

Numerična napaka

6. VAJA

6.1 Numerična napaka-kvadratna enačba

Izračunaj kvadratno enačbo $Ax^2 + Bx + C = 0$

Rešitve izračunaj na dva načina:

$$A = 1; \quad B = 1, 10, 100, \dots, 10^7; \quad C = -3.$$

$$x_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}.$$

$$x_{1,2} = \frac{-2C}{B \pm \sqrt{B^2 - 4AC}}.$$

Izpeljava:

$$x_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}.$$

$$x_{1,2} = \frac{(-B \pm \sqrt{B^2 - 4AC})(-B \mp \sqrt{B^2 - 4AC})}{2A(-B \mp \sqrt{B^2 - 4AC})} =$$

$$x_{1,2} = \frac{B^2 - B^2 + 4AC}{2A(-B \mp \sqrt{B^2 - 4AC})} = \frac{2C}{-B \mp \sqrt{B^2 - 4AC}}$$

Naredi preizkus tako, da rešitve vstaviš v kvadratno enačbo.
Kaj ugotoviš?

* Primer je vzet iz knjige J. Petrišič, Reševanje enačb, FS

6.1 Numerična napaka-kvadratna enačba

```
C:\vaje NM IR\numerične\nm\vaja 8\Untitled14.m*
File Edit Text Go Cell Tools Debug Desktop Window Help
1      %napaka kvadratne enačbe
2      clc; clear all; format long
3      a=1;b=1;c=-3;
4      disp '          B          x1          x2          r plus          r minus'
5      for i=1:9
6          %rešitev prvi način
7          x1=(-b+sqrt(b*b-4*a*c))/(2*a);
8          x2=(-b-sqrt(b*b-4*a*c))/(2*a);
9          %preizkus
10         r1=a*x1^2+b*x1+c;
11         r2=a*x2^2+b*x2+c;
12         fprintf('%11i  %+15.9f  %+15.9e  %+15.9f  %+15.9f \n',b,x1,x2,r1,r2)
13         b=b*10;
14     end
15     disp ' '
16     disp '          B          x1          x2          r plus          r minus'
17     b=1;
18     for i=1:9
19         %rešitev drugi način
20         y1=-2*c/(b+sqrt(b*b-4*a*c));
21         y2=-2*c/(b-sqrt(b*b-4*a*c));
22         %preizkus
23         k1=a*y1^2+b*y1+c;
24         k2=a*y2^2+b*y2+c;
25         fprintf('%11i  %+15.9f  %+15.9e  %+15.9f  %+15.9f \n',b,y1,y2,k1,k2)
26         b=10^i;
27     end
script Ln 27 Col 4 OVR
```

6.1 Numerična napaka-kvadratna enačba

Rezultati

```
Command Window
File Edit Debug Desktop Window Help
  B      x1      x2      r plus      r minus
  1      +1.302775638 -2.302775638e+000 +0.000000000 +0.000000000
  10     +0.291502622 -1.029150262e+001 +0.000000000 +0.000000000
  100    +0.029991005 -1.000299910e+002 +0.000000000 +0.000000000
  1000   +0.002999991 -1.000003000e+003 +0.000000000 +0.000000000
  10000  +0.000300000 -1.000000030e+004 -0.000000004 -0.000000015
  100000 +0.000030000 -1.000000000e+005 +0.000000289 +0.000000000
  1000000 +0.000003000 -1.000000000e+006 +0.000022843 +0.000000000
  10000000 +0.000000300 -1.000000000e+007 -0.001141310 +0.000000000
  100000000 +0.000000030 -1.000000000e+008 -0.019767761 +1.000000000

  B      x1      x2      r plus      r minus
  1      +1.302775638 -2.302775638e+000 +0.000000000 +0.000000000
  10     +0.291502622 -1.029150262e+001 +0.000000000 -0.000000000
  100    +0.029991005 -1.000299910e+002 +0.000000000 -0.000000001
  1000   +0.002999991 -1.000003000e+003 +0.000000000 -0.000009027
  10000  +0.000300000 -1.000000031e+004 +0.000000000 +0.132542491
  100000 +0.000030000 -9.999999040e+004 +0.000000000 -963.038169861
  1000000 +0.000003000 -9.999923856e+005 +0.000000000 -7614380.410278320
  10000000 +0.000000300 -1.000380581e+007 +0.000000000 +38072620860.609375000
  100000000 +0.000000030 -1.006632960e+008 +0.000000000 +66769561583613.000000000
>> |
```

