Limita funkcij:

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to 0}\frac{x}{a^x}=0,\quad a>1$$

$$\lim_{x\to\infty}\frac{e^x-1}{x}=1$$

$$\lim_{x\to 0}\frac{a^x-1}{x}=\log a \qquad za\ a>1$$

$$\lim_{x\to 0}(1+\frac{1}{x})^x=e$$

$$\lim_{x\to\infty}\frac{\log x}{x^\alpha}=0 \qquad \alpha>0$$

$$\lim_{x\to\infty}\frac{(n-1)}{(1+nk)^2}=0$$

$$\lim_{x\to 0}x^x=1$$

$$\lim_{x\to 0}x\log x=0$$

$$\lim_{x\to\infty}\sqrt[n]{n}=1$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$$

$$A=\lim_{x\to x_0}f(x)\quad \text{in}\quad B=\lim_{x\to x_0}g(x)$$

$$\lim_{x\to x_0}(f(x)+g(x))=A+B$$

$$\lim_{x\to x_0}C\cdot f(x)=C\cdot A$$

$$\lim_{x\to x_0}f(x)\cdot g(x)=A\cdot B$$

$$\lim_{x\to x_0}f(x)/g(x)=A/B \quad \text{za } B\neq 0$$

*

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=\lim_{x\to 0}\frac{1+x-1}{x(\sqrt{1+x}+1)}=\lim_{x\to 0}\frac{1}{\sqrt{1+x}+1}=$$

*

$$\lim_{x\to 0}\frac{\sqrt[n]{1+x}-\sqrt[n]{1-x}}{x}=\lim_{x\to 0}\frac{(\sqrt[n]{1+x}-\sqrt[n]{1-x})(\sqrt[n]{1+}}{x(\sqrt[n]{1+x}^{\,n-1}+}$$

$$\lim_{x\to 0}\frac{1+x-1+x}{x(\sqrt[n]{1+x}^{\,n-1}+\ldots+\sqrt[n]{1-x}^{\,n-1})}=\frac{2}{n}$$

* Par + primerov

$$\lim_{x\to 0}\frac{1-\cos^3 ax}{x\cdot\sin bx}=\lim_{x\to 0}\frac{(1-\cos ax)(1+\cos ax+\cos^2 ax)}{x\cdot\sin bx}=3\lim_{x\to 0}$$

$$\lim_{x\to\infty}\frac{\sqrt[3]{x^2+1}-4x}{6x+1}=\lim_{x\to\infty}\frac{\frac{\sqrt[3]{x^2+1}}{x}-\frac{4x}{x}}{\frac{6x}{x}+\frac{1}{x}}=\lim_{x\to\infty}\frac{-4}{6}=-\frac{2}{3}$$

$$\lim_{x\to\infty}\frac{1+2+3+\ldots+n}{n^2}=\lim_{x\to\infty}\frac{\frac{n(n+1)}{2}}{n^2}=\lim_{x\to\infty}\frac{n+1}{2n}=\lim_{x\to\infty}\frac{1+\frac{1}{n}}{2}=$$

$$\lim_{x\to\infty}\frac{3^{n+1}-2^{n+1}}{3^n-2^n}=\lim_{x\to\infty}\frac{3^{n+1}(1-\frac{2^{n+1}}{3^{n+1}})}{3^n(1-\frac{2^n}{3^n})}=3\lim_{x\to\infty}\frac{1-\frac{2^{n+1}}{3^{n+1}}\to 0}{1-\frac{2^n}{3^n}\to 0}=3$$

Odvodi:

$$(x^\alpha)'=\alpha x^{\alpha-1}$$

$$(e^x)'=e^x$$

$$(a^x)'=a^x\cdot\log a$$

$$(x^x)'=x^x\cdot\log(x+1)$$

$$(\log x)'=\frac{1}{x}$$

$$(\sin x)'=\cos x$$

$$(\cos x)'=-\sin x$$

$$(tgx)'=\frac{1}{\cos^2 x}=1+tg^2 x$$

$$(ctgx)'=-\frac{1}{\sin^2 x}=-(1+ctg^2 x)$$

$$(\arcsin x)'=\frac{1}{\sqrt{1-x^2}}$$

(arccos x)

