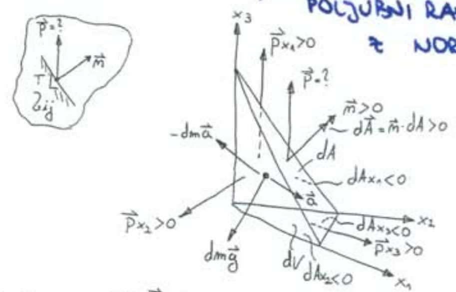
1 VEKTOR NAPETOSTI NA POLJUBNI RAVNINI DEF. Z NORMALO $\vec{n}$



$dV=dx_1\,dx_2\,dx_3$
$dA_{x_1}=dx_2\cdot dx_3$
$\vdots$

$\sum d\vec{F}_i=0 \rightarrow \underline{d\vec{F}_A=\vec{f}_A\,dA}$

$$\vec{p}\,dA=\sum_{i=1}^{3}\vec{p}_{x_i}\cdot dA_{x_i}+\underbrace{\vec{g}\,\overset{\rho dV}{dm}-\vec{a}\,\rho\,dV}_{\text{zanemarimo}}=0$$

$$\boxed{\vec{p}=\sum_{i=1}^{3}\vec{p}_{x_i}\cdot\frac{dA_{x_i}}{dA}}$$

$d\vec{A}=\vec{m}\,dA \qquad \vec{m}=(m_{x_1},m_{x_2},m_{x_3})$

$$\vec{m}=\frac{d\vec{A}}{dA}=\left(\frac{dA_{x_1}}{dA},\frac{dA_{x_2}}{dA},\frac{dA_{x_3}}{dA}\right)=\frac{dA_{x_i}}{dA}$$

$$\vec{p}=\vec{p}_{x_1}\cdot\frac{dA_{x_1}}{dA}+\vec{p}_{x_2}\cdot\frac{dA_{x_2}}{dA}+\frac{dA_{x_3}}{dA}\cdot\vec{p}_{x_3}$$

$$\vec{p}=\vec{p}_{x_1}\cdot m_{x_1}+\vec{p}_{x_2}\cdot m_{x_2}+\vec{p}_{x_3}\cdot m_{x_3}$$

$$\vec{p}=\vec{p}_x m_x+\vec{p}_y m_y+\vec{p}_z m_z$$

$$\vec{p}=(\sigma_x,\tau_{xy},\tau_{xz})m_x+(\tau_{yx},\sigma_y,\tau_{yz})m_y+(\tau_{zx},\tau_{zy},\sigma_z)m_z$$

$$\vec{p}=\begin{pmatrix}\sigma_x & \tau_{yx} & \tau_{zx}\\ \tau_{xy} & \sigma_y & \tau_{zy}\\ \tau_{xz} & \tau_{yz} & \sigma_z\end{pmatrix}\begin{pmatrix}m_x\\ m_y\\ m_z\end{pmatrix}=\begin{pmatrix}p_x\\ p_y\\ p_z\end{pmatrix}$$

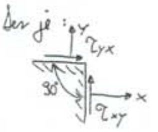
matematični zapis

$\vec{p}=[\sigma]^T\vec{m}$

tenzorski zapis

$p_i=\sigma_{ij}^T\cdot m_j$

Ker je:



$\tau_{xy}=\tau_{yx}$

velja simetrija: $\sigma_{ij}^T=\sigma_{ij}$

dobim: $\underline{\vec{p}=[\sigma]\vec{m}}$ ali $\underline{p_i=\sigma_{ij}\cdot m_j}$

2 SPEC. SPREMEMBA VOLUMNA IN GOSTOTE

$V=V(p,t)$; $m=\rho\cdot V=$ konst.

$$dV=\left(\frac{\partial V}{\partial T}\right)_p dT+\left(\frac{\partial V}{\partial p}\right)_T dp \;/ :V$$

$$\frac{dV}{V}=\underbrace{\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p}_{\beta}dT+\underbrace{\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T}_{-\lambda}dp$$

