_{n\to\infty} \frac{1}{n(1-e^{-\frac{1}{n}})} =$
$= (e-1) \lim\limits_{n\to\infty} \frac{1}{n(1-(1-\frac{1}{n}+\frac{1}{2n^2}-\frac{1}{6n^3}+\ldots)} = (e-1) \lim\limits_{n\to\infty} \frac{1}{1-\frac{1}{2n}+\frac{1}{6n^2}-\ldots} = e-1$.

105. $t = 4x-3$: $\int\limits_1^3 \sqrt{4x-3}dx = \int\limits_1^9 \sqrt{t}\frac{1}{4}dt = \frac{1}{4}\left[\frac{2}{3}t^{\frac{3}{2}}\right]_1^9 = \frac{1}{4} \cdot \frac{2}{3} \cdot 27 - \frac{1}{4} \cdot \frac{2}{3} \cdot 1 = \frac{13}{3}$.

106. $\int\limits_0^{2\pi} \sin x dx = [-\cos x]_0^{2\pi} = -\cos 2\pi + \cos 0 = -1+1 = 0$.

107. $\int\limits_0^{2\pi} \sin^2 x dx = \int\limits_0^{2\pi} \frac{1-\cos 2x}{2} dx = \left[\frac{x}{2} - \frac{\sin 2x}{4}\right]_0^{2\pi} = \pi$.

# III. Uporabne naloge iz integriranja

108. $(\sin^2 x)' = 2\sin x \cos x = \sin 2x$, $\sin^2 x = \int \sin 2x dx = \frac{-\cos 2x}{2} + C$. $C$ dobimo tako, da v enakost vstavimo kakšno točko, recimo $x=0$: $0 = \frac{-1}{2} + C$. Dobili smo znano enačbo $\sin^2 x = \frac{1}{2}(1-\cos 2x)$.

109. Krivulja je $y=0$. Sledi $y'=0$ in $l = \int\limits_0^1 \sqrt{1+y'^2}dx = \int\limits_0^1 1 dx = [x]_0^1 = 1$.

110. Krivulja je $y = \frac{b}{a}x$. Sledi $y' = \frac{b}{a}$ in $l = \int\limits_0^a \sqrt{1+y'^2}dx = \int\limits_0^a \sqrt{1+\frac{b^2}{a^2}}dx = \sqrt{1+\frac{b^2}{a^2}}\,[x]_0^a =$
$= a\sqrt{1+\frac{b^2}{a^2}} = \sqrt{a^2+b^2}$.

111. $y = \sqrt{x}^3$, $y' = \frac{3}{2}x^{\frac{1}{2}}$, $\sqrt{1+y'^2} = \sqrt{1+\frac{9}{4}x}$, $l = \int\limits_0^1 \sqrt{1+\frac{9}{4}x}dx = \left[\frac{4}{9}\frac{2}{3}\left(1+\frac{9}{4}x\right)^{\frac{3}{2}}\right]_0^1 =$
$= \frac{8}{27}\frac{13}{4}\frac{\sqrt{13}}{2} - \frac{8}{27} = \frac{13\sqrt{13}-8}{27}$.

112. $y = \sqrt{x}$, $y' = \frac{1}{2\sqrt{x}}$, $\sqrt{1+y'^2} = \sqrt{1+\frac{1}{4x}}$. Najprej se lotimo nedoločenega integrala. Uvedemo novo spremenljivko $t = \sqrt{1+\frac{1}{4x}}$. Izrazimo $x = \frac{1}{4}\frac{1}{t^2-1}$ in $dx = \frac{1}{4}\frac{-2t}{(t^2-1)^2}$.
$\int \sqrt{1+\frac{1}{4x}}dx = -\int t\frac{1}{2}\frac{t}{(t^2-1)^2}dt = -\frac{1}{2}\int \frac{t^2}{(t-1)^2(t+1)^2}dt =$
$= -\frac{1}{2}\int \left(\frac{\frac{1}{4}}{t-1} + \frac{\frac{1}{4}}{(t-1)^2} + \frac{-\frac{1}{4}}{t+1} + \frac{\frac{1}{4}}{(t+1)^2}\right) dt =$

