**Rešitve 1. izpita pri predmetu Teorija programskih jezikov**

**1. naloga (25 točk)**

Za vsakega od naslednjih *MiniML* programov ugotovite, ali ima tip in katerega. Nato ugotovite še, ali program divergira, blokira ali se evaluira v vrednost. Če se evaluira v vrednost, v katero? (*MiniML* je neučakan programski jezik.)

**a)** `if` (`if false then false else true`) `then false else true`

Tip: `bool`, vrednost: `false`.

**b)** (`if` $0 = 1$ `then false else` $14) * 3$

Ni tipa (različna tipa pri `if`), vrednost: 42.

**c)** (`fun` $f(g : \texttt{bool} \to \texttt{int}) : \texttt{int is}\ g\ \texttt{false}$) (`fun` $h(x : \texttt{int}) : \texttt{int is}\ x + 1$)

Tip: `bool` $\to$ `int`, je že vrednost.

**d)** `fun` $f(b : \texttt{bool}) : \texttt{int is if}\ b\ \texttt{then}\ 0\ \texttt{else}\ 1$

Ni tipa (funkcija ima drugačen tip kot argument), blokira (v koraku `false` $+ 1$).

**e)** (`fun` $f(b : \texttt{bool}) : \texttt{int is if}\ f\ b\ \texttt{then}\ 42\ \texttt{else}\ 42$) `false`

Ni tipa ($f\ b$ je tipa `int`, vendar se uporabi kot pogoj za `if`), divergira (vsakič se kliče $f$ `false`).

**2. naloga (25 točk)**

Programski jezik *MiniML+error* je *MiniML* razširjen s konstanto `error`, ki sproži izjemo. Jeziku želimo dodati še izraz oblike

$$\texttt{try}\ e_1\ \texttt{catch}\ e_2$$

Intuitivni pomen $\texttt{try}\ e_1\ \texttt{catch}\ e_2$ je: "Evaluiraj $e_1$ v vrednost $v_1$. Če je $v_1 = \texttt{error}$, evaluiraj $e_2$, sicer je rezultat $v_1$."

**a) (8 točk)** Zapišite pravila za preverjanje tipov $\texttt{try}\ e_1\ \texttt{catch}\ e_2$.

$$\frac{\Gamma \mid e_1 : \tau \qquad \Gamma \mid e_2 : \tau}{\Gamma \mid \texttt{try}\ e_1\ \texttt{catch}\ e_2 : \tau}$$

**b) (9 točk)** Zapišite pravila za semantiko malih korakov $\texttt{try}\ e_1\ \texttt{catch}\ e_2$.

$$\frac{}{\texttt{try error catch}\ e_2 \mapsto e_2}$$

$$\frac{v_1 \neq \texttt{error}}{\texttt{try}\ v_1\ \texttt{catch}\ e_2 \mapsto v_1}$$

$$\frac{e_1 \mapsto e_1'}{\texttt{try}\ e_1\ \texttt{catch}\ e_2 \mapsto \texttt{try}\ e_1'\ \texttt{catch}\ e_2}$$

**c) (8 točk)** Zapišite korake v evaluaciji programa $2 \cdot (\texttt{try}\ (2 + \texttt{error})\ \texttt{catch}\ (3 \cdot 7))$.

$$\begin{aligned}
& 2 \cdot (\texttt{try}\ (2 + \texttt{error})\ \texttt{catch}\ (3 \cdot 7)) \\
\mapsto\ & 2 \cdot (\texttt{try error catch}\ (3 \cdot 7)) \\
\mapsto\ & 2 \cdot (3 \cdot 7) \\
\mapsto\ & 2 \cdot 21 \\
\mapsto\ & 42
\end{aligned}$$

**3. naloga (25 točk)**

V ukaznem programskem jeziku dokažite naslednjo *popolno* pravilnost:

$$
\begin{array}{l}
[\ n \geq 0 \wedge (3 \mid n)\ ] \\
v := 0; \\
\texttt{while}\ n > 0\ \texttt{do} \\
\quad v := v + (n \bmod 10); \\
\quad n := n \operatorname{div} 10 \\
\texttt{done} \\
[\ (3 \mid v)\ ]
\end{array}
$$

Zapis $x \mid y$ pomeni, da število $x$ deli število $y$. Operaciji div in mod sta celoštevilsko deljenje in ostanek, torej velja $x = (x \operatorname{div} y) \cdot y + (x \bmod y)$ za vse $x$ in $y \neq 0$.

