

MEHANIKA FLUIDOV

TLAK

$$\Delta p = \rho \cdot g \cdot \Delta h$$

$$p^h = p - p_0$$

$$p^p = p_0 - p$$

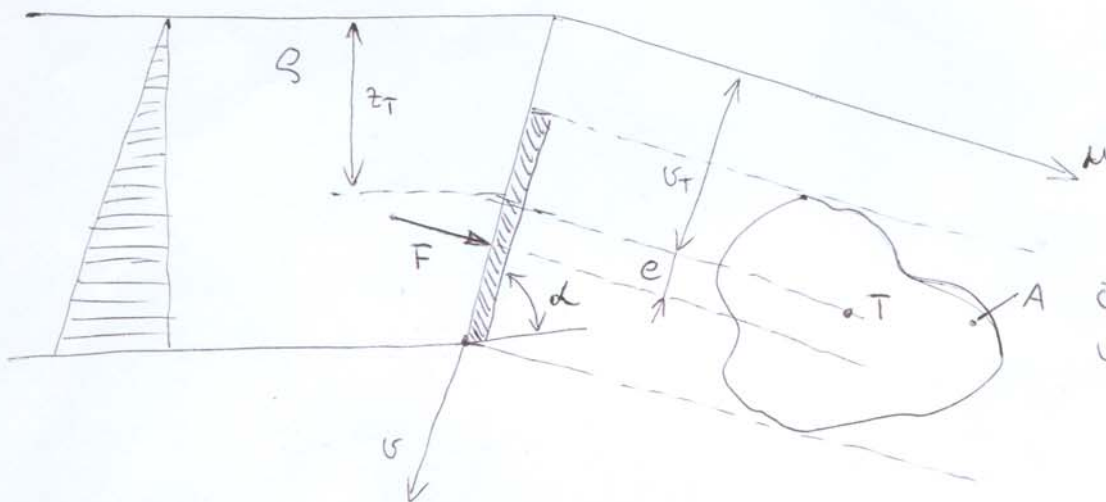
$$p^h = - p^p$$

stisljivost pri velikih tlakah:

$$Q(h) = \frac{E \cdot S_0}{E - \rho_0 \cdot g \cdot h}$$

$$\Delta p = E \cdot \ln \left(\frac{E}{E - \rho_0 \cdot g \cdot h} \right)$$

ZAPORNICE



$$F = \rho \cdot g \cdot z_T \cdot A$$

$$e = \frac{I_{Tt}}{A} > \emptyset$$

$I_{Tt} \rightarrow$ KSP 116

če je v posodi nad tlak:

