1. naloga: ali je dana urejena množica domena?

a) $((0,1), \leq)$

1. ✓ delno urejena

Ali obstaja dno?

2. za $\forall x \in (0,1)$ velja $\frac{x}{2} \in (0,1)$, $\frac{x}{2} < x$

$\Rightarrow ((0,1), \leq)$ ni domena

b) $([0,1], \leq)$

1. ✓

2. $\bot = 0$

Ali imajo vse verige supremum?

3. $\sup_n(a_n) = \lim_{n\to\infty} a_n \leq 1$ ker linearno urejeno $\in [0,1]$

$\Rightarrow ([0,1], \leq)$ je domena

c)

$\left([0,1) \cup (2,3], \leq\right)$

1. ✓
2. $\perp = 0$
3. $a_n = 1 - \frac{1}{n}$ to zaporedje nima sup v D

$\Rightarrow \left([0,1) \cup (2,3], \leq\right)$ ni domena

d)

$\left([0,1) \cup [2,3], \leq\right)$

3. $\sup_n \left(1 - \frac{1}{n}\right)_n = 2.$ $\quad \sup(a_n) = \begin{cases} 2, & \text{če } \lim_n (a_n) = 1 \\ \lim_n (a_n), & \text{sicer} \end{cases}$

$\Rightarrow$ je domena.

e)

$$(([0,1] \cap \mathbb{Q}), \leq)$$

1. ✓

2. $\bot = 0$

3. $a_n = \dfrac{\left(1 - \frac{1}{n}\right)^n}{3}$ $\quad \lim\limits_{n \to \infty} a_n = \dfrac{e}{3} \notin \mathbb{Q}$

ni domena

f) $D = (\mathbb{N}, \leq)$

1. ✓

2. $\bot = 1$

3. $a_n = n$

$\Rightarrow D$ ni domena

g) $D' = (\mathbb{N}, \geq)$

1. ✓

2. $\forall x \in \mathbb{N} \, \exists y \in \mathbb{N} : \neg (x \geq y)$

$\Rightarrow D'$ ni domena

2. naloga

$$([\mathbb{N} \to \mathbb{N}], \sqsubseteq)$$

$$f \sqsubseteq g \iff \forall n \in \mathbb{N}: f(n) \leq g(n)$$

Ali je delno urejena?

1) $\cdot\ \forall f: f \sqsubseteq f \Rightarrow \forall n \in \mathbb{N}: f(n) \leq f(n)$ ✓

$\cdot\ \forall f, g: f \sqsubseteq g \land g \sqsubseteq f \Rightarrow g = f$

$$\forall n \in \mathbb{N}: \begin{matrix} f(n) \leq g(n) \\ g(n) \leq f(n) \end{matrix} \Rightarrow f(n) = g(n)$$

$\cdot\ \forall f, g, h: f \sqsubseteq g \land g \sqsubseteq h \Rightarrow f \sqsubseteq h$

$$\forall n \in \mathbb{N}: \begin{matrix} f(n) \leq g(n) \\ g(n) \leq h(n) \end{matrix} \Rightarrow f(n) \leq h(n)\ ✓$$

Ali je domena?

2) $(f(n) = 1) = \bot$

3) $(f_i(n) = n + i)_i$

$\Downarrow$

nima sup

$\Downarrow$

ni domena

Nadomestimo N z [0, 1]:

za delno urejenost imamo podoben dokaz kot prej

$$([[0,1] \to [0,1]], \sqsubseteq)$$

$$\bot(x) = 0$$

$$(\bigvee_n f_n)(x) = \bigvee_n f_n(x)$$

Obstaja

JE DOMENA

Bolj splošno: s tako urejenostjo na funkcijah iz domene D v domeno E je množica [D -> E] prav tako domena.

3. naloga: preveri, ali so funkcije zvezne

$([0,1], \leq)$

$$f(x) = \begin{cases} 0 \; ; \; x < \frac{1}{2} \\ 1 \; ; \; x \geq \frac{1}{2} \end{cases}$$

monotona: ✓

$a_n = \frac{1}{2} - \frac{1}{n} \; ; \; n \geq 2$

$\bigvee_n a_n = \frac{1}{2}$

$f(a_n) = 0 \quad \forall n \in \mathbb{N} \Rightarrow \bigvee_n f(a_n) = 0$

$f(\bigvee_n a_n) = 1$

f ni zvezna

$$g(x) = \begin{cases} 0 \; ; \; x \leq \frac{1}{2} \\ 1 \; ; \; x > \frac{1}{2} \end{cases}$$

a_n neka veriga

$$g(\bigvee_n a_n) = \begin{cases} 0 \; ; \; \text{če } a_n \leq \frac{1}{2} \; \forall n \in \mathbb{N} \\ 1 \; ; \; \text{če } \exists n \in \mathbb{N}. \, a_n > \frac{1}{2} \end{cases}$$