| | | |
|---|---|---|
| 1 | $[\ n \geq 0 \wedge (3 \mid n)\ ]$ | |
| 2 | $v := 0;$ | |
| 3 | $[\ n \geq 0 \wedge (3 \mid n) \wedge v = 0\ ]$ | [EQ, CSP, CA] |
| 4 | $[\ n \geq 0 \wedge (3 \mid n + v)\ ]$ | [SK, SP, CA] |
| 5 | $\texttt{while}\ n > 0\ \texttt{do}$ | |
| 6 | $\quad [\ n \geq 0 \wedge (3 \mid n + v) \wedge n > 0 \wedge n = m\ ]$ | |
| 7 | $\quad v := v + (n \bmod 10);$ | |
| 8 | $\quad [\ n \geq 0 \wedge (3 \mid n + v - (n \bmod 10)) \wedge n > 0 \wedge n = m\ ]$ | [AS] |
| 9 | $\quad [\ n \operatorname{div} 10 \geq 0 \wedge (3 \mid (n \operatorname{div} 10) \cdot 10 + v) \wedge n \operatorname{div} 10 < m\ ]$ | [SK, SP, CA] |
| 10 | $\quad n := n \operatorname{div} 10$ | |
| 11 | $\quad [\ n \geq 0 \wedge (3 \mid 10n + v) \wedge n < m\ ]$ | [AS] |
| 12 | $\quad [\ n \geq 0 \wedge (3 \mid n + v) \wedge n < m\ ]$ | [SK, SP, CA] |
| 13 | $\texttt{done}$ | |
| 14 | $[\ n \geq 0 \wedge (3 \mid n + v) \wedge n \leq 0\ ]$ | [WHP] |
| 15 | $[\ (3 \mid v)\ ]$ | [SK, SP, CA] |

Korak $(3 \Rightarrow 4)$: Ker je $v = 0$ in $(3 \mid n)$, je $n = n + v$ in torej $(3 \mid n + v)$. Dobljeni pogoj je invarianta $I$.

Korak (6): Uvedemo količino $m = n$; zaradi $n > 0$ velja $m > 0$.

Korak $(8 \Rightarrow 9)$: Ker je $n \geq 0$, je tudi $n \operatorname{div} 10 \geq 0$. Ker je $n > 0$ in $n = m$, je $n \operatorname{div} 10 < m$. Ker je $n = (n \operatorname{div} 10) \cdot 10 + (n \bmod 10)$, je $n + v - (n \bmod 10) = (n \operatorname{div} 10) \cdot 10 + v$.

Korak $(11 \Rightarrow 12)$: Ker velja $10n + v = n + v + 3 \cdot 3n$ in $(3 \mid 10n + v)$, velja tudi $(3 \mid n + v)$. Dobili smo invarianto $I$, poleg tega pa je vrednost $n$ manjša od količine $m$.

Korak $(14 \Rightarrow 15)$: Ker je $n \geq 0$ in $n \leq 0$, velja $n = 0$ in torej $n + v = v$.

**4. naloga (25 točk)**

Za vsakega od danih programov v programskem jeziku *Poly* izračunajte njegov glavni tip ali pa dokažite, da izraz nima glavnega tipa.

**a)** $\texttt{fun}\ f \to \texttt{fun}\ g \to \texttt{fun}\ x \to f\ (g\ (f\ x))$

$$
\begin{array}{rlcr}
 & x : \alpha & & \\
f \text{ sprejme } x\text{, torej:} & f : \alpha \to \beta & \Rightarrow & f\ x : \beta \\
g \text{ sprejme } f\ x\text{, torej:} & g : \beta \to \gamma & \Rightarrow & g\ (f\ x) : \gamma \\
f \text{ sprejme } g\ (f\ x)\text{, torej:} & \gamma = \alpha & \Rightarrow & f\ (g\ (f\ x)) : \beta
\end{array}
$$

Celoten izraz je funkcija, ki sprejme $f : \alpha \to \beta$ in vrne funkcijo, ki sprejme $g : \beta \to \alpha$ in vrne funkcijo, ki sprejme $x : \alpha$ in vrne izraz $f\ (g\ (f\ x)) : \beta$. Glavni tip je torej

$$\texttt{fun}\ f \to \texttt{fun}\ g \to \texttt{fun}\ x \to f\ (g\ (f\ x)) : (\alpha \to \beta) \to (\beta \to \alpha) \to \alpha \to \beta.$$

**b)** $\texttt{rec}\ f\ \texttt{is}\ \texttt{fun}\ g \to \texttt{fun}\ x \to f\ (g\ (f\ x))$

$$
\begin{array}{rlcr}
 & x : \alpha & & \\
f \text{ sprejme } x\text{, torej:} & f : \alpha \to \beta & \Rightarrow & f\ x : \beta \\
g \text{ sprejme } f\ x\text{, torej:} & g : \beta \to \gamma & \Rightarrow & g\ (f\ x) : \gamma \\
f \text{ sprejme } g\ (f\ x)\text{, torej:} & \gamma = \alpha & \Rightarrow & f\ (g\ (f\ x)) : \beta
\end{array}
$$