$$H^* = \frac{p^h}{\rho \cdot g}$$

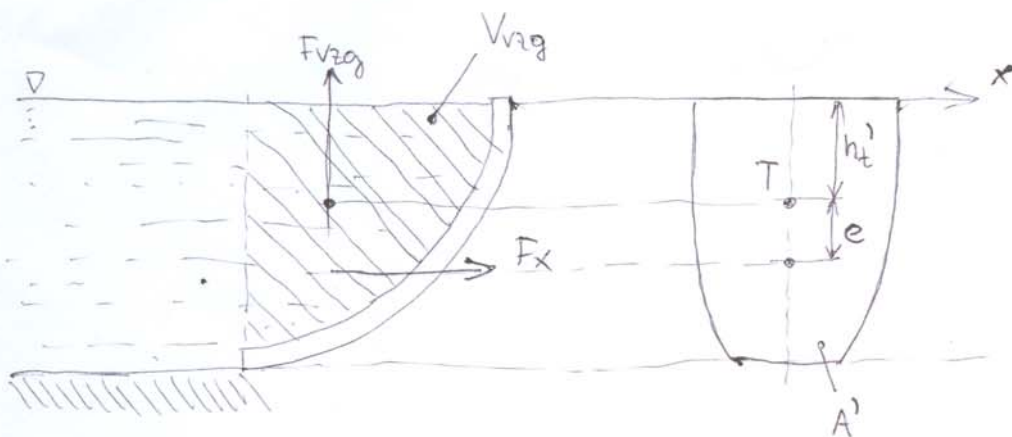
SILA NA POTOPLJENO UKRIVLJENO PLOSKEV

VZGON

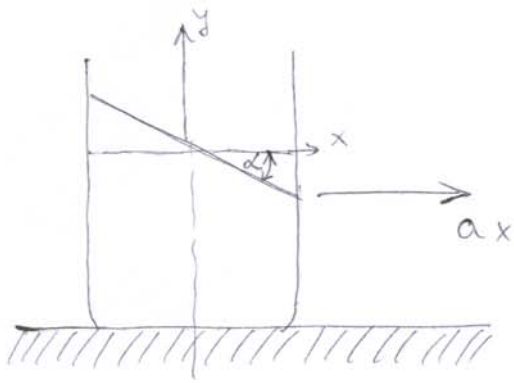
$$F_{vzg} = \rho \cdot g \cdot V_{vzg}$$

$$F_x = \rho \cdot g \cdot h_t' \cdot A'$$

$$e = \frac{I_{xT'}}{A' \cdot h_t'}$$



RELATIVNO MIROVANJE



$$\tan \alpha = -\frac{a_x}{g + a_y}$$

$$y = -\frac{a_x}{g + a_y} \cdot x + y_0$$

$$\frac{\partial p}{\partial x} = -\rho \cdot a_x \quad \frac{\partial p}{\partial y} = -\rho \cdot (g + a_y)$$

$$p(x, y) = -\rho \cdot a_x \cdot x - \rho(a_y + g) \cdot y$$

SILA NA KOLENO

$$\vec{R} = \underbrace{\sum_i \vec{p}_i \cdot \vec{n}_i A_i}_{\text{za odprte tokove} = \emptyset} + \sum_i \rho \cdot \vec{v}_i \cdot (\vec{v}_i \cdot \vec{n}_i) A_i - \underbrace{m \cdot \vec{g}}_{\vec{g}: \oplus (\ominus \cdot \ominus)}$$

za odprte tokove = \emptyset

$\vec{g}: \oplus (\ominus \cdot \ominus)$

$$\vec{F} = -\vec{R} = (-R_x, -R_y)$$

za odprte tokove = \emptyset

$$R_x = Q \cdot \rho (v_1 \cos \alpha_1 - v_2 \cos \alpha_2) + \rho_1 A_1 \cos \alpha_1 - \rho_2 A_2 \cos \alpha_2$$

$$R_y = Q \cdot \rho (v_1 \sin \alpha_1 - v_2 \sin \alpha_2) + \rho_1 A_1 \sin \alpha_1 - \rho_2 A_2 \sin \alpha_2$$

KONTINUITETNA ENAČBA

ZAKON O OHRANITVI MASE

$$Q = \text{koust.} : v_1 A_1 = v_2 A_2$$

$$\dot{m} = \rho \cdot Q$$

IMPULZNI STAVEK

ZAKON O OHRANITVI GIBALNE KOLIČINE

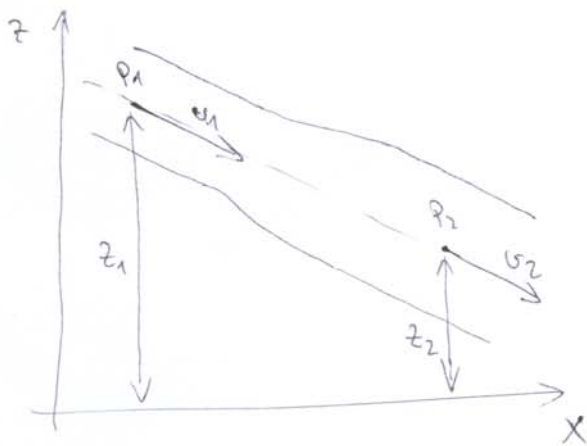
$$\vec{G} = \int \rho \cdot \vec{v} dV$$