6.1 Numerična napaka-kvadratna enačba

Na primeru vidimo, da dobimo pravilne rezultate pri

$$x_1 = \frac{-B - \sqrt{B^2 - 4AC}}{2A} \quad \text{in} \quad x_2 = \frac{-2C}{B + \sqrt{B^2 - 4AC}}.$$

Kako bi združili oba rezultata pri reševanju kvadratne enačbe?

Opomba: funkcija $\text{sign}()$ je definirana kot:

$$\text{sign}(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$$

6.1 Numerična napaka-kvadratna enačba

The screenshot shows the MATLAB 7.6.0 (R2008a) interface. The Command Window displays the results of a script: x1 = -10000000.000000300 and x2 = +0.000000300. The Editor window shows the script 'vaja8_3.m' which solves a quadratic equation with parameters a=1, b=1.e7, and c=-3. The script uses the quadratic formula to calculate x1 and x2, and then displays the results with their corresponding function values r1 and r2.

```
1  
2 %rešitev kvadratne enačbe s kombinacijo dveh enačb  
3 -  
4 - clear all;  
5  
6  
7 - a=1;  
8 - b=1.e7;  
9 - c=-3;  
10  
11 - x1=(-b-sign(b)*sqrt(b*b-4*a*c))/(2*a);  
12 - r1=a*x1^2+b*x1+c;  
13  
14 - x2=-2*c/(b+sign(b)*sqrt(b*b-4*a*c));  
15 - r2=a*x2^2+b*x2+c;  
16 - disp '          x1          vrednost'  
17 - fprintf('%+25.9f  %+15.9f \n',x1,r1)  
18 - disp '          x2          vrednost'  
19 - fprintf('%+25.9f  %+15.9f \n',x2,r2)  
20
```

Command Window Output:

```
x1          vrednost  
-10000000.000000300  +0.000000000  
x2          vrednost  
+0.000000300      +0.000000000  
>>
```

$$x_1 = \frac{-B - \text{sign}(B)\sqrt{B^2 - 4AC}}{2A}.$$

$$x_2 = \frac{-2C}{B + \text{sign}(B)\sqrt{B^2 - 4AC}}.$$

6.2 Numerična napaka-integral

Izračunajte integral $I_n = \int_0^1 e^{1-t} t^n dt$ s pomočjo rekurzijske formule

$$I_{n+1} = (n+1)I_n - 1; \quad n = 0, 1, 2, \dots \quad I_0 = e - 1$$

Izpeljava (integriranje po delih $u = e^{-t}; dv = t^n dt$):

$$I_n = e \int_0^1 e^{-t} t^n dt = \frac{e}{n+1} \left(t^{n+1} e^{-t} \Big|_0^1 + \int_0^1 e^{-t} t^{n+1} dt \right) = \frac{1}{n+1} (1 + I_{n+1}) \Rightarrow I_{n+1} = (n+1)I_n - 1$$

Izračunajte integral tudi pri obrnjeni rekurzijski formuli

$$I_n = \frac{I_{n+1} + 1}{n+1}$$

Opomba: Matlabova funkcija `single()` računa z manjšo natančnostjo (4 byti)

Matlabova funkcija `double()` računa z večjo natančnostjo (8 bytov)

6.2a Numerična napaka-integral

The image shows the MATLAB 7.6.0 (R2008a) environment. The Command Window displays the results of a script, showing the values of variables `Ie` and `Id` for `n` from 0 to 22. The Editor window shows the script code.

