1. Dokaz za inverznost:

F(G(y)) = y
G(F(x)) = x

2. Limita zaporedja:

- zaporedja:

$$A(x,y)=\frac{x+y}{2}\ge G(x,y)=\sqrt{x\cdot y}\ge H(x,y)=\frac{\cdots}{\frac{1}{x}\cdots}$$

- primeri:

* $a_n=\frac{2}{n}\quad n\ge 1\qquad 2,\ 1,\ \frac{2}{3},\frac{1}{2},\ldots$

1° Zaporedje je padajoče:

$$a_{n+1}-a_n=\frac{2}{n+1}-\frac{2}{n}=\frac{2n-2n-2}{(n+1)n}=-\frac{2}{(n+1)n}<\ldots$$

2° Omejeno navzdol: $a_n=\frac{2}{n}>0$

3° Konvergira: $\lim\limits_{n\to\infty}a_n=\lim\limits_{n\to\infty}\frac{2}{n}=0$

* $a_0=0\qquad a_{n+1}=\frac{1}{3}a_n^2+\frac{2}{3}$

1° Dokaz, da je $0\le a\le 1$: matematičaa indukcija

(i) $\mathrm{n}=1\Rightarrow a_1=\frac{1}{3}\cdot 0+\frac{2}{3}=\frac{2}{3}<1$

(ii) velja za $\mathrm{a_n}<1$, dokaz za $\mathrm{a_{n+1}}<1: a_{n+1}=\frac{1}{3}\ldots$

2° Zaporedje je naraščaraše: $\mathrm{a_{n+1}}-a_n=\frac{1}{3}a_n^2+\ldots$

3° Limita zap: $\lim\limits_{\mathrm{n}\to\infty}a_n=a$

$$\lim_{n\to\infty}(a_{n+1})=\lim_{n\to\infty}\left(\frac{1}{3}a_n^2+\frac{2}{3}\right)\Rightarrow a=\frac{1}{3}a^2+\ldots$$

* Indukcija po korakih:

$0<a_1<1\qquad a_{n+1}=1-\sqrt{1-a_n}$

$1^\circ Dokaz: 0<a_n<1$

(i) $\mathrm{n}=1\Rightarrow 0<a_1<1$

(ii) $0<a_n<1\Rightarrow 0>-a_n>-1\Rightarrow 1>1-a_n>\ldots$

2° Padajoce zap: $a_{n+1}-a_n=1-\sqrt{1-a_n}-a_n=\ldots$

$$3^\circ\ \lim_{\mathrm{n}\to\infty}a_n=a\Rightarrow\lim_{\mathrm{n}\to\infty}a_{n+1}=\lim_{\mathrm{n}\to\infty}(1-\sqrt{1-a_n}\ldots$$

$$\lim_{\mathrm{n}\to\infty}\frac{a_{n+1}}{a_n}=\lim_{\mathrm{n}\to\infty}\frac{1-\sqrt{1-a_n}}{a_n}=\lim_{\mathrm{n}\to\infty}\frac{1-\ldots}{a_n(1-\ldots}$$

3. Vrste:

* $$\sum_{k=1}^{\infty}\frac{1}{(2k-1)(2k+1)}=\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\ldots$$

$$S_n=\sum_{k=1}^{\infty}\frac{1}{(2k-1)(2k+1)}\Rightarrow\frac{1}{(2k-1)(2k+1)}=\frac{a}{2k-1}+\frac{b}{2k+1}$$

$$S_n=\sum_{k=1}^{\infty}\left(\frac{1}{2(2k-1)}-\frac{1}{2(2k+1)}\right)=\frac{1}{2}\left(1-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\frac{1}{5}-\ldots+\right.$$

$$S=\lim_{n\to\infty}S_n=\frac{1}{2}\lim_{n\to\infty}\left(1-\overset{\longrightarrow 0}{\frac{1}{2n+1}}\right)=\frac{1}{2}$$

- <u>Primerjalni kriterij:</u>

$$\sum_{k\to 1}a_k=\text{konvergira, če }a_k\le b_k\ \ in\ \sum_{k\to 1}b_k\ \ konvergira.\ Isto\ velja\ldots$$