$$\vec{R} = \int_V \frac{d(\rho \cdot \vec{v})}{dt} dV + \int_A p \cdot \vec{n} dA + \int_A \rho \cdot \vec{v} (\vec{v} \cdot \vec{n}) dA - \int_V \rho \cdot \vec{f}_m dV$$

masne sile zanemarimo

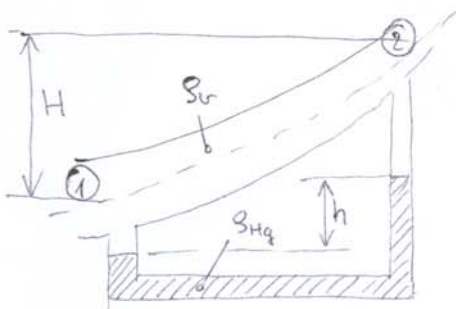
BERNOULIJEVA ENAČBA

ZAKON O OHRANITVI ENERGIJE



$$\frac{v_1^2}{2g} + \frac{p_1}{\rho \cdot g} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\rho \cdot g} + z_2$$

če je $p < 0 \rightarrow v = 0$ (NE TEČE)
POTREBNO JE CEV SKRAJŠATI



$$p_2^n = p_1^n - S_v \cdot g \cdot H - (S_{Hg} - S_v) \cdot g \cdot h$$

KINEMATIKA FLUIDOV

KONTINUITETNA ENAČBA \rightarrow NESTISLJIVOST!

$$\rho = \text{konst.} \quad \text{div}(\vec{v}) = 0 = \left(\frac{\partial v_x}{\partial x}, \frac{\partial v_y}{\partial y}, \frac{\partial v_z}{\partial z} \right)$$

$$\text{div}(\vec{v}) = \nabla \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \cdot \vec{v}) = 0$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + \frac{\partial \vec{v}}{\partial x} v_x + \frac{\partial \vec{v}}{\partial y} v_y + \frac{\partial \vec{v}}{\partial z} v_z$$

$$a_x = \frac{dv_x}{dt} = \frac{\partial v_x}{\partial x} v_x + \frac{\partial v_x}{\partial y} v_y + \frac{\partial v_x}{\partial z} v_z + \frac{\partial v_x}{\partial t}$$

$$a_y = \frac{dv_y}{dt} = \frac{\partial v_y}{\partial x} v_x + \frac{\partial v_y}{\partial y} v_y + \frac{\partial v_y}{\partial z} v_z + \frac{\partial v_y}{\partial t}$$

$$a_z = \frac{dv_z}{dt} = \frac{\partial v_z}{\partial x} v_x + \frac{\partial v_z}{\partial y} v_y + \frac{\partial v_z}{\partial z} v_z + \frac{\partial v_z}{\partial t}$$

ENAČBA TOKOVNIC:

$$\frac{dy}{v_y} = \frac{dx}{v_x} = \frac{dz}{v_z}$$

VRTINČNOST

$$\omega = \frac{1}{2} |\text{rot}(\vec{v})|, \quad \text{rot}(\vec{v}) = \vec{k} \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right)$$

če je tole ravninski, je rotor pravokoten na ravnino (ni vrtincev):

$$\text{rot}(\vec{v}) = 0$$

KOMPLEKSNJI TOKOVI

$$\vec{v} = (v_x, v_y)$$

$$\tilde{v} = v_x - i \cdot v_y \rightarrow \text{KOMPLEKSNJA HITROST}$$

$$\tilde{f} = \varphi + i \cdot \psi \rightarrow \text{KOMPLEKSNJI POTENCIAL}$$

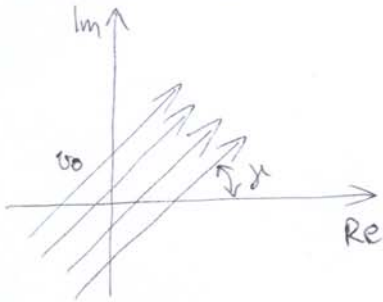
$$z = x + i \cdot y = r(\cos \alpha + i \cdot \sin \alpha) = r \cdot e^{i \cdot \alpha}$$

$$\tilde{v} = \tilde{v}_1 + \tilde{v}_2 + \dots$$

$$\tilde{f} = \tilde{f}_1 + \tilde{f}_2 + \dots$$

$$\frac{1}{z - z_0} = \frac{1}{(x - x_0) + i(y - y_0)} = \frac{(x - x_0) - i(y - y_0)}{(x - x_0)^2 + (y - y_0)^2}$$