$= \bigvee_n g(a_n)$

g je zvezna

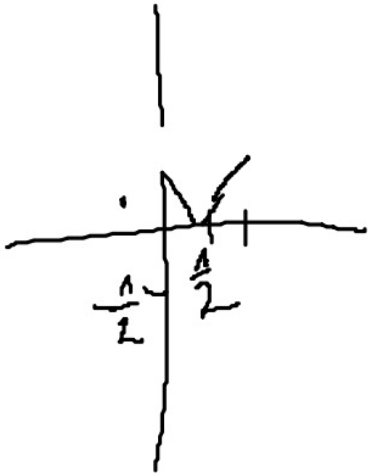
$([0,1], \leq)$

$h(x,y) = \frac{x+y}{2}$ $\qquad$ $k(x,y) = |x-y|$

h: monotona $(x_1,y_1) \leq (x_2,y_2) \Rightarrow \frac{x_1+y_1}{2} \leq \frac{x_2+y_2}{2}$ ✓

(x_n,y_n) veriga $\bigvee_n (x_n,y_n) = \left(\bigvee_n x_n, \bigvee_n y_n\right)$

$$h\left(\bigvee(x_n,y_n)\right) = h\left(\bigvee x_n, \bigvee_n y_n\right) = \frac{\bigvee x_n + \bigvee y_n}{2} = \bigvee_n \frac{x_n+y_n}{2} = \bigvee_n h(x_n,y_n)$$



k ni zvezna

h zvezna

4. naloga: katere funkcije so zvezne?

$$([0,1) \cup [2,3], \leq)$$

$$f(x) = \min\{2, x\}$$

$$g(x) = \min\{2+x, 5-x\}$$

$$h(x) = \min\{3, 2+\tfrac{x}{2}\}$$

$$f\left(\bigvee_n a_n\right) = \min\{2, \bigvee_n a_n\}$$

$$\bigvee_n f(a_n) = \bigvee_n \min\{2, a_n\}$$

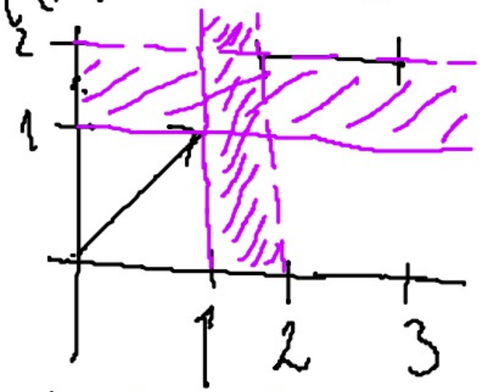
$\Rightarrow f$ je zvezna

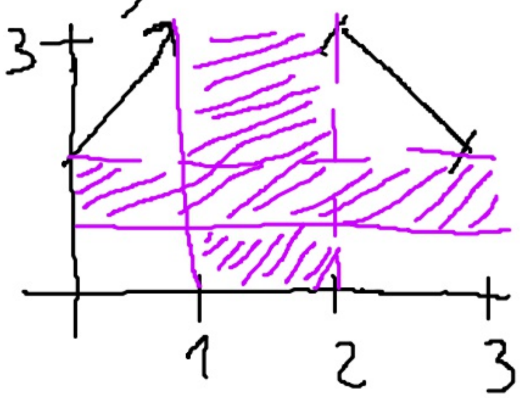
$$a_n = 1-\frac{1}{n} \quad \bigvee_n a_n = 2$$

$$\bigvee_n h(a_n) = \bigvee_n \min\left\{3, 2+\frac{1-\frac{1}{n}}{2}\right\} = \bigvee_n 2+\frac{1-\frac{1}{n}}{2} = 2\tfrac{1}{2}$$

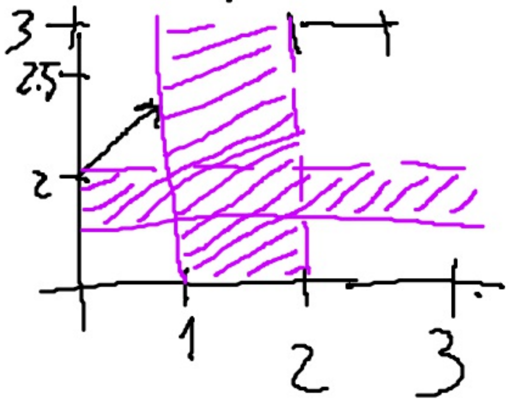
$$h\left(\bigvee_n a_n\right) = h(2) = 3$$

$\Rightarrow h$ ni zvezna





g ni monotona => ni zvezna



Območja, označeno z vijolično, v naši domeni ni - če bi ga na grafu izpustili, bi videli, da je funkcija f zvezna tudi v običajnem smislu, funkcija h pa ne.