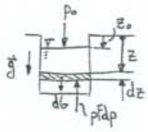
$$\underline{\frac{dV}{V}=\beta\,dT-\lambda\,dp} \qquad E=\frac{1}{\lambda}$$

λ... koef. stisljivosti
E.... modul stisljivosti
β.... koef. specif. volumske temp. dilatacije

$m=\rho\cdot V$

$$dm=0=d\rho\cdot V+\rho\,dV \;/\cdot\frac{1}{\rho V}$$

$$\frac{d\rho}{\rho}+\frac{dV}{V}=0\longrightarrow \underline{-\frac{dV}{V}=\frac{d\rho}{\rho}=-\beta\,dT+\lambda\,dp}$$



$$\sum d\vec{F}_g=0:\; Ap-(p+dp)A+dG=0$$

$$dp\,A=dG=g\cdot\rho\cdot dz\cdot A$$

$T=$ konst $\rightarrow dT=0$

$$\frac{d\rho}{\rho}=\lambda\,dp=\lambda\,\rho\,g\,dz$$

$$\int_{\rho_0}^{\rho}\frac{d\rho}{\rho^2}=\lambda g\int_{z_0}^{z}dz$$

$$-\frac{1}{\rho}\Big|_{\rho_0}^{\rho}=\lambda g(z-z_0)\;/\cdot(-1)$$

$$\frac{1}{\rho}-\frac{1}{\rho_0}=\lambda g(z_0-z)$$

$$\underline{\rho(z)=\frac{\rho_0}{1+\lambda g\cdot\rho_0(z_0-z)}}$$

$$dp=\rho\cdot g\cdot dz$$

$$dp=g\cdot\frac{\rho_0\cdot dz}{1+\lambda g\cdot\rho_0\cdot(z_0-z)}$$

$t=t(z)$
$t_0=1$

$$\int_{p_0}^{p}dp=p-p_0=\rho_0\cdot g\int_{z_0}^{z}\frac{dz}{\underbrace{1+\lambda\rho_0 g(z_0-z)}_{t}}=$$

$$=-\frac{1}{\lambda}\int_{1}^{t(z)}\frac{dt}{t}=\underline{-\frac{1}{\lambda}\ln t(z)}$$

$p\cdot v=R\cdot T$
$T=T_0=$ konst
$v=\frac{1}{\rho}$
$\frac{p}{\rho}=R\cdot T_0=k$
$p=k\cdot\rho$

$dp=k\cdot d\rho$
$dT=\varnothing$

$$\frac{d\rho}{\rho}=-\beta\,dT+\lambda\,dp$$

$$\frac{dp}{p}=\frac{d\rho}{\rho}=\lambda\,dp$$

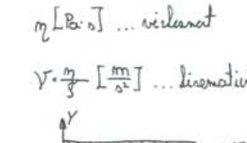
$$\boxed{\lambda=\frac{1}{p}\longrightarrow E=p}$$

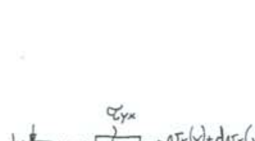
3 VISKOZNOST IN KINEMATIČNA VISKOZNOST

Viskoznost drugače pa povemo žilavost ali notranje trenje.

$\eta\,[Pa\cdot s]$ ... viskoznost

$\nu=\frac{\eta}{\rho}\,\left[\frac{m^2}{s}\right]$ ... kinematična viskoznost





$$\frac{dv_x(y)}{dy}=\frac{1}{dy}\cdot\frac{\partial v_x}{\partial y}dy$$

$$\underline{\tau_{xy}=\eta\cdot\frac{\partial v_x(y)}{\partial y}}$$

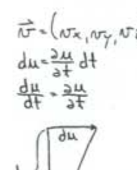
$\vec{u}=(u,v,w)=\vec{u}(\vec{r},t)$

$$\frac{d\vec{u}}{dt}=\left(\frac{du}{dt},\frac{dv}{dt},\frac{dw}{dt}\right)$$

$\vec{v}=(v_x,v_y,v_z)=\vec{v}(\vec{r},t)$

$du=\frac{\partial u}{\partial t}dt$

$\frac{du}{dt}=\frac{\partial u}{\partial t}$



$$\operatorname{tg}\gamma=\gamma=\frac{du}{dy}=\frac{\frac{\partial u}{\partial y}dy}{dy}=\frac{\partial u}{\partial y}$$

