Višji odvodi, Taylorjev polinom:

Leibnizova formula: $(f\cdot g)^{(n)}=\sum_{k=0}^{n}\binom{n}{k}f^{(k)}\cdot g^{(n-k)}$ Primer:

[illegible]

Taylorjev polinom: $T_n(x)=\sum_{k=0}^{n}\frac{f^{(k)}_{(x_0)}}{k!}\cdot(x-x_0)^k$ Primera:

$$(I)\quad f(x)=\frac{e^x}{1-x},\ \ T_n(x)=?,\quad x_0=0$$

$$(1-x)\cdot f(x)=e^x\Rightarrow k-krat\ odvajmo\Rightarrow\sum_{i}^{k}\binom{k}{i}($$

$$\binom{k}{0}(1-x)\cdot f^{(k)}_{(x)}+\binom{k}{1}(-1)\cdot f^{(k-1)}_{(x)}=e^x\Rightarrow x=0=$$

$$\frac{f^{(k)}_{(0)}}{k!}=\frac{f^{(k-1)}_{(0)}}{(k-1)!}+\frac{1}{k!}\Rightarrow \text{uporabis}: a_k=\frac{f^{(k)}_{(0)}}{k!}\Rightarrow$$

$$a_0=f(0)=1,\ a_1=a_0+\frac{1}{1!}=1+\frac{1}{1!},\ a_2=a_1+\frac{1}{2}$$

$$\text{Rešitev}\quad T_n(x)=\sum_{k=0}^{n}a_k\cdot x^k=\sum_{k=0}^{n}(1+\frac{1}{1!}+\frac{1}{2!}+..$$

$$(II)\quad f(x)=\frac{1}{x^2-x+1},\ T_{2n}(x)=?,\ x_0=0,\ f^{(}_{(0}$$

$$(x^2-x+1)\cdot f(x)=1\Rightarrow k-krat\ odvajmo\Rightarrow\sum_{i}^{k}($$

$$\binom{k}{0}(x^2-x+1)\cdot f^{(k)}_{(x)}+\binom{k}{1}(2x-1)\cdot f^{(k-1)}_{(x)}+\binom{k}{2}(2$$

$$f^{(k)}_{(0)}-k\cdot f^{(k-1)}_{(0)}+k(k-1)\cdot f^{(k-2)}_{(0)}=0\quad \text{delimo s}$$

$$\frac{f^{(k)}_{(0)}}{k!}-\frac{f^{(k-1)}_{(0)}}{(k-1)!}+\frac{f^{(k-2)}_{(0)}}{(k-2)!}=0\Rightarrow \text{vstavimo}\ a_k=$$

$$a_0=f(0)=1,\ a_1=f'(1)=1,\ a_2=a_1-a_0=0,$$

$$\text{Re}\,\check{s}itev:\quad T_n(x)=\sum_{k}^{n}a_k\cdot x^k=1+x-x^3-x^4+$$

$$a_{20}=\frac{f^{(20)}_{(0)}}{20!}\Rightarrow f^{(20)}_{(0)}=a_{20}\cdot 20!=0\cdot$$

Posebnosti:

$$f(x)=\cos x,\ x_0=0,\ T_{2n}(x)=\sum_{k=0}^{n}(-1)^k\frac{x^{2k}}{(2k)!}=$$

$$f(x)=\sin x,\ x_0=0,\ T_{2n}(x)=\sum_{k=0}^{n}(-1)^{k+1}\frac{x^{2k+}}{(2k+}$$

Integrali:

f(x)	F(x)
x^n	$\frac{x^{n+1}}{n+1}+C$
$\frac{1}{x}$	log x + C
a^x	$\frac{a^x}{\log x}+C$
e^{ax}	$\frac{e^{ax}}{a}+C$
sin x	- cos x + C
cos x	sin x + C
$\frac{1}{\sin^2 x}$	- ctg x +C
$\frac{1}{\cos^2 x}$	tg x + C
tg x	- log(cos x) + C
ctg x	log(sin x) + C
$\frac{1}{\sqrt{a^2-x^2}}$	arcsin $\frac{x}{a}$ + C
$\frac{1}{\sqrt{x^2+a^2}}$	$\text{arcsin}h\frac{x}{a}+C=\log(x+$
$\frac{1}{\sqrt{x^2-a^2}}$	$\text{arccos}h\frac{x}{a}+C=\log(x+$
$\frac{1}{x^2+1}$	arctg x + C
sinh x	cosh x + C
cosh x	sinh x + C
$\frac{1}{\sin x}$	$\ln(tg\frac{x}{2})+C$
$\frac{1}{\sinh x}$	$\ln(tgh\frac{x}{2})+C$
$\frac{1}{(x-a)^n}$	$-\frac{1}{(n-1)(x-a)^{n-1}}$
$\frac{1}{x^2-1}$	$\frac{1}{2}\log(\frac{x-1}{x+1})+C$