Celoten izraz ima tip telesa rekurzivne definicije – to je funkcija, ki sprejme $g : \beta \to \alpha$ in vrne funkcijo, ki sprejme $x : \alpha$ in vrne izraz $f\ (g\ (f\ x)) : \beta$. Ker mora biti $f$ enakega tipa, dobimo enačbo $\alpha \to \beta = (\beta \to \alpha) \to \alpha \to \beta$, od koder izpeljemo sistem enačb

$$
\begin{aligned}
\alpha &= \beta \to \alpha, \\
\beta &= \alpha \to \beta.
\end{aligned}
$$

Ker v obeh enačbah ena od spremenljivk nastopa na obeh straneh, sistema ne moremo rešiti, tako da izraz nima glavnega tipa.

**4. naloga na dolgo**

**a)**

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Gamma(f)=\alpha}{\Gamma \mid f:\alpha \ /_{\emptyset}\ \emptyset} \quad \dfrac{\dfrac{\Gamma(g)=\beta}{\Gamma \mid g:\beta \ /_{\emptyset}\ \emptyset} \quad \dfrac{\dfrac{\Gamma(f)=\alpha}{\Gamma \mid f:\alpha \ /_{\emptyset}\ \emptyset} \quad \dfrac{\Gamma(x)=\gamma}{\Gamma \mid x:\gamma \ /_{\emptyset}\ \emptyset}}{\Gamma \mid f\ x:\delta \ /_{\{\alpha,\gamma,\delta\}}\ \{\alpha=\gamma\to\delta\}}}{\Gamma \mid g\ (f\ x):\epsilon \ /_{\{\alpha,\beta,\gamma,\delta,\epsilon\}}\ \{\alpha=\gamma\to\delta,\beta=\delta\to\epsilon\}}}{\Gamma=x:\gamma,g:\beta,f:\alpha \mid f\ (g\ (f\ x)):\zeta \ /_{\{\alpha,\beta,\gamma,\delta,\epsilon\}}\ \{\alpha=\gamma\to\delta,\beta=\delta\to\epsilon,\alpha=\epsilon\to\zeta\}}}{g:\beta,f:\alpha \mid \texttt{fun}\ x\to f\ (g\ (f\ x)):\gamma\to\zeta \ /_{\{\alpha,\beta,\gamma,\delta,\epsilon\}}\ \{\alpha=\gamma\to\delta,\beta=\delta\to\epsilon,\alpha=\epsilon\to\zeta\}}}{f:\alpha \mid \texttt{fun}\ g\to\texttt{fun}\ x\to f\ (g\ (f\ x)):\beta\to\gamma\to\zeta \ /_{\{\alpha,\beta,\gamma,\delta,\epsilon\}}\ \{\alpha=\gamma\to\delta,\beta=\delta\to\epsilon,\alpha=\epsilon\to\zeta\}}}{\mid \texttt{fun}\ f\to\texttt{fun}\ g\to\texttt{fun}\ x\to f\ (g\ (f\ x)):\alpha\to\beta\to\gamma\to\zeta \ /_{\{\alpha,\beta,\gamma,\delta,\epsilon\}}\ \{\alpha=\gamma\to\delta,\beta=\delta\to\epsilon,\alpha=\epsilon\to\zeta\}}$$

Reševanje sistema enačb:

1. $r=[],\mathcal{E}=\{\alpha=\gamma\to\delta,\beta=\delta\to\epsilon,\alpha=\epsilon\to\zeta\}$
2. $r=[\alpha=\gamma\to\delta],\mathcal{E}=\{\beta=\delta\to\epsilon,\gamma\to\delta=\epsilon\to\zeta\}$
3. $r=[\alpha=\gamma\to\delta,\beta=\delta\to\epsilon],\mathcal{E}=\{\gamma\to\delta=\epsilon\to\zeta\}$
4. $r=[\alpha=\gamma\to\delta,\beta=\delta\to\epsilon],\mathcal{E}=\{\gamma=\epsilon,\delta=\zeta\}$
5. $r=[\alpha=\epsilon\to\delta,\beta=\delta\to\epsilon,\gamma=\epsilon],\mathcal{E}=\{\delta=\zeta\}$
6. $r=[\alpha=\epsilon\to\zeta,\beta=\zeta\to\epsilon,\gamma=\epsilon,\delta=\zeta],\mathcal{E}=\emptyset$