*

$$\sum_{k=1}^{\infty}\frac{1}{k^2}\Rightarrow\frac{1}{k^2}\le\frac{1}{k(k-1)}\quad in\quad\sum_{k=2}^{\infty}\frac{1}{k(k-1)}\ \ konvergira.\Rightarrow\sum_{k=1}^{\infty}\frac{1}{k^2}\ldots$$

* $$\sum_{k=1}^{\infty}\frac{1}{1+a^k}\qquad a>0$$

$$\text{Če je }a>1\Rightarrow\frac{1}{1+a^k}\le\frac{1}{a^k}\quad in\quad\sum_{k=1}^{\infty}\frac{1}{a^k}\ \ je\ konvergentna.$$

$$\text{Če je }a=1\Rightarrow\sum_{k=1}^{\infty}\frac{1}{1+1^k}=\sum_{k=1}^{\infty}\frac{1}{2}-divergira$$

$$\text{Če je }0<a<1\Rightarrow\lim_{k\to\infty}\frac{1}{1+a^k}=1-\text{ni izpolnjen potreben pogoj}\ldots$$

- <u>Kvocientni kriterij:</u>

$$\sum_{k=1}^{\infty}a_k,\qquad a_k>0\qquad l=\lim_{k\to\infty}\frac{a_{k+1}}{a_k}\Rightarrow\begin{array}{l}\text{če je }0\le l<1\text{ - kon}\ldots\\ \text{če je }l>1\text{ - dive}\ldots\end{array}$$

*

$$\sum_{k=1}^{\infty}\frac{k}{(k+1)!}\Rightarrow l=\lim_{k\to\infty}\frac{\frac{k+1}{(k+2)!}}{\frac{k}{(k+1)!}}=\lim_{k\to\infty}\frac{(k+1)(k+1)!}{(k+1)!(k+2)k}=\lim_{k\to\infty}\frac{\ldots}{k(\ldots}$$

- <u>Korenski kriterij:</u>

$$\sum_{k=1}^{\infty}a_k\qquad a_k>0\qquad l=\lim_{k\to\infty}\sqrt[k]{k^k}\Rightarrow\begin{array}{l}\text{če je }0\le l<1-konve\ldots\\ \text{če je }l>1-diverg\ldots\end{array}$$

*

$$\sum_{k=1}^{\infty}\frac{1}{(\log k)}\Rightarrow l=\lim_{k\to\infty}\sqrt[k]{\frac{1}{(\log k)^k}}=\lim_{k\to\infty}\frac{1}{\log k}=0-konvergira$$

Korenski in kvocientni kriterij za l = 1 nista definirana.

- <u>Cauchyjev kriterij</u>:

$$\sum_{k=1}^{\infty}a_k\ \text{konvergira takrat, ko}\sum_{k=1}^{\infty}2^k a_{2^k}\ \text{konvergira.}$$

*

$$\sum_{k=1}^{\infty}\frac{1}{k}\qquad a_k=\frac{1}{k}\Rightarrow a_{2^k}=\frac{1}{2^k}\quad\Rightarrow\sum_{k=0}^{\infty}2^k a_{2^k}=\sum_{k=0}^{\infty}2^k\frac{1}{2^k}=\sum\ldots$$

- <u>Alternirajoče vrste:</u>

$$\sum_{k=1}^{\infty}(-1)^{k+1}a_k\text{ - konvergira, ko }a_k\text{ padajoc in }\lim_{\mathrm{k}\to\infty}a_k=0$$