$= -\frac{1}{8}\log|t-1| + \frac{1}{8}\frac{1}{t-1} + \frac{1}{8}\log|t+1| + \frac{1}{8}\frac{1}{t+1} + C = \frac{1}{8}\log|\frac{t+1}{t-1}| + \frac{1}{8}\frac{t+1+t-1}{(t-1)(t+1)} + C =$
$= \frac{1}{8}\log|\frac{t^2+2t+1}{t^2-1}| + \frac{1}{4}\frac{t}{t^2-1} + C = \frac{1}{8}\log|\frac{1+\frac{1}{4x}+2\sqrt{1+\frac{1}{4x}}+1}{\frac{1}{4x}}| + \frac{1}{4}\frac{\sqrt{1+\frac{1}{4x}}}{\frac{1}{4x}} + C =$
$= \frac{1}{8}\log|8x+1+8x\sqrt{1+\frac{1}{4x}}| + x\sqrt{1+\frac{1}{4x}} + C$. Torej je
$l = \int_0^1 \sqrt{1+y'^2}dx = \left[\frac{1}{8}\log|8x+1+8x\sqrt{1+\frac{1}{4x}}| + x\sqrt{1+\frac{1}{4x}}\right]_0^1 = \frac{1}{8}\log|9+4\sqrt{5}| + \frac{\sqrt{5}}{2}$.

113. $y = x^2$, $y' = 2x$, $\sqrt{1+y'^2} = \sqrt{1+4x^2}$. Najprej se lotimo nedoločenega integrala. Substitucija $t = 2x$ nam da $\int\sqrt{1+4x^2}dx = \frac{1}{2}\int\sqrt{1+t^2}dt = \frac{1}{2}\int\frac{1}{\sqrt{1+t^2}}dt + \frac{1}{2}\int\frac{t^2}{\sqrt{1+t^2}}dt$. Prvi integral je elementaren, drugega se lotimo po delih ($u = t$, $dv = \frac{t}{\sqrt{1+t^2}}dt$):
$\frac{1}{2}\int\frac{1}{\sqrt{1+t^2}}dt + \frac{1}{2}\int\frac{t^2}{\sqrt{1+t^2}}dt = \frac{1}{2}\log|t+\sqrt{1+t^2}| + \frac{1}{2}\left[t\sqrt{1+t^2} - \int\sqrt{1+t^2}dt\right]$. Dobili smo enačbo $\frac{1}{2}\int\sqrt{1+t^2}dt = \frac{1}{2}\log|t+\sqrt{1+t^2}| + \frac{1}{2}t\sqrt{1+t^2} - \frac{1}{2}\int\sqrt{1+t^2}dt$, od koder sledi $\frac{1}{2}\int\sqrt{1+t^2}dt = \frac{1}{4}\log|t+\sqrt{1+t^2}| + \frac{1}{4}t\sqrt{1+t^2} + C$. Torej je
$\int\sqrt{1+4x^2}dx = \frac{1}{4}\log|2x+\sqrt{1+4x^2}| + \frac{1}{2}x\sqrt{1+4x^2} + C$ in zato
$l = \int_0^1\sqrt{1+y'^2}dx = \left[\frac{1}{4}\log|2x+\sqrt{1+4x^2}| + \frac{1}{2}x\sqrt{1+4x^2}\right]_0^1 = \frac{1}{4}\log|2+\sqrt{5}| + \frac{\sqrt{5}}{2}$. Preveri, da je ta rezultat enak rezultatu prejšnje naloge, ter premisli, ali je to le naključje ali pa morata biti rezultata res enaka.

114. Četrtino obsega enotskega kroga tvori krivulja $y = \sqrt{1-x^2}$ od točke $(0,1)$ do točke $(1,0)$. $y' = \frac{-x}{\sqrt{1-x^2}}$ in $\sqrt{1+y'^2} = \sqrt{1+\frac{x^2}{1-x^2}} = \sqrt{\frac{1}{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$. Torej je $l = \int_0^1\sqrt{1+y'^2}dx =$
$= \int_0^1\frac{1}{\sqrt{1-x^2}}dx = [\arcsin x]_0^1 = \frac{\pi}{2} - 0 = \frac{\pi}{2}$. Obseg celega enotskega kroga je potemtakem $4\cdot\frac{\pi}{2} = 2\pi$.