Glavni tip:

$$\texttt{fun}\ f\to\texttt{fun}\ g\to\texttt{fun}\ x\to f\ (g\ (f\ x)):(\epsilon\to\zeta)\to(\zeta\to\epsilon)\to\epsilon\to\zeta$$

**b)**

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Gamma(f)=\alpha}{\Gamma \mid f:\alpha \ /_{\emptyset}\ \emptyset} \quad \dfrac{\dfrac{\Gamma(g)=\beta}{\Gamma \mid g:\beta \ /_{\emptyset}\ \emptyset} \quad \dfrac{\dfrac{\Gamma(f)=\alpha}{\Gamma \mid f:\alpha \ /_{\emptyset}\ \emptyset} \quad \dfrac{\Gamma(x)=\gamma}{\Gamma \mid x:\gamma \ /_{\emptyset}\ \emptyset}}{\Gamma \mid f\ x:\delta \ /_{\{\alpha,\gamma,\delta\}}\ \{\alpha=\gamma\to\delta\}}}{\Gamma \mid g\ (f\ x):\epsilon \ /_{\{\alpha,\beta,\gamma,\delta,\epsilon\}}\ \{\alpha=\gamma\to\delta,\beta=\delta\to\epsilon\}}}{\Gamma=x:\gamma,g:\beta,f:\alpha \mid f\ (g\ (f\ x)):\zeta \ /_{\{\alpha,\beta,\gamma,\delta,\epsilon\}}\ \{\alpha=\gamma\to\delta,\beta=\delta\to\epsilon,\alpha=\epsilon\to\zeta\}}}{g:\beta,f:\alpha \mid \texttt{fun}\ x\to f\ (g\ (f\ x)):\gamma\to\zeta \ /_{\{\alpha,\beta,\gamma,\delta,\epsilon\}}\ \{\alpha=\gamma\to\delta,\beta=\delta\to\epsilon,\alpha=\epsilon\to\zeta\}}}{f:\alpha \mid \texttt{fun}\ g\to\texttt{fun}\ x\to f\ (g\ (f\ x)):\beta\to\gamma\to\zeta \ /_{\{\alpha,\beta,\gamma,\delta,\epsilon\}}\ \{\alpha=\gamma\to\delta,\beta=\delta\to\epsilon,\alpha=\epsilon\to\zeta\}=\mathcal{E}_1}}{\mid \texttt{rec}\ f\ \texttt{is}\ \texttt{fun}\ g\to\texttt{fun}\ x\to f\ (g\ (f\ x)):\beta\to\gamma\to\zeta \ /_{\{\alpha,\beta,\gamma,\delta,\epsilon\}}\ \mathcal{E}_1\cup\{\alpha=\beta\to\gamma\to\zeta\}}$$

Reševanje sistema enačb:

1. $r=[],\mathcal{E}=\{\alpha=\gamma\to\delta,\beta=\delta\to\epsilon,\alpha=\epsilon\to\zeta,\alpha=\beta\to\gamma\to\zeta\}$
2. $r=[\alpha=\gamma\to\delta],\mathcal{E}=\{\beta=\delta\to\epsilon,\gamma\to\delta=\epsilon\to\zeta,\gamma\to\delta=\beta\to\gamma\to\zeta\}$
3. $r=[\alpha=\gamma\to\delta,\beta=\delta\to\epsilon],\mathcal{E}=\{\gamma\to\delta=\epsilon\to\zeta,\gamma\to\delta=(\delta\to\epsilon)\to\gamma\to\zeta\}$
4. $r=[\alpha=\gamma\to\delta,\beta=\delta\to\epsilon],\mathcal{E}=\{\gamma=\epsilon,\delta=\zeta,\gamma\to\delta=(\delta\to\epsilon)\to\gamma\to\zeta\}$
5. $r=[\alpha=\epsilon\to\delta,\beta=\delta\to\epsilon,\gamma=\epsilon],\mathcal{E}=\{\delta=\zeta,\epsilon\to\delta=(\delta\to\epsilon)\to\epsilon\to\zeta\}$
6. $r=[\alpha=\epsilon\to\zeta,\beta=\zeta\to\epsilon,\gamma=\epsilon,\delta=\zeta],\mathcal{E}=\{\epsilon\to\zeta=(\zeta\to\epsilon)\to\epsilon\to\zeta\}$
7. $r=[\alpha=\epsilon\to\zeta,\beta=\zeta\to\epsilon,\gamma=\epsilon,\delta=\zeta],\mathcal{E}=\{\epsilon=\zeta\to\epsilon,\zeta=\epsilon\to\zeta\}$

Ker ni naslednjega koraka, sistem enačbe nima rešitve in izraz nima tipa.