(arctgx) =

(arcctgx)

(sinh x) =

(cosh x) =

(tghx) =

(ctghx) =

(arcsinh x)

(arcctghx)

L'Hospitalovo pravilo:

$$\lim_{x\to x_0}\frac{f(x)}{g(x)}\ {}^{\to 0,\infty}_{\to 0,\infty}=\lim_{x\to x_0}\frac{f'(x)}{g'(x)};\ \text{in če je to še vedno } \frac{0}{0} \text{ oz } \frac{\infty}{\infty},$$

Če je $f'(x)=0$, je za vsak x funkcija konstanta $f(x)=C$.

*

$$\lim_{x\to 0}(\cos x)^{\frac{1}{x^2}}=\lim_{x\to 0}e^{\frac{1}{x^2}\log(\cos x)}\Rightarrow\lim_{x\to 0}\frac{\log(\cos x)}{x^2}\ {}^{\to 0}_{\to 0}=\lim_{x\to 0}\frac{\frac{1}{\cos x}}{}$$

$$=\lim_{x\to 0}\frac{\sin x}{x}(-\frac{1}{2\cos x})=1\cdot(-\frac{1}{2})\Rightarrow\lim_{x\to 0}e^{\frac{1}{x^2}\log(\cos x)}=e^{-\frac{1}{2}}$$

Odvod inverzne funkcije:

f(x) mora biti strogo naraščajoča, tako je izpolnjen pogoj, da je injektivna. g'(y) je njen inverz. y = f(x) in x = g(y)

$$g'(y)=\frac{1}{f'(x)}=\frac{1}{f'(g(y))}$$

* $x\in(0,2),\ f(x)=\arccos(1-x)-\sqrt{(2-x)x}$

Pokaži. da je funkcija strogo naraščajoča in ima inverz:

$$f'(x)>0\Rightarrow f'(x)=\frac{-1(-1)}{\sqrt{1-(1-x)^2}}-\frac{2-2x}{2\sqrt{(2-x)x}}=-$$

Naj bo g(y) njen inverz. Izračunaj $g'(\frac{\pi}{4}-\frac{\sqrt{2}}{2})$.

$$g'(y)=\frac{1}{f'(x)};x=g(y);y=f(x)$$

$$\frac{\pi}{4}-\frac{\sqrt{2}}{2}=\arccos(1-x)-\sqrt{(2-x)x}\Rightarrow\frac{\pi}{4}=ac\cos$$

$$\text{Pr}\,eizkus:\sqrt{(2-x)x}=\sqrt{(2-\frac{2-\sqrt{2}}{2})\frac{2-\sqrt{2}}{2}}=\sqrt{}$$

Asimptota:

Predpostavimo, da je asimptota y = kx + n

$$k=\lim_{x\to\infty}\frac{f(x)}{x}$$

$$n=\lim_{x\to\infty}(f(x)-k\cdot x)$$

*

$$f(x)=arctg(1+\frac{1}{x})$$

$$y=kx+n\Rightarrow k=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{arctg(1+}{x}$$

Naraščanje: $f'(x)>0$; Minimum: $f''(x)>0$

Konkavnost: $f''(x)\le 0$

Padanje: $f'(x)<0$ Maksimum: $f''(x)<0$ Konveksnost: $f''(x)\ge 0$

Uporabne naloge:

* Med vsemi pravokotniki z obsegom *l* poišči tistega z največjo površino:

$$l=2a+2b\Rightarrow b=\frac{l-2a}{2}$$

$$S=a\cdot b\Rightarrow S(a)=a\frac{l-2a}{2}=\frac{1}{2}(la-2a^2)$$

$$S'(a)=\frac{1}{2}(l-4a)=0\Rightarrow a=\frac{l}{4}$$

$$b=\frac{l-\frac{l}{2}}{2}=\frac{l}{4}$$

$$\text{Dokaz za maksimum}:$$

$$S''(a)=-2<0-maksimum$$