Integriranje po delih:

$$\int F(x)\cdot g(x)dx=F(x)\cdot G(x)-\int f(x)\cdot G(x)dx$$

Primeri:

$$(I)\qquad \int\log x dx=\int 1\cdot\log x dx=x\cdot\log x-\int x\cdot\frac{1}{x}dx=x\cdot\log x$$

$$(II)\qquad \int_0^1(1-x^2)e^{-2}dx=(1-x^2)(-\frac{1}{2}e^{-2x})\Big|_0^1-\int_0^1-2x(-\frac{1}{2}e^{-}$$

$$\frac{1}{2}-(x(-\frac{1}{2}e^{-2x})\Big|_0^1-\int_0^1-\frac{1}{2}e^{-2x}dx)=\frac{1}{2}+\frac{1}{2}e^{-2}-\frac{1}{2}(-\frac{1}{2}e^{-2x}$$

Nova spremenljivka: Primeri

$(I) \qquad \int (1-x^2)^2 xdx = (u = x^2, du = 2xdx) = \int ($

$(II) \qquad \int_{-1}^{1} \frac{dx}{\sqrt{x^2+x+1}} = \int_{1}^{-1} \frac{dx}{\sqrt{(x+1/2)^2+3/4}} = ($

$$\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{du}{(1+u)^2} = \arcsin hu \Big|_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} = \ldots$$

Ko integriramo racionalne funkcije, dobimo čelne : $\int \frac{Ax+B}{(x^2+ax+b)^m} dx$, kjer kvadratni polinom v imenovalcu ne moremo razstaviti. Vedno zapišemo:

$$x^2+ax+b = (x+\frac{a}{2})^2 + (b-\frac{a^2}{4}) =$$

$= (x+\frac{a}{2})^2 + c^2$, kjer damo novo spremenljivko: $x+\frac{a}{2} = c \cdot u$

Integracija racionalnih funkcij: razcep na parcialne ulomke: Primeri:

$$(I) \qquad f(x) = \frac{1}{x^2(x^2+1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{(1+x^2)}$$

$$1 = Ax(1+x^2)^2 + B(1+x^2)^2 + (Cx+D)x^2(1+x^2)$$

$$x^5: A+C=0;\ x^4: B+D=0;\ x^3: 2A+C+E=$$

$$\text{Razberemo}: A=0, B=1, C=0, D=-1, E=0, F=$$

$$\text{Sledi}: \frac{1}{x^2(x^2+1)^2} = \frac{1}{x^2} - \frac{1}{(1+x^2)} - \frac{1}{(1+x^2)^2} =$$

Integrali racionalni izrazov:

Vedno deluje nova spremenljivka: $u = \tan\frac{x}{2} \Rightarrow dx = \frac{2du}{1+u^2}$, in iz tega sledi , da je:

$\sin x = \frac{2u}{1+u^2} \quad \text{in} \quad \cos x = \frac{1-u^2}{1+u^2}$ *Obstaja tudi*:

$$u = \tanh\frac{x}{2}, du = \frac{1-u^2}{2} dx, \sinh x = \frac{2u}{1-u^2}$$

Izlimitirani integrali: Primeri:

$$(I) \qquad \int_{-\infty}^{\infty} e^{-x^2} e^{-(a-x)^2} dx = e^{-a^2} \int_{-\infty}^{\infty} e^{-2x^2+2ax} = e^{-}$$

$$(\sqrt{2}(x-\frac{a}{2}) = u, \sqrt{2}dx = du) = e^{-\frac{a^2}{2}} \int_{-\infty}^{\infty} e^{-u^2} \frac{du}{\sqrt{2}}$$

Uporaba integralov:
Dolžina krivulje:

$$L = \int_a^b \sqrt{\dot{x}_{(t)}^2 + \dot{y}_{(t)}^2}\, dt = \int_a^b \sqrt{1+(\dot{f}(x))^2}\, dx$$

Vektor v smeri tangente na krivuljo v točki

$$(x(t), y(t))\ je (\dot{x}(t), \dot{y}(t))$$

Ploščina: $A = -\int_a^b y \cdot \dot{x}\, dt = \int_a^b x \cdot \dot{y}\, dt$ Volumen:

$$V = \pi \int_a^b (f(x))^2$$

Površina: $P = 2\pi \int_a^b f(x) \cdot \sqrt{1+(\dot{f}(x))^2}\, dx$ Ukrivljenost:

$$k = \frac{\dot{y}\ddot{x} - \dot{x}\ddot{y}}{(\dot{x}^2+\dot{y}^2)^{3/2}}$$

Polarne koordinate: $x = r \cdot \cos\phi \quad r = \sqrt{x^2+y^2}$

$y = r \cdot \sin\phi \quad \phi = \arctan\frac{y}{x}$

Površina: $A = \int_\alpha^\beta \frac{1}{2} r^2(\phi) d\phi$ Dolžina krivulje:

$$L = \int_\alpha^\beta \sqrt{\dot{r}^2 + r^2}\, d\phi$$