*

$$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{k(k+2)}\qquad a_k=\frac{1}{k(k+2)}\ge a_{k+1}=\frac{1}{(k+1)(k\ldots}$$

$$\lim_{k\to\infty} a_k = \lim_{k\to\infty} \frac{1}{k(k+2)} = 0 - konvergira$$

$$S_n = \sum_{k=1}^{n} (-1)^{k+1} \frac{1}{k(k+2)} \Rightarrow \frac{1}{k(k+2)} = \frac{a}{k} + \frac{b}{k+2}$$

$$S_n = \frac{1}{2} \sum_{k=1}^{n} (-1)^{k+1} (\frac{1}{k} - \frac{1}{k+2}) = \frac{1}{2} ((1-\frac{1}{3}) - (\frac{1}{2} -$$

$$S = \lim_{n\to\infty} S_n = \frac{1}{2}(1-\frac{1}{2}) = \frac{1}{4}$$

Vrsta $\sum_{k=1}^{\infty} (-1)^{k+1} a_{2^k}$ je absolutno konvergentna, če je vrsta $\sum_{k=1}^{\infty} a_k$ konvergentna.

4. Limita funkcij:

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{x}{a^x} = 0, \quad a > 1$$

$$\lim_{x\to 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x\to 0} \frac{a^x - 1}{x} = \log a \qquad za\ a > 1$$

$$\lim_{x\to 0} (1 + \frac{1}{x})^x = e$$

$$\lim_{x\to\infty} \frac{\log x}{x^\alpha} = 0 \qquad \alpha > 0$$

$$A = \lim_{x\to x_0} f(x) \quad \text{in} \quad B = \lim_{x\to x_0} g(x)$$

$$\lim_{x\to x_0} (f(x) + g(x)) = A + B$$

$$\lim_{x\to x_0} C \cdot f(x) = C \cdot A$$

$$\lim_{x\to x_0} f(x) \cdot g(x) = A \cdot B$$

$$\lim_{x\to x_0} f(x) / g(x) = A/B \qquad \text{za } B \neq 0$$

*

$$\lim_{x\to 0} \frac{\sqrt{1+x} - 1}{x} = \lim_{x\to 0} \frac{1+x-1}{x(\sqrt{1+x}+1)} = \lim_{x\to 0} \frac{1}{\sqrt{1+x}+1} =$$

*

$$\lim_{x\to 0} \frac{\sqrt[n]{1+x} - \sqrt[n]{1-x}}{x} = \lim_{x\to 0} \frac{(\sqrt[n]{1+x} - \sqrt[n]{1-x})(\sqrt[n]{1+}}{x(\sqrt[n]{1+x}^{\,n-1} +}$$

$$\lim_{x\to 0} \frac{1+x-1+x}{x(\sqrt[n]{1+x}^{\,n-1} + \ldots + \sqrt[n]{1-x}^{\,n-1})} = \frac{2}{n}$$

* Par + primerov

$$\lim_{x\to 0} \frac{1-\cos^3 ax}{x \cdot \sin bx} = \lim_{x\to 0} \frac{(1-\cos ax)(1+\cos ax + \cos^2 ax)}{x \cdot \sin bx} = 3 \lim_{x\to 0}$$

$$\lim_{x\to\infty} \frac{\sqrt[3]{x^2+1} - 4x}{6x+1} = \lim_{x\to\infty} \frac{\frac{\sqrt[3]{x^2+1}}{x} - \frac{4x}{x}}{\frac{6x}{x} + \frac{1}{x}} = \lim_{x\to\infty} \frac{-4}{6} = -\frac{2}{3}$$

$$\lim_{x\to\infty} \frac{1+2+3+\ldots+n}{n^2} = \lim_{x\to\infty} \frac{\frac{n(n+1)}{2}}{n^2} = \lim_{x\to\infty} \frac{n+1}{2n} = \lim_{x\to\infty} \frac{1+\frac{1}{n}}{2}$$

$$\lim_{x\to\infty} \frac{3^{n+1} - 2^{n+1}}{3^n - 2^n} = \lim_{x\to\infty} \frac{3^{n+1}(1-\frac{2^{n+1}}{3^{n+1}})}{3^n(1-\frac{2^n}{3^n})} = 3 \lim_{x\to\infty} \frac{1-\frac{2^{n+1}}{3^{n+1}} \to 0}{1-\frac{2^n}{3^n} \to 0} =$$

Adicijski izreki:

sin (x + y) = sin x cos y + sin y cos x
cos (x + y) = cos x cos y – sin x sin y