$\gamma=\gamma(\vec{r},t)$

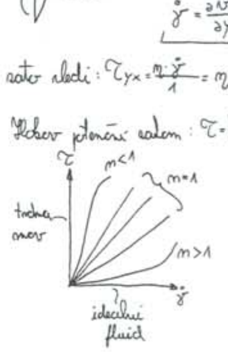
$$\frac{d\gamma}{dt}=\dot{\gamma}=\frac{d}{dt}\underbrace{\left(\frac{\partial u}{\partial y}\right)}_{v_x}=\frac{\partial v_x(y)}{\partial y}$$

$$\underline{\dot{\gamma}=\frac{\partial v_x(y)}{\partial y}}$$

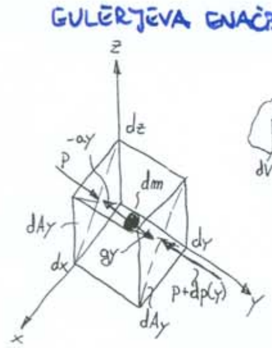
$\dot{\gamma}$... hitrost spreminjanja kota

zato sledi: $\tau_{yx}=\frac{\eta\cdot\dot{\gamma}}{1}=\eta\cdot\dot{\gamma}$

Ostwaldov potenčni zakon: $\tau=k(\eta)\cdot\dot{\gamma}^m$



4 EULERJEVA ENAČBA STATIKE FLUIDOV



$dm=\rho\cdot dV$

$$\sum dF_{y_i}=0:\; p\,dA_y-[(p+dp(y))]dA_y+g_y\,dm-a_y\,dm=0$$

$$\underline{-dp(y)\cdot dA_y+g_y\cdot\rho\cdot dV-a_y\rho\,dV=0}$$

$dV=dx\,dy\,dz \qquad dp(y)=\frac{\partial p}{\partial y}dy$ } vstavim v
$dA_y=dx\,dz \qquad dV=dy\,dA_y$ } zgornjo enačbo

$$-\frac{\partial p}{\partial y}dy\,dA_y+g_y\rho\,dy\,dA_y-a_y\rho\,dy\,dA_y=0$$

smer x: $-\frac{\partial p}{\partial x}+\rho g_x=\rho a_x \;/\cdot\vec{i}$
smer y: $-\frac{\partial p}{\partial y}+\rho g_y=\rho a_y \;/\cdot\vec{j}$ $\oplus$
smer z: $-\frac{\partial p}{\partial z}+\rho g_z=\rho a_z \;/\cdot\vec{k}$

Dobim naslednjo enačbo:

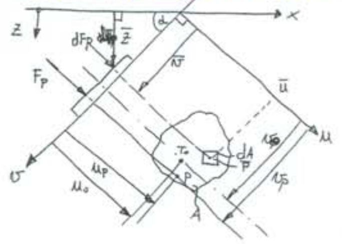
$$-\underbrace{\left(\frac{\partial p}{\partial x}\vec{i}+\frac{\partial p}{\partial y}\vec{j}+\frac{\partial p}{\partial z}\vec{k}\right)}_{\vec{\nabla}p}+\rho\cdot\underbrace{\left(g_x\vec{i}+g_y\vec{j}+g_z\vec{k}\right)}_{\vec{g}}=\rho\cdot\underbrace{(a_x\vec{i}+a_y\vec{j}+a_z\cdot\vec{k})}_{\vec{a}}$$