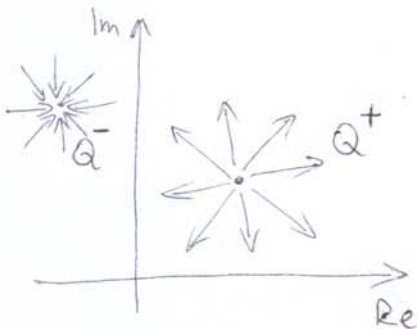
PARALELNI TOK :



$$\tilde{f} = v_0 \cdot z \cdot e^{-i \cdot \alpha}$$

$$\tilde{v} = v_0 \cdot e^{-i \cdot \alpha} = v_0 (\cos \alpha - i \cdot \sin \alpha)$$

IZVOR ALI PONOR :



⊕ IZVOR

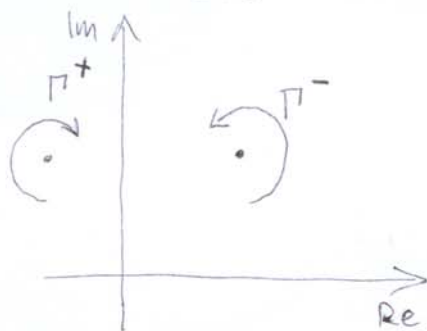
⊖ PONOR

$$Q > 0$$

$$\tilde{f} = \pm \frac{Q}{2\pi} \cdot \ln(z - z_0)$$

$$\tilde{v} = \pm \frac{Q}{2\pi} \cdot \frac{1}{z - z_0}$$

CIRKULACIJA :



$$\Gamma > 0$$

$$\tilde{f} = \pm \frac{i \cdot \Gamma}{2\pi} \ln(z - z_0)$$

$$\tilde{v} = \pm \frac{i \cdot \Gamma}{2\pi} \cdot \frac{1}{z - z_0}$$

DOLOČITEV OBLIKE TELESA :

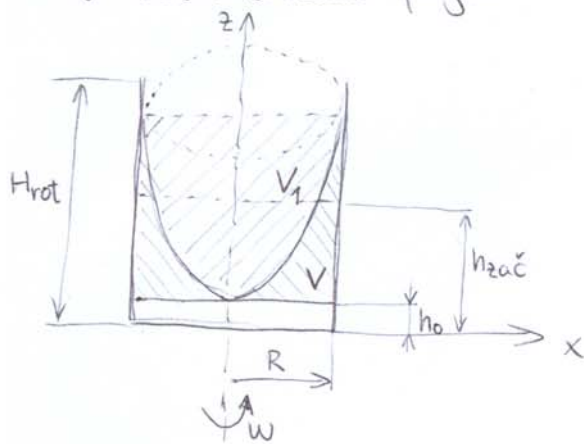
- zastopna točka ($\tilde{v} = 0, v_x = 0, v_y = 0$)
- kompleksni potencial \tilde{f}
- rešitev je Im del $\psi = \text{konst.}$
- robni pogoj z.t. ($z = \dots \rightarrow r, \alpha$)
- ψ z r in $\alpha = \text{konst.}$
(REŠITEV = konst.)