Command Window Output:

n	I(enojna natančnost)	I(dvojna natančnost)
0	1.718282	1.718282
1	0.718282	0.718282
2	0.436563	0.436564
3	0.309690	0.309691
4	0.238762	0.238764
5	0.193810	0.193819
6	0.162857	0.162916
7	0.139999	0.140415
8	0.119995	0.123323
9	0.079956	0.109911
10	-0.200439	0.099112
11	-3.204834	0.090234
12	-39.458008	0.082808
13	-513.954102	0.076508
14	-7196.357422	0.071119
15	-107946.359375	0.066784
16	-1727142.750000	0.068539
17	-29361428.000000	0.165170
18	-528505696.000000	1.973062
19	-10041608192.000000	36.488184
20	-200832172032.000000	728.763690
21	-4217475694592.000000	15303.037482
22	-92784463708160.000000	336665.824594

Script Code (vaja8_4.m):

```
1 - clc;
2 - clear all;
3 - format long
4 - n=0;
5 - Ie=single(exp(1))-1; %enojna natančnost 4 byti
6 - Id=double(exp(1))-1; %dvojna natančnost 8 bytov
7 - fprintf(' n I(enojna natančnost) I(dvojna natančnost)\r');
8 - fprintf('%3i %25.6f %25.6f\r',n,Ie,Id);
9 - for n=0:21
10 - Ie=(n+1)*single(Ie)-1; %enojna natančnost 4 byti
11 - Id=(n+1)*double(Id)-1; %dvojna natančnost 8 bytov
12 - fprintf('%3i %25.6f %25.6f\r',n+1,Ie,Id);
13 - end
14 - disp(' ')
15 - disp('Spremenljivka Ie je zapisana v enojni natančnosti')
16 - whos Ie
17 - disp('Spremenljivka Id je zapisana v dvojni natančnosti')
18 - whos Id
19
```

Variable Information:

Spremenljivka Ie je zapisana v enojni natančnosti

Name	Size	Bytes	Class	Attributes
Ie	1x1	4	single	

Spremenljivka Id je zapisana v dvojni natančnosti

Name	Size	Bytes	Class	Attributes
Id	1x1	8	double	

6.2b Numerična napaka-integral

The image shows the MATLAB 7.6.0 (R2008a) environment. The Command Window displays the results of running a script, showing the value of I for different values of n . The Editor window shows the script code, which includes a function `Glavni` and a function `Integral(n)`. The script calls `Integral(6)` and `Integral(11)`. The output shows that as n increases, the value of I converges to approximately 1.718282.

```
Command Window
```

n	I(enojna natančnost)
6	0.000000
5	0.166667
4	0.233333
3	0.308333
2	0.436111
1	0.718056
0	1.718055

n	I(enojna natančnost)
11	0.000000
10	0.090909
9	0.109091
8	0.123232
7	0.140404
6	0.162915
5	0.193819
4	0.238764
3	0.309691
2	0.436564
1	0.718282
0	1.718282

```
Editor - C:\vaje NM IR\numerične\nm\vaja 8\vaja8_4.m*
```

```
1 function Glavni
2     clc;
3     clear all;
4     Integral(6);
5     Integral(11);
6     end
7
8 function Integral(n)
9     I=0;
10    fprintf('  n      I(enojna natančnost) \r');
11    fprintf('%3i %25.6f \r',n,I);
12    n=n-1;
13    for n=n:-1:0
14        I=(single(I)+1)/(n+1); %enojna natančnost 4 byti
15        fprintf('%3i %25.6f \r',n,I);
16    end
17    disp ' '
18    end
19
20
```

Start | Glavni / Integral | Ln 9 Col 5 | OVR.

