

Diferencialna enačba I. reda z ločljivima spremenljivkama:

$$y' = f(x) \cdot g(y) \quad ; \quad y(a) = \alpha \quad \Rightarrow \quad \int \frac{dy}{g(y)} = \int f(x) dx$$

Homogena diferencialna enačba I. reda:

$$y' = f\left(\frac{y}{x}\right) \quad ; \quad y(a) = \alpha \quad \Rightarrow \quad u(x) = \frac{y}{x} \rightarrow y' = x \cdot u'(x) + u(x) \quad ; \quad u(a) = \frac{\alpha}{a}$$

Linearna diferencialna enačba I. reda:

$$y' + f(x) \cdot y = g(x) \quad ; \quad y(a) = \alpha$$

$$1. \text{ homogena enačba: } y' + f(x) \cdot y = 0 \quad \Rightarrow \quad \frac{y'}{y} = -f(x) \rightarrow y_H = e^{-\int f(x) dx}$$

$$2. \text{ partikularna enačba: } y_P = C(x) \cdot y_H = \int \frac{g(x)}{y_H} dx \cdot y_H$$

$$3. \text{ rešitev: } y = y_P + C \cdot y_H$$

Bernoulijeva enačba:

$$y' + f(x) \cdot y = g(x) \cdot y^n \quad ; \quad y(a) = \alpha$$

$$\frac{y'}{y^n} + f(x) \cdot \frac{y}{y^n} = g(x) \quad ; \quad \frac{y}{y^n} = \underline{\underline{z}}$$

$$z' + f(x) \cdot (-n+1) = g(x) \cdot (-n+1) \quad \Leftarrow \text{ linearna d.e. I. reda } \rightarrow \text{ izraziš } y = z^{\frac{1}{-n+1}}$$

Linearna diferencialna enačba II. reda s konstantnimi koeficienti:

$$y = y(x) \quad ay'' + by' + cy = g(x)$$

$$1. \text{ homogena enačba: } ay'' + by' + cy = 0 \quad \text{z nastavkom } y = e^{\lambda x}$$

$$\lambda_1 = a \quad \lambda_2 = b \quad \Rightarrow \quad y_1 = e^{ax} \quad y_2 = e^{bx}$$

$$\lambda_{1,2} = a \quad \Rightarrow \quad y_1 = e^{ax} \quad y_2 = x e^{ax}$$

$$\lambda_{1,2} = a \pm bi \quad \Rightarrow \quad y_1 = e^{ax} \cdot \cos(bx) \quad y_2 = e^{ax} \cdot \sin(bx)$$

$$2. \text{ partikularna enačba: } y_P = -y_1(x) \int \frac{y_2(u) \cdot g(u)}{W(u)} du + y_2(x) \int \frac{y_1(u) \cdot g(u)}{W(u)} du$$

$$3. \text{ rešitev: } y = y_P + C_1 \cdot y_1 + C_2 \cdot y_2$$

$$\Rightarrow W \neq 0$$

$$y_1 = e^x \quad y_1 = e^{-x} \quad y_1 = \sin x \quad y_1 = \cos x$$

$$f(x) = e^x \cdot \cos x \rightarrow e^{(1+i)x} = e^x \cdot e^{ix} = e^x \cdot (\cos x + i \sin x) = \text{Re}(e^{(1+i)x})$$

$$y_P = A \cdot e^{(1+i)x}$$

Nastavki za d.e. II. reda:

$$\underline{y_1 = e^x} \quad \underline{y_2 = e^{2x}}$$

$$f(x) = 2x^2 + 3x + 5$$

$$\underline{y_P = ax^2 + bx + c}$$

$$f(x) = \sin x$$

$$\underline{y_P = a \sin x + b \cos x}$$

$$f(x) = \sin^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x$$

$$y_{P1} = A; \text{ če } y_1 = x e^x \rightarrow y_P = Ax$$

$$\underline{y_{P2} = a \sin x + b \cos x}$$

$$f(x) = e^{3x}$$

$$\underline{y_P = a e^{3x}}$$

$$f(x) = e^x$$

$$\underline{y_P = a \cdot x \cdot e^x}$$

$$\underline{y_1 = \sin x} \quad \underline{y_2 = \cos x}$$

$$f(x) = \sin x$$

$$\underline{y_P = x(a \sin x + b \cos x)}$$

Linearno neodvisne rešitve:

$$\Rightarrow W \neq 0$$

$$\Rightarrow \frac{u(x)}{v(x)} \neq \text{konst}$$

$$W = y_1 \cdot y_2' - y_1' \cdot y_2$$

$$e^{ix} = \cos x + i \sin x$$

$$e^{x(a+ib)} = e^{xa} (\cos bx + i \sin bx)$$

$$\int (x^n) dx = \frac{x^{n+1}}{n+1} + c$$

$$\int (e^{ax}) dx = \frac{e^{ax}}{a} + c$$

$$\int (a^x) dx = \frac{a^x}{\log a} + c$$

$$\int \left(\frac{1}{x}\right) dx = \log x + c$$

$$\int \left(\frac{1}{x-a}\right) dx = \log(x-a) + c$$

$$\int \left(\frac{1}{(x-a)^m}\right) dx = -\frac{1}{(m-1)(x-a)^{m-1}} + c$$

$$\int \left(\frac{x}{(1+x^2)^m}\right) dx = -\frac{1}{2^{m-1}(1+x^2)^{m-1}} + c$$

$$\int \left(\frac{x}{(1+x^2)}\right) dx = \frac{1}{2} \log(1+x^2) + c$$

$$\int \left(\frac{1}{\cos^2 x}\right) dx = \int (1 + \tan^2 x) dx = x + \tan x + c$$

$$\int \left(\frac{1}{\sqrt{1-x^2}}\right) dx = \arcsin x + c$$

$$\int \left(\frac{1}{1+x^2}\right) dx = \arctan x + c$$

$$\int \left(\frac{1}{\sqrt{a^2-x^2}}\right) dx = \arcsin \frac{x}{a} + c$$

$$\int \cosh x dx = \sinh x + c$$

$$\int \sinh x dx = \cosh x + c$$

$$\int \frac{1}{\sqrt{1+x^2}} dx = \operatorname{arcsinh} x + C$$

$$= \log(x + \sqrt{1+x^2})$$

$$\int \left(\frac{1}{x^2-1}\right) dx = \frac{1}{2} \log\left(\frac{x-1}{x+1}\right) + c$$

$$\int \frac{f'(x)}{f(x)} dx = \log(f(x))$$

$$\int \cos 2x dx = \frac{1}{2} \sin 2x$$

$$f(x)^{g(x)} = e^{g(x) \cdot \log f(x)}$$

$$(x^\alpha)' = \alpha \cdot x^{\alpha-1}$$

$$(e^{ax})' = ae^{ax}$$

$$(a^x)' = a^x \cdot \log a$$

$$(\log x)' = \frac{1}{x}$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \frac{1}{\cos^2 x} = 1 + \tan^2 x$$

$$(\cot x)' = -\frac{1}{\sin^2 x} = -(1 + \cot^2 x)$$

$$(\cosh x)' = \sinh x$$

$$(\sinh x)' = \cosh x$$

$$(\operatorname{tgh} x)' = \frac{1}{\cosh^2 x} = 1 - \operatorname{tgh}^2 x$$

$$(\operatorname{csch} x)' = -\frac{1}{\sinh^2 x} = -(1 + \operatorname{csch}^2 x)$$

$$e^{\frac{y}{x}} = e^{\frac{y}{x}} \cdot e^{-\frac{y}{x}} \rightarrow e^{\frac{y}{x}} \cdot e^{-\frac{y}{x}}$$

$$e^{\frac{y}{x}} = e^{\frac{y}{x}} \cdot e^{-\frac{y}{x}} \rightarrow e^{\frac{y}{x}}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\operatorname{arctan} x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$$

$$(\operatorname{arcsinh} x)' = \frac{1}{\sqrt{1+x^2}}$$

$$(\operatorname{arcosh} x)' = \frac{1}{\sqrt{x^2-1}}$$

$$(\operatorname{arcsin} x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\sin^3 x = \sin^2 x \cdot \sin x = (1 - \cos^2 x) \cdot \sin x$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^4 x = \left(\frac{1}{2}(1 + \cos 2x)\right)^2 = \frac{1}{4}(1 + 2\cos 2x + \cos^2 2x)$$

$$\cos^4 x = \frac{1}{4}(1 + 2\cos 2x + \cos^2 2x)$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$(x^3 + 1) = (x+1)(x^2 - x + 1)$$

$$x^3 + 3x^2 + 3x + 1 = (x+1)^3$$

$$\int_0^{2\pi} \sin^2 x \cdot dx = \int_c^{2\pi} \cos^2 x \cdot dx = \dots$$

$$\int \frac{1}{x^2 + y^2} dy = \frac{1}{x} \arctan\left(\frac{y}{x}\right)$$

$$\int \frac{1}{(c+z)^3} dz = -\frac{1}{2(c+z)^2}$$

$$\int \frac{1}{\sqrt{1+x^2}} dx = x + \sqrt{1+x^2}$$

$$\int (\sin x) dx = -\cos x$$

$$\int (\cos x) dx = \sin x$$

$$\sin' x = \cos x$$

$$\cos' x = -\sin x$$

$$\sin 2x = 2 \cdot \sin x \cdot \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\sin^2(\arccos x) = 1 - \cos^2(\arccos x) = 1 - x^2$$

$$\cos(\arccos x) = x$$

$$\cos^2(\arccos x) = [\cos(\arccos x)]^2 = x^2$$