TLAK IN HITROST V NESKONČNOSTI :

$$\frac{\rho \cdot v_A^2}{2} + p_A = \frac{\rho \cdot v_\infty^2}{2} + p_\infty$$

ROTACIJA POSOD

nestisljiva kapljevina pri enakomerni rotaciji: okrog vertikalne osi v težnostnem polju:



$$a_n = -\omega^2 R$$

$$V = V_1$$

$$\omega = 2\pi n$$

$$z_1 = \frac{\omega^2}{2g} r_1^2$$

$$z_2 = \frac{\omega^2}{2g} r_2^2$$

ENAČBA GLADINE:

$$\frac{\omega^2(x^2 + y^2)}{2} + g(h_0 - z) = 0, \quad x^2 + y^2 = R^2$$

$$\frac{\omega^2}{2} R^2 + g(h_0 - z) = 0$$

- volumen kapljevine pri mirovanju $V = \pi R^2 \cdot h_{zac}$
 - volumen rotacijskega paraboloida $V_1 = \frac{\pi}{2} R^2 (H_{rot} - h_0)$
 - volumen kapljevine pri enako. rotaciji: $V = \pi R^2 \cdot H_{rot} - V_1$
- $$H_{rot} - h_{zac} = \frac{1}{2} (H_{rot} - h_0)$$

ZAPRTA POSODA

$$r_p^2 = \frac{2gH}{\omega^2} \rightarrow \Delta \text{ velja, ko je rot. posoda zaprta in bi, če bi bila odprta, tekočina stekla ven}$$

r_p = polmer rot. paraboloida

$$R_2 = R \sqrt{\frac{2(H-h)}{H}} \rightarrow \text{radij kroga, ki ga voda ne pokriva na pokrovu posode}$$

$$\omega_0 = \frac{1}{R_2} \sqrt{2gH} = \frac{H}{R} \sqrt{\frac{g}{H-h}} \rightarrow \text{pri tej } \omega \text{ se prikaže dna pri rotaciji.}$$

ODPRTA POSODA

$$\omega_{pre} = \frac{2}{R} \sqrt{g(H-h)} \rightarrow \omega \text{ pri kateri začne tekočina iztekati ven}$$

$$\omega_{dna} = \frac{2}{3} \sqrt{gH} \rightarrow \omega \text{ pri kateri se gladina dotakne dna}$$

PORAZDELITEV TLAKA V TEKOČINI

$$P - P_0 = \frac{\rho}{2} (\omega^2 r^2 - 2g \cdot z)$$

ČRPAVKĚ

$$\frac{v_0^2}{2g} + \frac{P_0^{(n)}}{\rho g} + z_0 = \frac{v_1^2}{2g} + \frac{P_1^{(n)}}{\rho g} + z_1 + h_{itzg} - H\check{c}$$

$$H\bar{c} = H + h_{itzg} = \frac{\Delta P}{\rho g}$$

= ∅ (c̄ črpalke ni na poti)

$$h_{itzg} = \frac{v^2}{2g} (\xi_{lku} + \xi_{lin})$$

↓
KOLĚNA!
x · ξ_{lk} + y · ξ_{st}...

↓
Količnik lin. izgub
ξ_{lin} = λ · $\frac{l}{d}$

$$\xi = k, \text{ KSP, str. 143}$$

$$Re = \frac{v \cdot d_n}{\nu}, \quad d_n = \frac{4A}{O}, \quad v = \frac{Q}{A}$$

$$\nu = 14,9 \cdot 10^{-6} \text{ m}^2/\text{s} \quad \text{ZRAK}$$

$$\nu = 1,5 \cdot 10^{-6} \text{ m}^2/\text{s} \quad \text{VODA}$$

$$Re < 2320 \rightarrow \text{LAMINAREN}$$

$$Re < 2320 \rightarrow \text{TURBOLENTEN}$$

TURBOLENTEN TOK:

$$\lambda = \frac{0,309}{\left[\log(0,143 \cdot Re) \right]^2} \quad (\xi = \emptyset)$$

$$\lambda = \frac{0,25}{\left[\log\left(\frac{15}{Re} + 0,269 \frac{\xi}{d} \right) \right]^2} \quad (\xi \neq \emptyset)$$

MOČ ČRPAVKĚ:

$$P_{\check{c}} = \frac{\rho \cdot v \cdot \pi d^2 H\bar{c} \cdot g}{4 \cdot \eta_{\check{c}}} = \Delta P_{\check{c}} \frac{Q}{\eta_{\check{c}}} = \frac{Q \cdot g \cdot H\bar{c}}{\eta}$$