

Naravna dedukcija

Izjavni račun: izjava $A, B ::= \perp \mid \top \mid A \wedge B \mid A \Rightarrow B \mid A \vee B$

$\neg A$ okrajšava $A \Rightarrow \perp$

$A \Leftrightarrow B$ okrajšava za $(A \Rightarrow B) \wedge (B \Rightarrow A)$

$$A \wedge B \vee C \equiv (A \wedge B) \vee C$$

$$A \vee B \Rightarrow C \equiv (A \vee B) \Rightarrow C$$

$$A \Rightarrow B \Rightarrow C \equiv A \Rightarrow (B \Rightarrow C)$$

Pravilo sklepanja:

$$\frac{J_1 \quad J_2 \quad \dots \quad J_n}{J}$$

če velja/smo dokazali
 J_1, \dots in J_n ,
potem velja tudi J

J sodba, V našem primeru:

$$\underbrace{B_1, B_2, \dots, B_n}_{\text{hipoteze}} \vdash A$$

\uparrow izjava

"A velja, pri hipotezah B_1, \dots, B_n "

$$\Gamma \vdash A$$

Pravila sklepanja

$$\frac{}{B_1, B_2, \dots, B_n \vdash B_i} \quad (1 \leq i \leq n) \quad \text{HYP}$$

Konjunkcija:

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \quad \text{I}\wedge$$

$$\frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A} \quad \text{E}\wedge$$

$$\frac{\Gamma \vdash A \wedge B}{\Gamma \vdash B} \quad \text{E}\wedge$$

Implikacija:

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \Rightarrow B} \quad \text{I}\Rightarrow$$

$$\frac{\Gamma \vdash A \Rightarrow B \quad \Gamma \vdash A}{\Gamma \vdash B} \quad \text{E}\Rightarrow$$

Disjunkcija:

$$\frac{\Gamma \vdash A}{\Gamma \vdash A \vee B} \quad \text{I}\vee$$

$$\frac{\Gamma \vdash B}{\Gamma \vdash A \vee B} \quad \text{I}\vee$$

$$\frac{\Gamma \vdash A \vee B \quad \Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma \vdash C} \quad \text{E}$$

Resnica:

$$\frac{}{\Gamma \vdash \top} \quad \text{IT}$$

Neresnica:

$$\frac{\Gamma \vdash \perp}{\Gamma \vdash A} \quad \text{E}\perp$$

Negacija $\neg A \equiv A \Rightarrow \perp$ izpeljena pravila

$$\frac{\Gamma, A \vdash \perp}{\Gamma \vdash \neg A} \text{I}\neg$$

$$\frac{\Gamma \vdash \neg A \quad \Gamma \vdash A}{\Gamma \vdash \perp} \text{E}\neg$$

Primer dokaza:

$$\frac{\frac{\frac{}{A \wedge B \vdash A \wedge B} \text{HYP}}{A \wedge B \vdash B} \text{E}_2 \wedge \quad \frac{\frac{\frac{}{A \wedge B \vdash A \wedge B} \text{HYP}}{A \wedge B \vdash A} \text{E}_1 \wedge}}{A \wedge B \vdash B \wedge A} \text{I}\wedge$$

$$\vdash A \wedge B \Rightarrow B \wedge A \quad \text{I}\Rightarrow$$

Primer

$$\begin{array}{c}
 \frac{}{A, \neg A \vdash \neg A} \text{HYP} \qquad \frac{}{A, \neg A \vdash A} \text{HYP} \\
 \frac{}{A, \neg A \vdash \perp} \text{E}\neg \\
 \frac{}{A \vdash \neg(\neg A)} \text{I}\neg \\
 \hline
 \vdash A \Rightarrow \neg\neg A \qquad \text{I}\Rightarrow
 \end{array}$$

Zakon o izključenih tretji možnosti

$$\overbrace{\vdash A \vee \neg A}^{\text{LEM}}$$

Coq :

$$B_1, B_2, B_3 \vdash A$$

$$H_1 : B_1$$

$$H_2 : B_2$$

$$H_3 : B_3$$

$$A$$