6.3 Numerična napaka-zaporedje

Imamo zaporedje $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots, \frac{1}{3^n} = p_n, n = 0, 1, 2, \dots$

Zapiši zaporedje na tri načine:

-z Matlabovim vektorskim zapisom

-kot zaporedje: $p_{n+1} = \frac{1}{3} p_n, p_0 = 1, n = 0, 1, 2, \dots$

-kot zaporedje: $p_{n+1} = p_{n-1} - \frac{8}{3} p_n, p_0 = 1, p_1 = \frac{1}{3}$

Opazuj napako pri rekurzivnem odštevanju.

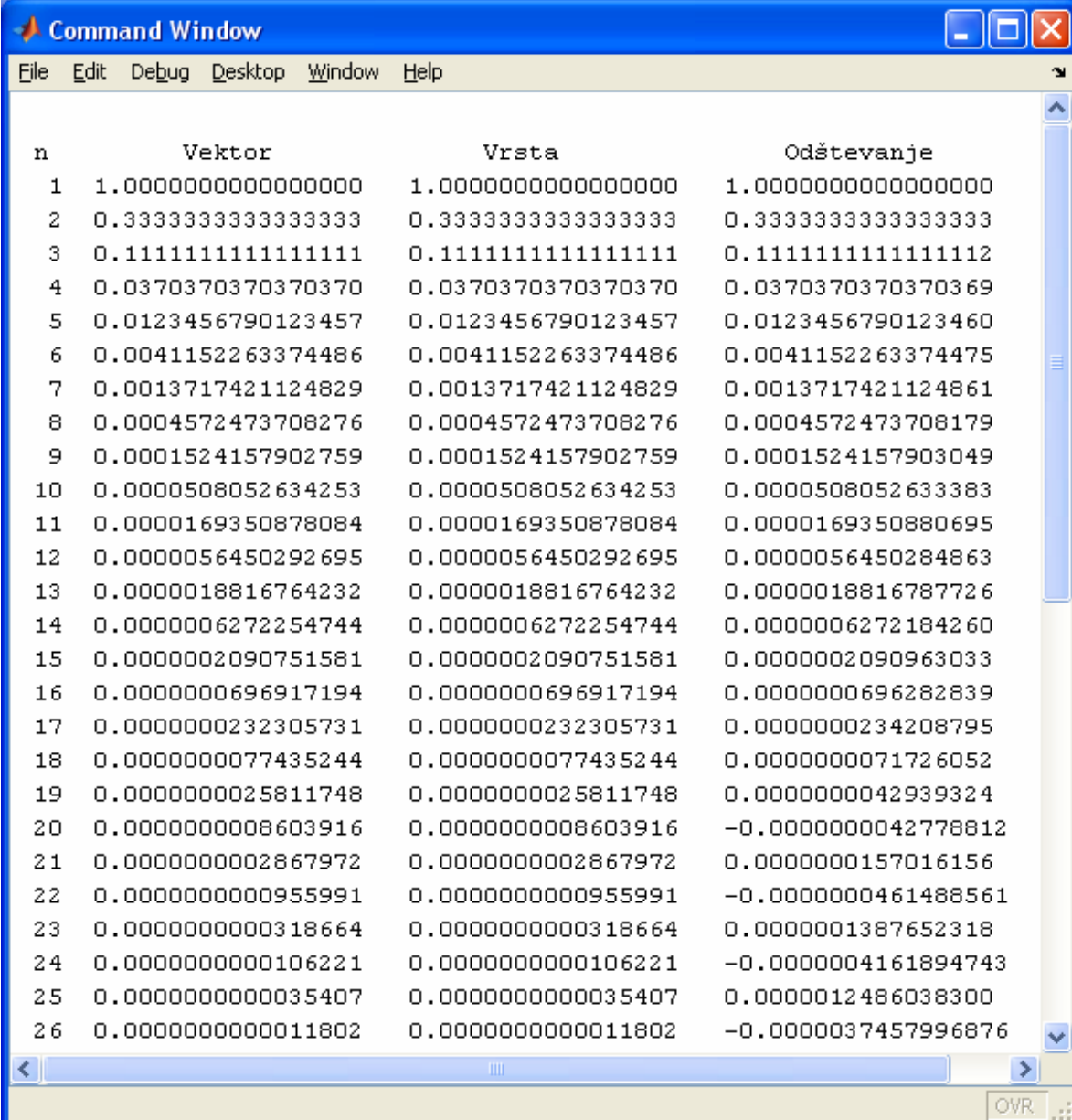
6.3 Numerična napaka-zaporedje

```
C:\vaje NM IR\numerične\nm\vaja 8\vaja8_7.m
File Edit Text Go Cell Tools Debug Desktop Window Help
1 -   clc;clear all;format long
2 -   N=26;
3 -   p1 = (1/3).^(0:N); %matlab izračun v vektorju
4
5 -   p2(1) = p1(1); %začetne vrednosti
6 -   p2(2) = p1(2);
7 -   p3(1) = p1(1);
8 -   p3(2) = p1(2);
9
10 -  for n = 1:N
11 -      p2(n+1) = (1/3)*p2(n); %izračun zaporedja 1/3^n
12
13 -  end
14 -      disp ' '
15 -  for n = 2:N,
16 -      p3(n+1) = p3(n-1)-8/3*p3(n); % izračun z odštevanjem
17
18 -  end
19 -      disp ' n          Vektor          Vrsta          Odštevanje'
20 -  for i=1:N
21 -      fprintf('%3i %18.16f %18.16f %18.16f\n',i,p1(i),p2(i),p3(i))
22 -  end
23
24 -      plot([p2',p3'])
25
```