in končna oblika:

$$\boxed{-\vec{\nabla}p+\rho\vec{g}=\rho\vec{a}}$$

6

VELIKOST, LEGA SILE NA POŠEVNO RAVNO PLOSKEV



$$\boxed{F_p = \rho_0 g \int_A \bar z\, dA = \rho_0 g \sin\alpha \int_A \bar v\, dA}$$

$$S_u = \int_A \bar v\, dA = v_0 A \rightarrow F_p = \rho_0 g \sin\alpha\, S_u$$

$$M_u = \int_{F_p} \bar v\, dF_p = \rho_0 g \int_A \bar v \bar z\, dA = \rho_0 g \sin\alpha \int_A \bar v^2\, dA$$

$$J_u = \int_A \bar v^2\, dA = J_{uT} + v_0^2 A$$

$$\bar z = \bar v \sin\alpha$$

$$M_u = \rho_0 \cancel{g \sin\alpha}\, J_u = v_p F_p = v_p \rho_0 \cancel{g \sin\alpha}\, S_u$$

$$v_p S_u = J_u \rightarrow v_p = \frac{J_u}{S_u}$$

$$\boxed{v_p = \frac{J_{uT} + v_0^2 A}{v_0 A} = \frac{J_{uT}}{v_0 A} + v_0}$$

$$M_v = \int_{F_p} \bar u\, dF_p = \rho_0 g \sin\alpha \int_A \bar u \bar v\, dA$$

$$\int_A \bar u \bar v\, dA = J_{uv} = J_{uTvT} + u_0 v_0 A$$

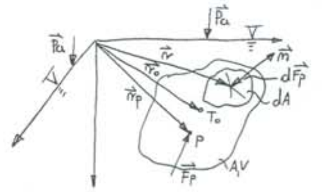
$$M_v = \rho_0 \cancel{g \sin\alpha}\, J_{uv} = u_p F_p = u_p \rho_0 \cancel{g \sin\alpha}\, S_u$$

$$u_p = \frac{J_{uv}}{S_u} = \frac{J_{uTvT} + u_0 v_0 A}{v_0 A}$$

$$\boxed{u_p = \frac{J_{uTvT}}{v_0 A} + u_0}$$

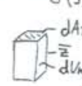
5

HIDROSTATIČNE S. NA POTOPLJENO PLOSKEV



$$d\vec F_p = -\rho_0 g z\, d\vec A = -\rho_0 g z (dA_x, dA_y, dA_z) = -\rho_0 g \int_A z\, d\vec A = -\rho_0 g \int_A z (dA_x, dA_y, dA_z)$$

$$\vec F_p = -\rho_0 g \left(\int_{A_x} z\, dA_x, \int_{A_y} z\, dA_y, \int_{A_z} z\, dA_z \right)$$



$$dV_k = \bar z\, dA_z$$

$$V_k = \int_{A_z} \bar z\, dA_z$$

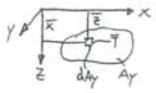
$$\vec F_p = -\rho_0 g (S_x, S_y, V_k)$$

$$\boxed{\vec F_p = -\rho_0 g (z_0 A_y, z_0 A_x, V_k)}$$

$$\vec M_p = \vec r_p \times \vec F_p = \int_{F_p} \vec r \times d\vec F_p$$

Najprej izračunam $\vec r_p \times \vec F_p$, nato pa še izpeljem enačbe za razdalje od prijemališča sile do osi:

$$x_p F_{py} = \int_{F_{py}} \bar x\, dF_{py} = -x_p(-\rho_0 g S_x) = \int_{A_y} \bar x (-\rho_0 g \bar z\, dA_y) \Rightarrow x_p S_x = \int_{A_y} \bar x \bar z\, dA_y = J_{xz}^{A_y}$$



$$x_p = \frac{J_{xz}^{A_y}}{S_x} = \frac{J_{xTzT}^{A_y} + x_0 z_0 A_y}{z_0 A_y}$$

