1. naloga: s pomočjo karakteristične funkcije dokaži enakost

$$\chi(A,x)=\begin{cases}1 ; & x\in A\\ 0 ; & x\notin A\end{cases}$$

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$$|A|=\sum_{x\in\varphi}\chi(A,x)$$

a)

$$|A\cup B|=|A|+|B|-|A\cap B|=\sum_{x\in\varphi}\chi(A,x)+\sum_{x\in\varphi}\chi(B,x)-\sum_{x\in\varphi}\chi(A\cap B,x)$$

$$=\sum_{x\in\varphi}\big(\chi(A,x)+\chi(B,x)-\chi(A,x)\cdot\chi(B,x)\big)=\sum_{x\in\varphi}\chi(A\cup B,x)$$

1	0	1		0
0	1	0	0	1
1	1	1	1	1

c)

$$|A \cup B \cup C| = |A| + |A^c \cap B| + |A^c \cap B^c \cap C| =$$

$$= \sum_{x \in S} \Big(\chi(A,x) + \big(1-\chi(A,x)\big) \cdot \chi(B,x)$$

$x \in A$	1		
$x \notin A,\ x \in B$	0	1	0
$x \notin A,\ x \notin B,\ x \in C$	0	0	1

$$+ \big(1-\chi(A,x)\big) \cdot \big(1-\chi(B,x)\big) \cdot \chi(C,x) \Big) =$$

$$= \sum_{x \in S} \chi(A \cup B \cup C, x)$$

2. naloga: določi število udeležencev ankete

S - vsi sadje

M - mental

G - grozdje

A - anketiranci

$|A| = 50$

$|S| = 22$

$|M| = 25$

$|G| = 39$

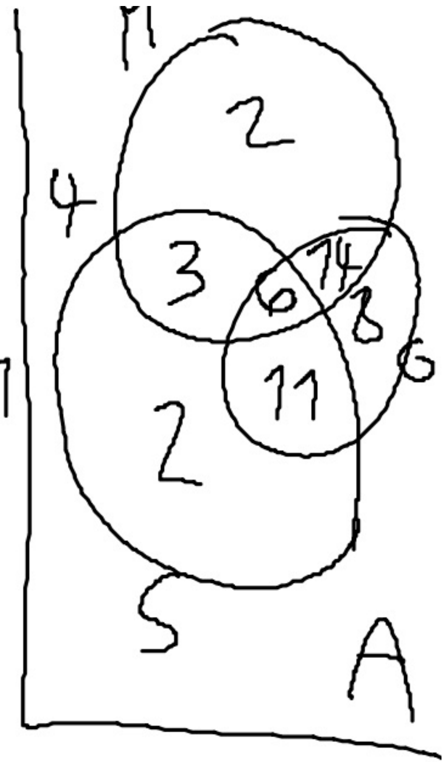
$|M \cap S| = 9$

$|S \cap G| = 17$

$|M \cap G| = 20$

$|M \cap G \cap S| = 6$

$|M^c \cap G^c \cap S^c| = 4$



$$|A| = |S \cup M \cup G| + |(S \cup M \cup G)^c| =$$

$$= |S| + |M| + |G| - |S \cap M| - |S \cap G| - |M \cap G| + |S \cap M \cap G| + |S^c \cap M^c \cap G^c| = 22 + 25 + 39 - 9 - 17 - 20 + 6 + 4 = 86 - 46 + 10 = 50$$

3. naloga

Koliko št. med 1 in 1000 je deljivih z vsaj enim od števil 6, 7 ali 10.