script Ln 24 Col 16 OVR

6.3 Numerična napaka-zaporedje

Rezultat



n	Vektor	Vrsta	Odštevanje
1	1.0000000000000000	1.0000000000000000	1.0000000000000000
2	0.3333333333333333	0.3333333333333333	0.3333333333333333
3	0.1111111111111111	0.1111111111111111	0.1111111111111112
4	0.0370370370370370	0.0370370370370370	0.0370370370370369
5	0.0123456790123457	0.0123456790123457	0.0123456790123460
6	0.0041152263374486	0.0041152263374486	0.0041152263374475
7	0.0013717421124829	0.0013717421124829	0.0013717421124861
8	0.0004572473708276	0.0004572473708276	0.0004572473708179
9	0.0001524157902759	0.0001524157902759	0.0001524157903049
10	0.0000508052634253	0.0000508052634253	0.0000508052633383
11	0.0000169350878084	0.0000169350878084	0.0000169350880695
12	0.0000056450292695	0.0000056450292695	0.0000056450284863
13	0.0000018816764232	0.0000018816764232	0.0000018816787726
14	0.0000006272254744	0.0000006272254744	0.0000006272184260
15	0.0000002090751581	0.0000002090751581	0.0000002090963033
16	0.0000000696917194	0.0000000696917194	0.0000000696282839
17	0.0000000232305731	0.0000000232305731	0.0000000234208795
18	0.0000000077435244	0.0000000077435244	0.0000000071726052
19	0.0000000025811748	0.0000000025811748	0.0000000042939324
20	0.0000000008603916	0.0000000008603916	-0.0000000042778812
21	0.0000000002867972	0.0000000002867972	0.0000000157016156
22	0.0000000000955991	0.0000000000955991	-0.0000000461488561
23	0.0000000000318664	0.0000000000318664	0.0000001387652318
24	0.0000000000106221	0.0000000000106221	-0.0000004161894743
25	0.0000000000035407	0.0000000000035407	0.0000012486038300
26	0.0000000000011802	0.0000000000011802	-0.0000037457996876