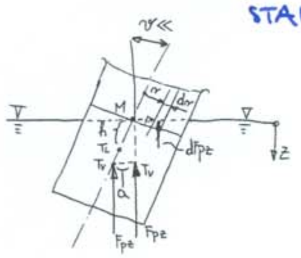
$$\boxed{x_p = \frac{J_{xTzT}^{A_y}}{z_0 A_y} + x_0}$$

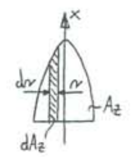
Na podoben način izračunam še za koordinati y in z, ter dobim rezultat:

$$\boxed{y_p = \frac{J_{yTzT}^{A_x}}{z_0 A_x} + y_0} \quad \text{in} \quad \boxed{z_p = \frac{J_{zT}^{A_y}}{z_0 A_y} + z_0}$$

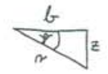
8

STABILNOST PLAVANJA





$$J_x = \int_{A_z} r^2\, dA_z$$



$$\sum M_{iM} = 0: \; -F_{pz} \cdot a + \int_{F_{pz}} b\, dF_{pz} = 0$$

$$a = m \sin\vartheta = m \tan\vartheta = m\vartheta$$

$$b = r \underbrace{\cos\vartheta}_{\approx 1} = r$$

$$z = r \sin\vartheta = r\vartheta$$

$$F_{pz} = \rho_0 g V_k$$

$$dF_{pz} = \rho_0 g z\, dA_z$$

$$\cancel{\rho_0 g} V_k m \cancel{\vartheta} = \int_{A_z} r \cancel{\rho_0 g}\, r \cancel{\vartheta}\, dA_z$$

$$m V_k = \int_{A_z} r^2\, dA_z = J_x$$

$$m = \frac{J_x}{V_k} = \overline{T_L T_V} + h$$

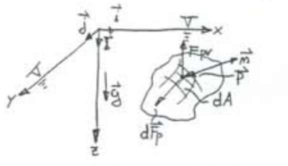
$$\boxed{h = \frac{J_x}{V_k} - \overline{T_L T_V}}$$

$h > 0$: stabilno plavanje
$h < 0$: nestabilno plavanje

M... metacenter
h... metacentrična višina
V_k... prostornina potopljenega dela telesa
J_x... vztrajnostni moment
$\overline{T_L T_V}$... razdalja med težiščem telesa L in prijemališčem sile vzgona V.

7

SILA STATIČNEGA VZGONA



$$d\vec F_p = -p\, d\vec A = -(p_a + \rho_0 g z)\, d\vec A$$

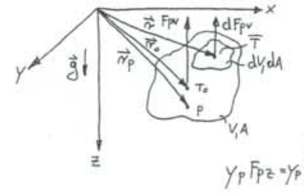
$$\vec F_p = -\int_V \vec\nabla (p_a + \rho_0 g z)\, dV$$

$$\vec\nabla(\,) = (0, 0, \rho_0 g)$$

$$\vec\nabla (p_a + \rho_0 g z) = \left(\frac{\partial(\,)}{\partial x}, \frac{\partial(\,)}{\partial y}, \frac{\partial(\,)}{\partial z} \right)$$

$$\vec F_p = -(0, 0, \rho_0 g) \int_{V_k} dV = (0, 0, -\rho_0 g V_k) = (F_{px}, F_{py}, F_{pz}) = 0\vec i + 0\vec j - \rho_0 g V_k \vec k$$

$$F_{pz} = |-\rho_0 g V_k| \qquad \boxed{F_{pv} = \rho_0 g V_k}$$



$$J_1 = \int_{A_z} \bar y (\rho_0 g) \underbrace{\bar z\, dA_z}_{dV}$$

$$y_p F_{pz} = \int_{F_{pz}} \bar y\, dF_{pz} = J_1$$

$$F_{pz} = \rho_0 g V_k$$

$$y_p F_{pz} = y_p \cancel{(\rho_0 g)} V_k = J_1 = \int_{V_k} \bar y\, dV \cancel{(\rho_0 g)}$$

