$$S_0^2 = \frac{1}{n}\sum_{k=1}^{n}\left(X_k - \overline{X}\right)^2 = \frac{1}{n}\sum_{k=1}^{n}\left(X_k^2 - 2X_k\cdot\overline{X} + \overline{X}^2\right) = \frac{1}{n}\cdot\sum_{k=1}^{n}X_k^2 - \frac{2}{n}\overline{X}\cdot\sum_{k=1}^{n}X_k + \overline{X}^2 =$$

$$= Z_2 - \overline{X}^2 = Z_2 - Z_1^2$$

2) Naj ima X enakomerno porazd. na $[a, b]$, kjer sta a in b neznana parametra.

Gostota $p(x; a, b) = \begin{cases} \frac{1}{b-a}, & \text{če } a \le x \le b \\ 0, & \text{sicer} \end{cases}$

$z_1 = E(X) = \frac{a+b}{2}$

$$z_2 = E(X^2) = \frac{1}{b-a}\int_a^b x^2\,dx = \frac{1}{b-a}\,\frac{x^3}{3}\Big|_a^b =$$

$$= \frac{b^3 - a^3}{3(b-a)} = \frac{(b-a)(b^2+ab+a^2)}{3(b-a)} = \frac{a^2+ab+b^2}{3}$$

Iz prve enačbe izrazimo $b = 2z_1 - a$ in

vstavimo v drugo: $3z_2 = a^2 + b(a+b) =$

$= a^2 + (2z_1 - a)\cdot 2z_1 = a^2 + 4z_1^2 - 2z_1\cdot a$

Kvadratna enačba za a:

$$a_1^2 - 2z_1\cdot a + 4z_1^2 - 3z_2 = 0$$

$$D = 4z_1^2 - 4(4z_1^2 - 3z_2) = 4z_1^2 - 16z_1^2 + 12z_2$$

$$= 12(z_2 - z_1^2)\,; \quad a_{1,2} = \frac{1}{2}\left(2z_1 \pm \sqrt{12(z_2 - z_1^2)}\right) =$$

$= z_1 \pm \sqrt{3(z_2 - z_1^2)}$

Zaradi simetrije a in b je torej

$a = z_1 - \sqrt{3(z_2 - z_1^2)}$ in $b = z_1 + \sqrt{3(z_2 - z_1^2)}$

Cenilka za a je $A = Z_1 - \sqrt{3(Z_2 - Z_1^2)} =$

$= \overline{X} - \sqrt{3 \cdot S_0^2} = \overline{X} - S_0 \cdot \sqrt{3}$, cenilka za b je $B = \overline{X} + S_0 \cdot \sqrt{3}$

Denimo, da imamo konkreten vzorec: $-2, 0, 1, 2, 4$.

$\bar{x} = \frac{-2+1+2+4}{5} = 1$, $s_0^2 = \frac{1}{5}\left((-3)^2 + (-1)^2 + 0^2 + 1^2 + 3^2\right)$

$= \frac{1}{5} \cdot 20 = 4$, $s_0 = 2$; vrednost za A je $1 - 2\sqrt{3} = -2'46$

za B pa $1 + 2\sqrt{3} = 4'46$