

1. naloga: izračunaj denotacijski pomen izraza v praznem kontekstu

$$\llbracket 0 \cdot (1/0) \rrbracket () = \llbracket 0 \rrbracket () \cdot \llbracket 1/0 \rrbracket () = 0 \cdot \perp = \perp$$

2. naloga: kateri izrazi imajo enak denotacijski pomen?

$$\llbracket x: \text{int} \mid (x \cdot x - 1) / (x - 1) \rrbracket = x \mapsto \begin{cases} x+1, & \text{če } x \neq 1 \\ \perp, & \text{če } x = 1 \end{cases}$$

$$\llbracket x: \text{int} \mid x+1 \rrbracket = x \mapsto x+1$$

$$\llbracket x: \text{int}, y: \text{int} \mid x+1 \rrbracket = (x, y) \mapsto x+1$$

$$\llbracket x: \text{int} \mid (x+1) \cdot (x+1) - x \cdot x - x \rrbracket = x \mapsto x+1$$

enak pomen

3. naloga: v PCF definiraj seštevanje

$add = \text{rec } add : \text{nat} \rightarrow \text{nat} \rightarrow \text{nat} \text{ is}$
 $\text{fun } x : \text{nat} \rightarrow \text{fun } y : \text{nat} \rightarrow$
 $\text{if iszero } y \text{ then } x \text{ else } add (\text{succ } x) (\text{pred } y)$

4. naloga: v PCF definiraj primerjavo (enakost) števil

$e\mathcal{L} = \text{rec } e\mathcal{L} : \text{nat} \rightarrow \text{nat} \rightarrow \text{nat} \text{ is}$
 $\text{fun } x : \text{nat} \rightarrow \text{fun } y : \text{nat} \rightarrow$
 $\text{if iszero } y \text{ then } x^{\text{iszero}} \text{ else}$
 $\text{if iszero } x \text{ then } f \text{ else}$
 $e\mathcal{L} (\text{pred } x) (\text{pred } y)$

5. naloga: pokaži, da je vsaka nekonstantna zvezna funkcija iz dviga v dvig množice A striktna

Ima $f : D \rightarrow E$; f zvezna

Funkcija je STRIKTNA: $f(\perp_D) = \perp_E$

$g : A_{\perp} \rightarrow A_{\perp}$ Pokaži: g zvezna in ni konstantna \Rightarrow
striktna.

$g(\perp) = a$; $a \in A$

$g(x) \geq g(\perp) = a \Rightarrow g(x) = a \Rightarrow g$ konstanta

$x \geq \perp$

6. naloga: ali je f zvezna funkcija? Izračunaj najmanjšo negibno točko.

$$D = ([0, 1], \leq)$$

$$f(x) = \begin{cases} \frac{1}{2} & ; x \leq \frac{1}{4} \\ \frac{2}{3} & ; \frac{1}{4} < x \leq \frac{3}{4} \\ 1 & ; \text{vsak} \end{cases}$$

$$f_{\text{fix}_0}(f) = \bigvee_{i \in \mathbb{N}} f^i(0)$$

$$0, \frac{1}{2}, \frac{2}{3}, \left(\frac{2}{3}\right), \dots$$

$$\bigvee \left\{ 0, \frac{1}{2}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \dots \right\} = \frac{2}{3}$$

7. naloga: izračunaj najmanjšo negibno točko funkcije F

$$F : [N_{\perp} \rightarrow N_{\perp}] \rightarrow [N_{\perp} \rightarrow N_{\perp}]$$

$$f(\#)(m) = \begin{cases} \perp & ; m = \perp \\ 0 & ; m = 0 \\ (2m+7) +_{\perp\perp} f(m-1) & ; \text{nič} \end{cases}$$

$$f(m) = \begin{cases} \perp & ; m = \perp \\ (m+7)^2 - 7 & ; \text{nič} \end{cases}$$

	\perp	0	1	2	3	4	5
f_0	\perp	\perp	\perp	\perp	\perp	\perp	\perp
f_1	\perp	0	\perp	\perp	\perp	\perp	\perp
f_2	\perp	0	3	\perp	\perp	\perp	\perp
f_3	\perp	0	3	8	\perp	\perp	\perp
f_4	\perp	0	3	8	15	\perp	\perp
f	\perp	0	3	8	15	24	35