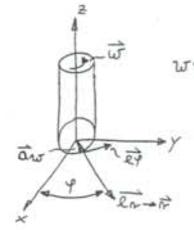
$$y_p = \frac{\int_{V_k} \bar y\, dV}{V_k} = y_0 \longrightarrow \boxed{\vec r_p = \vec r_0}$$

9_1

RELATIVNO MIROVANJE FLUIDA (TRANSLACIJA)

$-\vec{\nabla}p + \rho_0 \cdot \vec{g} = \rho_0 \cdot \vec{a}_{CEL}$

$\vec{a}_{CEL} = \vec{a}_{tr} + \vec{a}_{\omega} = \vec{a} + \vec{a}_{\omega}$



$\omega = konst = (0, 0, \omega_z)$ $\qquad \vec{a}_\omega = -\omega_z^2 \cdot \vec{r}$

$\frac{1}{\rho_0} \underbrace{\vec{\nabla}p \cdot d\vec{r}}_{dp} = \frac{1}{\rho_0} dp = \vec{g}\, d\vec{r} - \vec{a}\, d\vec{r} + \omega_z^2 \vec{r}\, d\vec{r}$

$-\frac{1}{\rho_0} \vec{\nabla}p = -\vec{g} + \vec{a} - \omega_z^2 \cdot \vec{r} \quad /\cdot(-1)$

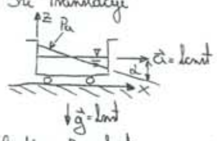
$\frac{1}{\rho_0} \vec{\nabla}p = \vec{g} - \vec{a} + \omega_z^2 \vec{r}$

$\vec{\omega} = konst.$

$\frac{1}{\rho_0} dp = (g_x dx + g_y dy + g_z dz) - (a_x dx + a_y dy + a_z dz) + \omega_z^2 (x dx + y dy + z dz)$

$$\underline{\frac{1}{\rho_0} dp = \vec{g}\vec{r} - \vec{a}\vec{r} + \frac{\omega_z^2}{2} r^2 + C}$$

a) Pri translaciji



$\downarrow \vec{g} = konst$

gladina $Pa = konst.$

$\vec{g} = (0, 0, -g)$ $\qquad dpa = \varnothing$

$\vec{a} = (a, 0, 0)$

$\frac{1}{\rho_0} dp = (\vec{g} - \vec{a}) d\vec{r} = -g\, dz - a\, dx$

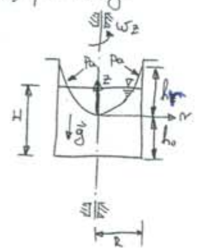
enačba gladine je torej: $\frac{1}{\rho_0} dpa = 0 = \underline{a\,dx - g\,dz = 0}$

$\frac{dz}{dx} = \tan\alpha = \frac{-a}{g} \longrightarrow z = -\frac{a}{g} \cdot x + C$

9_2

RELATIVNO MIROVANJE FLUIDA (ROTACIJA)

b) pri rotaciji



$\vec{g} = (0, 0, -g)$

$\vec{a} = 0$

$\frac{1}{\rho_0} p = -g \cdot z + \frac{\omega_z^2}{2} r^2 + C$

$\frac{1}{\rho_0} p(z=0, r=0) = \frac{1}{\rho_0} \cdot pa = C$

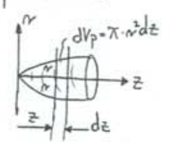
$\frac{1}{\rho_0}(p - p_0) = -gz + \frac{\omega_z^2}{2} r^2$ $\qquad$ gladina $p = p_0$

sledi enačba gladine: $\frac{\omega_z^2}{2} r^2 - gz = 0$

$z = \frac{\omega_z^2}{2g} r^2$ $\qquad h_r = \frac{\omega_z^2}{2g} R^2$

$V = konst = \pi R^2 \cdot H = \pi R^2 (h_0 + h_r) - V_p$

Gladina kapljevine je rotacijski paraboloid. Določim volumen tega paraboloida:



$V_p = \pi \int_0^{h_r} r^2 dz = \frac{2g}{\omega_z^2} \pi \int_0^{h_r} z\, dz = \frac{2 \cdot g}{\omega_z^2} \pi \frac{h_r^2}{2}$

$\frac{2g}{\omega_z^2} \cdot h_r = R^2$

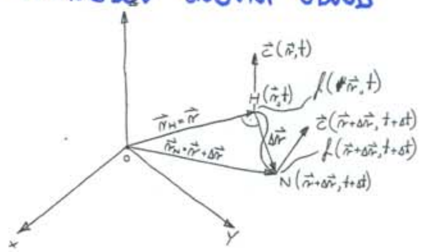
$\underline{V_p = \frac{\pi}{2} h_r \cdot R^2}$ $\qquad$ volumen rot. paraboloida

$\cancel{\pi R^2} H = \cancel{\pi R^2} (h_0 + h_r) - \frac{\cancel{\pi} h_r \cancel{R^2}}{2}$

$H = h_0 + h_r - \frac{h_r}{2} = h_0 - \frac{h_r}{2}$ $\qquad \Delta h = \frac{h_r}{2}$

10

STOKESOV SNOVNI ODVOD



$f(\vec{r}+\Delta\vec{r}, t+\Delta t) = f(\vec{r}, t) + \underbrace{\frac{\partial f}{\partial t}\Delta t + \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial z}\Delta z + R_m}_{\mathrm{I}}$

$\mathrm{I} = \Delta f = f(\vec{r}+\Delta\vec{r}, t+\Delta t) - f(\vec{r}, t)$

$\frac{Df}{Dt} = \lim_{\substack{\Delta t \to 0 \\ \Delta x \to 0 \\ \Delta y \to 0 \\ \Delta z \to 0}} \frac{\Delta f}{\Delta t} = \lim_{\Delta x \to 0} \frac{\frac{\partial f}{\partial t}\Delta t + \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial z}\Delta z}{\Delta t}$

$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} + v_z \frac{\partial f}{\partial z} = \frac{\partial f}{\partial t} + (\vec{v}\vec{\nabla}) f$

$\frac{D\vec{c}}{Dt} = \frac{\partial \vec{c}}{\partial t} + (\vec{v}\vec{\nabla})\vec{c}$

sledi lenina enačba: $\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v}\vec{\nabla})\vec{v} = \vec{a}$

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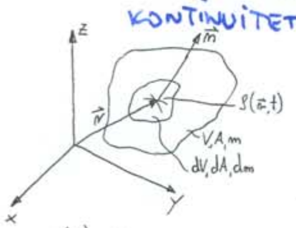
KONTINUITETNA ENAČBA



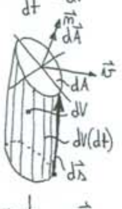
$dm = \rho \cdot dV$

$m = \int_V \rho \cdot dV \neq m(x)$

$\frac{dm}{dt} = \dot{m} = \frac{d}{dt}\int_V \rho \cdot dV(\vec{r}, t) = \int_V \frac{d}{dt}(\rho \cdot dV) = 0$

$= \int_V \left(\frac{\partial \rho}{\partial t} dV + \rho \frac{d(dV)}{dt}\right) = 0$

$\frac{d(dV)}{dt} = d\dot{V}$



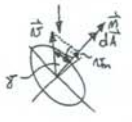
$dV = d\vec{s} \cdot d\vec{A}$

$\vec{v} = \frac{d\vec{s}}{dt}$

$d\vec{s} = \vec{v} \cdot dt$

$dV = (\vec{v} \cdot dt) \cdot d\vec{A} \quad /:dt$

$\frac{d(dV)}{dt} = \vec{v} \cdot d\vec{A} = d\dot{V}$



$\vec{v} \cdot d\vec{A} = v \cdot dA \cdot \cos\gamma = dA(v \cdot \cos\gamma) = v_m \cdot dA$