115. Četrtino enotskega kroga omejuje krivulja $y = \sqrt{1-x^2}$ od točke $(0,1)$ do točke $(1,0)$. $pl = \int_0^1 y\,dx = \int_0^1\sqrt{1-x^2}dx = \left[\frac{1}{2}\arcsin x + \frac{1}{2}x\sqrt{1-x^2}\right]_0^1 = \frac{\pi}{4}$. Ta integral smo namreč že izračunali (naloga 77). Potem je ploščina enotskega kroga $4\cdot\frac{\pi}{4} = \pi$.

116. Najprej poiščemo presečišča: $x^2 = \frac{1}{2}x + \frac{1}{2}$. Rešitvi sta $x = 1$ in $x = -\frac{1}{2}$. Ploščina lika je $pl = \int_{-\frac{1}{2}}^1\left(\frac{1}{2}x + \frac{1}{2} - x^2\right)dx = \left[\frac{1}{4}x^2 + \frac{1}{2}x - \frac{1}{3}x^3\right]_{-\frac{1}{2}}^1 = = \frac{1}{4} + \frac{1}{2} - \frac{1}{3} - \frac{1}{16} + \frac{1}{4} - \frac{1}{24} = \frac{9}{16}$.

117. Enačba $x^2 = 1 - x^2$ ima rešitvi $x = -\frac{1}{\sqrt{2}}$ in $x = \frac{1}{\sqrt{2}}$, zato je $pl = \int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}}(1-2x^2)\,dx =$
$= \left[x - \frac{2}{3}x^3\right]_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}} - \frac{2}{3}\frac{1}{2}\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{2}{3}\frac{1}{2}\frac{1}{\sqrt{2}} = \sqrt{2} - \frac{\sqrt{2}}{3} = \frac{2\sqrt{2}}{3}$.

118. Enačba $x^4 = 1 - x^4$ ima rešitvi $x = -2^{-\frac{1}{4}}$ in $x = 2^{-\frac{1}{4}}$, zato je $pl = \int_{-2^{-\frac{1}{4}}}^{2^{-\frac{1}{4}}}(1-2x^4)\,dx =$
$= \left[x - \frac{2}{5}x^5\right]_{-2^{-\frac{1}{4}}}^{2^{-\frac{1}{4}}} = 2^{-\frac{1}{4}} - \frac{2}{5}\frac{1}{2}2^{-\frac{1}{4}} + 2^{-\frac{1}{4}} - \frac{2}{5}\frac{1}{2}2^{-\frac{1}{4}} = 2^{\frac{3}{4}} - \frac{2^{\frac{3}{4}}}{5} = 4\frac{2^{-\frac{1}{4}}}{5}$.

119. Enačba $x^3 - x = x - x^3$ ima rešitve $x = -1$, $x = 0$ in $x = 1$. Zaradi simetrije je torej $pl = 2\int_0^1(x - x^3 - x^3 + x)\,dx = 4\left[\frac{1}{2}x^2 - \frac{1}{4}x^4\right]_0^1 = 2 - 1 = 1$.

120. Presečišča drugih dveh krivulj so rešitve enačbe $x^2 = 3 - 2x^3$, torej $2x^3 + x^2 - 3 = 0$. Edina ničla je 1, saj je $2x^3 + x^2 - 3 = (x-1)(2x^2+3x+3)$. Ploščina je torej
$pl = \int_0^1 (3 - 2x^3 - x^2)\,dx = \left[3x - \frac{1}{2}x^4 - \frac{1}{3}x^3\right]_0^1 = 3 - \frac{1}{2} - \frac{1}{3} = \frac{13}{6}$.