D_n = množica večkrat. št. n

$$|D_6 \cup D_7 \cup D_{10}| = |D_6| + |D_7| + |D_{10}| - \overbrace{|D_6 \cap D_7|}^{D_{42}} - \overbrace{|D_6 \cap D_{10}|}^{D_{30}}$$

$$- \underbrace{|D_7 \cap D_{10}|}_{D_{70}} + \underbrace{|D_6 \cap D_7 \cap D_{10}|}_{D_{210}}$$

$$= 166 + 142 + 100 - 24 - 33 - 14 + 4 = 412$$

$$\begin{array}{r} 412 \\ -71 \\ \hline 341 \end{array}$$

4. naloga

$[1, 18024]$

Koliko je takšnih, ki so deljiva z 21 ali 12 in niso deljiva z 26.

$$|(D_{12} \cup D_{21}) \setminus D_{26}|$$

$$= |D_{12}| + |D_{21}| - \underbrace{|D_{12} \cap D_{26}|}^{156} - \underbrace{|D_{21} \cap D_{26}|}^{546} - \underbrace{|D_{12} \cap D_{21}|}^{84}$$

$$+ |D_{12} \cap D_{21} \cap D_{26}| =$$

$$= 1502 + 858 - 115 - 33 - 214 + 16 = \underline{\underline{2014}}$$

$$\begin{array}{r|l} 12 & 2 \\ 6 & 2 \\ 3 & 3 \\ 1 & \end{array} \qquad \begin{array}{r|l} 21 & 3 \\ 7 & 7 \\ 1 & \end{array} \qquad \begin{array}{r|l} 26 & 2 \\ 13 & 13 \\ 1 & \end{array}$$

$$2 \cdot 2 \cdot 3 \cdot 7 \cdot 13 = 1092$$

5. naloga: pokaži, da je moč domene injektivne preslikave manjša ali enaka moči njene kodomene

$f: A \to B$ injektivna

$|A| \leq |B|$

$f(A) \subseteq B$

$g: A \to f(A)$

$g: x \mapsto f(x)$

g je bijektivna

$|A| = |f(A)| \leq |B|$

$|A| \leq |B|$

6. naloga: konstruiraj dobro urejenost racionalnih števil

$$|\mathbb{Q}| = |\mathbb{N}| = |\mathbb{N}^2|$$

$\frac{a}{b}$ $\quad a \perp b$

$\gcd(a, b) = 1$

	1	2	3	4	...
1	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	...
2	$\frac{2}{1}$		$\frac{2}{3}$		
3	$\frac{3}{1}$	$\frac{3}{2}$		$\frac{3}{4}$	
4	$\frac{4}{1}$		$\frac{4}{3}$		
⋮	⋮				

$$0, 1, -1, \frac{1}{2}, -\frac{1}{2}, 2, -2, \frac{1}{3}, -\frac{1}{3}, 3, -3, \frac{1}{4}, -\frac{1}{4}, \ldots$$

$$|2^{\mathbb{N}}| = |\mathbb{N}^{\mathbb{N}}| = |\mathbb{R}| = |\mathbb{N}^{\mathbb{R}}| = c$$

7. naloga: pokaži, da sta množici enako močni

$$\overset{A}{[0,1)} \sim \overset{B}{[0,1]}$$

$$f: A \to B \qquad f^{-1}: B \to A$$

$$a_n = 2^{-n}$$

$$f(x) = \begin{cases} 2x \,; & x = 2^{-n} \\ x \,; & x \neq 2^{-n} \end{cases} \qquad f^{-1}(x) = \begin{cases} \frac{1}{2}x \,; & x = 2^{1-n} \\ x \,; & x \neq 2 \end{cases}$$

$$n \in \mathbb{N} \setminus \{0\}$$

8. naloga: ali ima množica A enako moč kot množica realnih števil?

$$A = \{\underbrace{f : \mathbb{R} \to \mathbb{R}}_{\text{preslikave}}\} = \mathbb{R}^{\mathbb{R}}$$

$$f, g : A \to B$$

$$f = g \iff \forall x \in A : f(x) = g(x)$$

$$\underset{\overset{\shortparallel}{c}}{|\mathbb{R}|} < |A|$$

$f : \mathbb{R} \to A$ bijektivna

$f : x \mapsto f_x$

$g : \forall x \in \mathbb{R} : g \neq f_x$

$$g(x) = f_x(x) + 1$$

f ni surjektivna