

Nihanje

1) Kako v splošnem opišemo harmonično nihanje in kaj je zanj značilno?

Za nihanje je značilno, da amplitude poti in hitrosti ter pospeška spreminjajo s funkcijami sin in cos to pomeni, da se enake vrednosti, amplitude ponavljajo, kar pa ni značilno za premo gibanje. Harmonično nihanje lahko preide tudi v resonanco – pojav ko pride do zelo velikih amplitud.

$$\begin{aligned} \varphi(t) &= \varphi_0 + \omega t \\ x(t) &= x_0 \cdot \sin(\omega t + \varphi_0) \\ \dot{x}(t) = v(t) &= x_0 \cdot \omega \cdot \cos(\omega t + \varphi_0) \\ \ddot{x}(t) = a(t) &= -x_0 \cdot \omega^2 \cdot \sin(\omega t + \varphi_0) \end{aligned}$$

$\omega = \frac{2\pi}{T_0} = 2\pi\nu$
 $\nu = \frac{1}{T_0}; T = \frac{2\pi}{\omega}$

$x(t)$ pely (t) ferencalncos(ωbφ₀)T₀ rāz za frkvenco pri:
-nihalo na vijačno vzmet oz. spiralo:

$$\begin{aligned} F &= -kx = m \cdot \ddot{x} = ma \\ \ddot{x} + \omega^2 x &= 0 \\ \ddot{x} + \frac{k}{m} x &= 0 \\ \omega &= \sqrt{\frac{k}{m}} \\ W &= \frac{kx^2}{2} + \frac{mv^2}{2} = \frac{kx_0}{2} = \frac{mv_0^2}{2} \\ M &= -D\varphi = J \cdot \ddot{\varphi} \\ \ddot{\varphi} + \frac{D}{J} \varphi &= 0 \rightarrow \omega = \sqrt{\frac{D}{J}} \\ W &= \frac{1}{2} \omega J \omega_0^2 \varphi_0^2 = \frac{\varphi_0^2}{2} \end{aligned}$$

-fizično in matematično nihalo.

$$\begin{aligned} M &= -gml \sin \varphi = J \cdot \ddot{\varphi}; \sin \varphi = \varphi \quad J = ml^2 \\ \ddot{\varphi} + \frac{gml \varphi}{J} &= 0 \\ \omega &= \sqrt{\frac{gml}{J}} \\ W &= \frac{1}{2} J \omega_0^2 \varphi_0^2 = \frac{1}{2} mgl \varphi_0^2 \\ M &= -mgl \sin \varphi = J \cdot \ddot{\varphi} \\ \ddot{\varphi} + \frac{mgl}{ml^2} \varphi &= 0 \\ \omega &= \sqrt{\frac{g}{l}} \end{aligned}$$

-nihanje vode v u cevki:

$$\begin{aligned} x &= x_0 \sin \omega t \\ F &= ma = 2Sx\rho g = m\omega^2 x \\ \omega^2 &= \frac{2Sx\rho g}{mx\rho Sb} = \frac{2g}{b} \\ W &= Sb\rho g x_0 \end{aligned}$$

-el. nihajni krog

$$\begin{aligned} U_c &= U_0 \sin \omega t \\ \frac{d^2 U_c}{dt^2} + 2\beta \frac{dU_c}{dt} + \omega_0^2 U_c &= \omega_0^2 U_g \\ \omega_0 &= \sqrt{\frac{1}{LC}}; \beta = \frac{R}{2L} \\ W &= \frac{LI^2}{2} = \frac{CU_c^2}{2} = \frac{LI_0^2}{2} = \frac{CU_{c0}^2}{2} \end{aligned}$$

3) Izpelji diferencialno enačbo dušenega nihanja mase na vijačni vzmeti, ki je potopljena v tekočino. kako opišemo časovni potek nihanja in kaj je logaritemski dekrament? Kdaj je gibanje aperioidično?

-gibanje je periodično ko je $\beta > \omega$ in $\omega^2 < 0$

$$F = -kx - R\dot{x} = m\ddot{x}$$

$$\ddot{x} + \frac{R}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\ddot{x} + \frac{R}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$$

$$\omega = \sqrt{\frac{k}{m}}; \beta = \frac{R}{2m}$$

$$x(t) = x_0 \cos(\omega t - \varphi_0)$$

$$\omega^2 = \omega_0^2 - \beta^2; \beta = \frac{1}{\lambda}$$

$$x(t) = x_0(t) \cdot e^{-\beta t} \cos(\omega t - \varphi_0)$$

$$x_0(t) = \frac{x_0(T)}{x_0} = e^{-\lambda}$$

$$\lambda = \ln \frac{x_0(0)}{x_0(T)} = \log \text{aritmski dekrament}$$

4) Izpelji enačbo vsiljenega nihanja mase na prožni vijačni vzmeti, ki visi na premičnem drogu. Pojasni pojav resonance in skiciraj krivulji za amplitudo in fazni premik.

$$F = -k(x - x_1) - R\dot{x} = m\ddot{x} \quad \ddot{x} + 2\beta\dot{x} + \omega_0^2 x = \omega_0^2 x_1(t)$$

$$\ddot{x} + \frac{R}{m}\dot{x} + k(x - x_1) = 0$$

$$2\beta = \frac{R}{m} \rightarrow \omega_0^2 = \frac{k}{M}$$

$$\beta = \frac{R}{2M} \rightarrow \omega_0 = \sqrt{\frac{k}{M}}$$

$$x_1(t) = a \sin(\omega t)$$

$$\frac{x_0}{a} = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\beta^2 \omega^2}}$$

$$\text{tg } \varphi(\omega) = \frac{2\beta\omega}{\omega_0^2 - \omega^2}$$

5) Kako je sestavljen električni oscilator in kako pokažemo resonanco električnega nihajnega kroga?