

1. naloga

a) Pokaži, da je R delna urejenost

$$\mathbb{R}^2 \quad (x, y) R (z, w) \Leftrightarrow y \leq w \wedge (x - y) \leq (z - w)$$

REFLEKSIVNOST: ✓

$$(x, y) R (x, y)$$

ANTISIMETRIČNOST:

$$(x, y) R (z, w) \wedge (z, w) R (x, y)$$

$$y \leq w \wedge (x - y) \leq (z - w) \wedge w \leq y \wedge (z - w) \leq (x - y)$$

$$y \leq w \wedge w \leq y \quad (z - w) = (x - y)$$

$$y = w$$

$$z = x$$

$$\mathbb{R}^2 \quad (x, y) R (z, w) \Leftrightarrow y \leq w \wedge (x - y) \leq (z - w)$$

TRANZITIVNOST  $\checkmark$   
 $(x, y) R (i, j) \wedge (i, j) R (z, w) \Rightarrow (x, y) R (z, w)$

$$(x \leq j) \wedge (x - y) \leq (i - j) \wedge (j \leq w) \wedge (i - j) \leq (z - w) \\ \Rightarrow (x - y) \leq (z - w) \wedge (x \leq w)$$

$$(2, 1) R (2, 2) \Leftrightarrow 1 \leq 2 \wedge \underline{1 \leq 0}$$

$$(2, 2) R (2, 1) \Leftrightarrow \underline{2 \leq 1} \wedge 0 \leq 1$$

b) Ali je R linearna urejenost?

ni zavisna  
 $\Rightarrow$  ni lin. urejenost

c) Poišči neskončno množico A, ki bo imela prvi element glede na R.

$$\mathbb{R}^2 \quad (x, y) R (z, w) \Leftrightarrow y \leq w \wedge (x - y) \leq (z - w)$$

$$(i, j) R (x, y)$$

$$\forall (x, y) \in A \quad \exists (i, j) \in A \quad (i - j) \leq (x - y)$$

$$\forall (x, y) \in A \quad \exists (i, j) \in A \quad (i - y + 1) \leq (x - y)$$

$$A = \{(x, y) \mid x, y \in \mathbb{N} \wedge x \geq y\} \quad \left. \begin{array}{l} i + 1 \leq x \\ i \leq x - 1 \end{array} \right\}$$

d) Poišči infimum in supremum elementov  $(x,y)$  in  $(z,w)$  glede na  $R$ .

$$\mathbb{R}^2 \quad (x,y) R (z,w) \Leftrightarrow y \leq w \wedge (x-y) \leq (z-w)$$

$$\inf \{ (x,y), (z,w) \} = (i,j)$$

$$j \leq y \wedge (i-j) \leq (x-y)$$

$$j \leq w \wedge (i-j) \leq (z-w)$$

$$j = \min \{ y, w \}$$

$$i = \min \{ x-y, z-w \} + j$$

$$i \leq x-y + j$$

$$i \leq z-w + j$$

$$\sup \{ (x,y), (z,w) \} = (\max \{ x-y, z-w \} + \max \{ y, w \}, \max \{ y, w \})$$

5. naloga: pokaži veljavnost sklepa

a)

$$1. \forall x (P(x) \Rightarrow \forall y : (Q(y) \Rightarrow \neg R(x,y)))$$

$$2. \forall x : (P(x) \Rightarrow \exists y : (S(y) \wedge R(x,y))) \quad \vdash \exists x : (S(x) \wedge \neg Q(x))$$

$$3. \exists x : P(x)$$

$$4. P(x_0) \quad 3. \{x \rightarrow x_0\}$$

$$5. \forall y : (Q(y) \Rightarrow \neg R(x_0,y)) \text{ MP}(4,1)$$

$$6. \exists y : (S(y) \wedge R(x_0,y)) \text{ MP}(4,2)$$

$$7. S(y_0) \wedge R(x_0,y_0) \quad 6. \{y \rightarrow y_0\}$$

$$8. S(y_0) \quad P_0(7)$$

$$11. S(y_0) \wedge \neg Q(y_0) \text{ Zd}$$

$$9. R(x_0,y_0) \quad P_0(7)$$

$$12. \exists x : (S(x) \wedge \neg Q(x)) \quad (8,10)$$

$$10. \neg Q(y_0) \text{ MT}(9,5)$$

$$11. \{x \leftarrow y_0\}$$