121. Presečišča drugih dveh krivulj so rešitve enačbe $1 + x^2 = \frac{13}{6}x$. Rešimo kvadratno enačbo: $x^2 - \frac{13}{6}x + 1 = (x - \frac{2}{3})(x - \frac{3}{2})$. Zato je $pl = \int_0^{\frac{2}{3}} \left(1 + x^2 - \frac{13}{6}x\right) dx = \left[x + \frac{1}{3}x^3 - \frac{13}{12}x^2\right]_0^{\frac{2}{3}} =$
$= \frac{2}{3} + \frac{1}{3}\frac{8}{27} - \frac{13}{12}\frac{4}{9} = \frac{54+8-39}{81} = \frac{23}{81}$.

122. $pl = \int_{\frac{2}{3}}^{\frac{3}{2}} \left(\frac{13}{6}x - 1 - x^2\right) dx = \left[\frac{13}{12}x^2 - x - \frac{1}{3}x^3\right]_{\frac{2}{3}}^{\frac{3}{2}} = \frac{13}{12}\frac{9}{4} - \frac{3}{2} - \frac{1}{3}\frac{27}{8} + \frac{23}{81} = \frac{39-24-18}{16} + \frac{23}{81} =$
$= \frac{16 \cdot 23 - 3 \cdot 81}{16 \cdot 81} = \frac{125}{1296}$.

123. Krivulje, ki ga omejujejo, so $y = \frac{3}{2}x$, $y = \frac{7}{4}x$ in $y = 2x - 1$. Potem velja
$pl = \int_0^4 \frac{7}{4}x dx - \int_0^2 \frac{3}{2}x dx - \int_2^4 (2x-1)\,dx = \left[\frac{7x^2}{8}\right]_0^4 - \left[\frac{3x^2}{4}\right]_0^2 - [x^2 - x]_2^4 = 14-3-16+4+4-2 = 1$.

124. Krivulje, ki ga omejujejo, so $y = \frac{b}{a}x$, $y = \frac{d}{c}x$ in $y = \frac{d-b}{c-a}x + \frac{bc-ad}{c-a}$. Potem velja
$pl = \int_a^c \left(\frac{d-b}{c-a}x + \frac{bc-ad}{c-a}\right) dx - \int_a^0 \frac{b}{a}x dx - \int_0^c \frac{d}{c}x dx$ (premisli, zakaj je to vedno res). Sedaj le še poračunamo: $pl = \left[\frac{d-b}{c-a}\frac{x^2}{2} + \frac{bc-ad}{c-a}x\right]_a^c - \left[\frac{bx^2}{2a}\right]_a^0 - \left[\frac{dx^2}{2c}\right]_0^c =$
$= \frac{d-b}{c-a}\frac{(c^2-a^2)}{2} + bc - ad + \frac{ab}{2} - \frac{cd}{2} = \frac{cd+ad-ab-bc}{2} + bc - ad + \frac{ab}{2} - \frac{cd}{2} = \frac{bc-ad}{2}$. Če dobimo negativen rezultat, mu spremenimo predznak.

125. Izračunajmo, kolikšen del kroga leži nad polovico radija. Vzemimo enotski krog. Ploščina nad polovico radija je lik, ki ga omejujeta krivulji $y = \sqrt{1-x^2}$ in $y = \frac{1}{2}$. Enačba $\sqrt{1-x^2} = \frac{1}{2}$ ima rešitvi $x = -\frac{\sqrt{3}}{2}$ in $x = \frac{\sqrt{3}}{2}$. Zaradi simetrije je iskana ploščina
$pl = 2\int_0^{\frac{\sqrt{3}}{2}} \left(\sqrt{1-x^2} - \frac{1}{2}\right) dx = 2\left[\frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\arcsin x - \frac{1}{2}x\right]_0^{\frac{\sqrt{3}}{2}} =$
$= \frac{\sqrt{3}}{2}\frac{1}{2} + \arcsin\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$. Pri tem smo spet uporabili nalogo 77. Ker je ploščina enotskega kroga $\pi$, je delež praznega prostora v cisterni $\frac{\frac{\pi}{3} - \frac{\sqrt{3}}{4}}{\pi} = \frac{1}{3} - \frac{\sqrt{3}}{4\pi} \simeq 0,1955$. Ker je cisterna 3000 litrska, gre vanjo $3000 \cdot 0,1955 = 586,5$ litrov.

