

$$\int (x^n) dx = \frac{x^{n+1}}{n+1} + c$$

$$\int (e^{ax}) dx = \frac{e^{ax}}{a} + c$$

$$\int (a^x) dx = \frac{a^x}{\log a} + c$$

$$\int \left(\frac{1}{x}\right) dx = \log x + c$$

$$\int \left(\frac{1}{x-a}\right) dx = \log(x-a) + c$$

$$\int \left(\frac{1}{(x-a)^m}\right) dx = -\frac{1}{(m-1)(x-a)^{m-1}} + c$$

$$\int \left(\frac{x}{(1+x^2)^m}\right) dx = -\frac{1}{2(m-1)(1+x^2)^{m-1}} + c$$

$$\int \left(\frac{x}{(1+x^2)}\right) dx = \frac{1}{2} \ln(1+x^2) + c$$

$$\int (\sin x) dx = -\cos x + c$$

$$\int (\cos x) dx = \sin x + c$$

$$\int \left(\frac{1}{\cos^2 x}\right) dx = \int (1 + \tan^2 x) = \tan x + c$$

$$\int \left(\frac{1}{\sqrt{1-x^2}}\right) dx = \arcsin x + c$$

$$\int \left(\frac{1}{1+x^2}\right) dx = \arctan x + c$$

$$\int \left(\frac{1}{2-x^2}\right) dx = \arcsin \frac{x}{a} + c$$

r c

NOVA SPREMENLJIVKA: $\int_a^b F(g(x)) \cdot g'(x) dx = \int_a^b f(u) du$

$$\int \left(\frac{1}{\sqrt{1+x^2}}\right) dx = \operatorname{arcsinh} x + c = \log(x + \sqrt{1+x^2})$$

$$\int \left(\frac{1}{x^2-1}\right) dx = \frac{1}{2} \log\left(\frac{x-1}{x+1}\right) + c$$

$$\int \frac{f'(x)}{f(x)} dx = \log(f(x))$$

$$\int_0^{\pi} \sin 2x dx = \frac{1}{2} \sin 2x \Big|_0^{\pi} = 0$$

$$\int \frac{1}{x^2+y^2} dy = \frac{1}{x} \arctan\left(\frac{y}{x}\right)$$

$$\int \frac{1}{(x^2+y^2)^3} dz = \frac{1}{2(x^2+y^2)^2}$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\sin^3 x = \sin^2 x \cdot \sin x = (1 - \cos^2 x) \cdot \sin x \quad \cos x = u$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x) \quad -\sin x \cdot dx = du$$

$$\sin^4 x = \left(\frac{1}{2}(1 + \cos 2x)\right)^2 = \frac{1}{4}(1 + 2\cos 2x + \cos^2 2x)$$

$$\cos^4 x = \frac{1}{4}(1 + 2\cos 2x + \cos^2 2x)$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$(x^3 + 1) = (x+1)(x^2 - x + 1)$$

$$\sin 2x = 2 \cdot \sin x \cdot \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\sin^2(\arccos x) = 1 - \cos^2(\arccos x) = 1 - x^2$$

$$\cos(\arccos x) = x$$

$$\cos^2(\arccos x) = [\cos(\arccos x)]^2 = x^2$$

$$\text{Per - partes: } \int_a^b f(x) \cdot g(x) dx = [F(x) \cdot G(x)]_a^b - \int_a^b f(x) \cdot G(x) dx$$

Primer \Rightarrow nova spremenljivka:

$$\int R(x, \sqrt{x^2+a^2}) \Rightarrow x = a \cdot \sinh t ; x = a \cdot \tanh t$$

$$\int R(x, \sqrt{x^2-a^2}) \Rightarrow x = a \cdot \cosh t$$

$$\int R(x, \sqrt{a^2-x^2}) \Rightarrow x = a \cdot \sin t ; x = a \cdot \cos t$$

NOVA SPREMENLJIVKA: $\Phi(u, v) = (u\sqrt{uv}, v\sqrt{uv}) = (\Phi_1, \Phi_2)$

Jacobijeva determinanta:
$$J_\varphi = \begin{vmatrix} \frac{\partial \Phi_1}{\partial u} & \frac{\partial \Phi_1}{\partial v} \\ \frac{\partial \Phi_2}{\partial u} & \frac{\partial \Phi_2}{\partial v} \end{vmatrix} = \left(\frac{\partial \Phi_1}{\partial u} \cdot \frac{\partial \Phi_2}{\partial v} \right) - \left(\frac{\partial \Phi_1}{\partial v} \cdot \frac{\partial \Phi_2}{\partial u} \right)$$

$$\iint_{\Phi(Q)} f(x, y) dx dy = \iint_Q f(x(u, v), y(u, v)) \cdot |J_\varphi(u, v)| \cdot du dv$$

Površina ploskve nad funkcijo:
$$P = \iint_G \sqrt{1 + \left(\frac{\partial F}{\partial x} \right)^2 + \left(\frac{\partial F}{\partial y} \right)^2} dx dy$$

POLARNE KOORDINATE: ko so zadeve v krogih

$$\int xy \cdot dx dy = \int d\varphi \int r^2 \cos \varphi \sin \varphi \cdot r \cdot dr$$

$$\Phi(r, \varphi) = (r \cos \varphi, r \sin \varphi)$$

$$x = r \cos \varphi$$

$$y = r \sin \varphi$$

$$J_\varphi = r$$

$$\Phi(r, \varphi) = (r \cos \varphi, r \sin \varphi)$$

$$x - 1 = r \cos \varphi$$

$$y - 1 = r \sin \varphi$$

$$J_\varphi = r$$

krog premaknjen in zredišča:

CILINDRIČNE KOORDINATE:

$$x = r \cos \varphi$$

$$y = r \sin \varphi$$

$$z = z$$

$$J_\varphi = r$$

KROGELJNE KOORDINATE:

$$x = r \cdot \sin \Theta \cdot \cos \varphi \quad \Theta \in [0, \pi]$$

$$y = r \cdot \sin \Theta \cdot \sin \varphi \quad \varphi \in [0, 2\pi]$$

$$z = r \cdot \cos \Theta$$

$$J_\varphi = r^2 \cdot \sin \Theta$$

premaknjene koordinate:

$$\frac{x}{a} = r \cdot \sin \Theta \cdot \cos \varphi \quad \Theta \in [0, \pi]$$

$$\frac{y}{b} = r \cdot \sin \Theta \cdot \sin \varphi \quad \varphi \in [0, 2\pi]$$

$$\frac{z}{c} = r \cdot \cos \Theta$$

$$J_\varphi = abc \cdot r^2 \cdot \sin \Theta$$

$$x^2 + y^2 = r^2 \cdot \sin^2 \Theta$$

$$x^2 + y^2 + z^2 = r^2$$

ENAČBE:

valj: $x^2 + y^2 = R^2$

krogla: $x^2 + y^2 + z^2 = R^2$