126. Najprej si narišemo graf danih krivulj in izračunamo presečišča: $y = x + 2$ seka $y = 0$ v točki $x = -2$, $y = 2$ pa v točki $x = 0$. $y = x - 2$ seka $y = 0$ v točki $x = 2$ in $y = 2$ v točki $x = 4$. Sedaj je $pl = \int_{-2}^0 (x + 2 - 0)dx + \int_0^2 (2-0)dx + \int_2^4 (2 - (x-2))dx =$
$= \left[\frac{x^2}{2} + 2x\right]_{-2}^0 + [2x]_0^2 + \left[4x - \frac{x^2}{2}\right]_2^4 = [-2+4] + [4] + [16-8-8+2] = 8$.

127. Presečišča krivulj so: prvih dveh $x = -1$, prve in tretje $x = 0$ in zadnjih dveh $x = 1$. Potem je $pl = \int_{-1}^0 (x^2 + 2 - (x^2 - 2x))dx + \int_0^1 (x^2 + 2 - (x^2 + 2x))dx =$
$= [x^2 + 2x]_{-1}^0 + [-x^2 + 2x]_0^1 = [-1+2] + [-1+2] = 2$.

128. Presečišča krivulj so: prvih dveh $x = -1$, prve in tretje $x = -2$ in zadnjih dveh $x = 2$. Potem je $pl = \int_{-2}^{-1} (x^2 + x + 4 - (x^2 - 2x - 2))dx + \int_{-1}^{2} (x^2 + x + 4 - (x^2 + 2x + 2))dx =$
$= \left[3\frac{x^2}{2} + 6x\right]_{-2}^{-1} + \left[-\frac{x^2}{2} + 2x\right]_{-1}^{2} = \left[\frac{3}{2} - 6 - 6 + 12\right] + \left[-2 + 4 + \frac{1}{2} + 2\right] = 6.$

129. Presečišči sta rešitvi enačbe $x^4+x^3+x^2+x+1 = 3x^2+7x+5$ oziroma $x^4+x^3-2x^2-6x-4 = 0$ in sta $x = -1$ in $x = 2$, zato je
$pl = \int_{-1}^{2} (x^4 + x^3 + x^2 + x + 1 - (3x^2 + 7x + 5))dx = \left[\frac{x^5}{5} + \frac{x^4}{4} - 2\frac{x^3}{3} - 6\frac{x^2}{2} - 4x\right]_{-1}^{2} =$
$= \left(\frac{32}{5} + 4 - \frac{16}{3} - 12 - 8\right) - \left(-\frac{1}{5} + \frac{1}{4} + \frac{2}{3} - 3 + 4\right) = -\frac{333}{20}$. Ploščina je torej $\frac{333}{20}$.

130. Presečišči sta rešitvi enačbe $\frac{1}{x} = -x + \frac{5}{2}$ oziroma $2x^2 - 5x + 2 = 0$ in sta $x = \frac{1}{2}$ in $x = 2$, zato je $pl = \int_{\frac{1}{2}}^{2} (-x + \frac{5}{2} - \frac{1}{x})dx = \left[-\frac{x^2}{2} + \frac{5}{2}x - \log x\right]_{\frac{1}{2}}^{2} = -2 + 5 - \log 2 + \frac{1}{8} - \frac{5}{4} + \log\frac{1}{2} =$
$= \frac{15}{8} - \log 4.$

131. Presečišči sta rešitvi enačbe $x + \frac{1}{x} = \frac{10}{3}$ oziroma $3x^2 - 10x + 3 = 0$ in sta $x = \frac{1}{3}$ in $x = 3$, zato je $pl = \int_{\frac{1}{3}}^{3} (\frac{10}{3} - x - \frac{1}{x})dx = \left[\frac{10}{3}x - \frac{x^2}{2} - \log x\right]_{\frac{1}{3}}^{3} = 10 - \frac{9}{2} - \log 3 - \frac{10}{9} + \frac{1}{18} + \log\frac{1}{3} =$
$= \frac{40}{9} - \log 9.$

# Kazalo