$\int_V \frac{\partial \rho}{\partial t} dV + \int_{\dot{V}} \rho \cdot d\dot{V} = 0$

$\int_V \frac{\partial \rho}{\partial t} dV + \int_A \rho \vec{v}\, d\vec{A} = 0$ $\qquad d\vec{A} = \vec{m} \cdot dA$

$m: \int_A \rho \vec{v}\, d\vec{A} = \int_A (\rho\vec{v})\vec{m}\, dA = \int_V \vec{\nabla}(\rho\vec{v})\, dV \longrightarrow \int_V \left[\frac{\partial \rho}{\partial t} + \nabla(\rho \cdot \vec{v})\right] dV = \varnothing$

$dm: \frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho\vec{v}) = \varnothing$

sledi: IZVOR ÷ PONOR = $+q_1$; $-q_2$

Kontinuitetna enačba je:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho\vec{v}) = \sum_{i=1}^{m} q_i^i - \sum_{j=1}^{m} q_j^j$$

Za tok v elementarni tokovni cevi pa se glasi: $\boxed{\rho_i \cdot v_{mi} \cdot dA_i = konst.}$

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BERNOULIJEVA ENAČBA

Bernoulijevo enačbo izpeljemo s pomočjo integracije Eulerjeve gibalne enačbe med poljubnima točkama ① in ② na tokovnici:

$$\frac{1}{\rho}dp + g\,dz + v\,dv = 0$$

$$\int_1^2 \frac{dp}{\rho} + g(z_2 - z_1) + \frac{1}{2}(v_2^2 - v_1^2) = 0$$

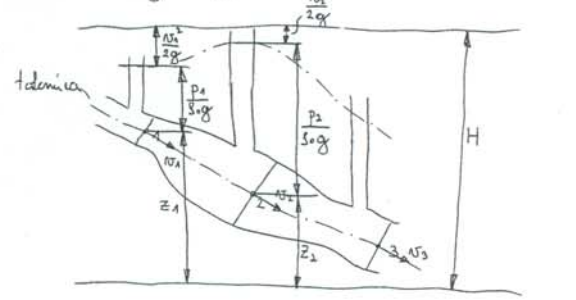
Za nestisljivo tekočino $\rho = \rho_0 = konst.$ je:

$$\frac{1}{\rho_0}(p_2 - p_1) + g(z_2 - z_1) + \frac{1}{2}(v_2^2 - v_1^2) = 0$$

To zvezo zapišemo v obliki energijskih višin za točki ① in ② na tokovnici:

$$\frac{p_1}{\rho_0 g} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\rho_0 g} + z_2 + \frac{v_2^2}{2g}$$

splošno: $\frac{p}{\rho_0 g} + z + \frac{v^2}{2g} = konst. = H$



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NAVIER-STOKESOVE ENAČBE

Te enačbe nestacionarnega gibanja newtonske viskozne in stisljive tekočine izpeljemo iz splošne gibalne enačbe ob upoštevanju zakona tečenja.

v komponentni obliki:

$$x:\ \rho\cdot\frac{Dv_x}{Dt} = \rho\cdot f_{mx} - \frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left[\eta\cdot\left(2\cdot\frac{\partial v_x}{\partial x} - \frac{2}{3}\vec{\nabla}\cdot\vec{v}\right)\right] + \frac{\partial}{\partial y}\left[\eta\cdot\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left[\eta\left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right)\right]$$

y: enačba je popolnoma ista, samo zamenjajo se indeksi: $x \to y$ in $y \to z$ in $z \to x$

z: -II- : $y \to z$ in $z \to x$ in $x \to y$

Te enačbe skupaj s kontinuitetno enačbo,

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho\cdot\vec{v}) = 0$$

energijsko enačbo

$$\rho\cdot c_p\frac{DT}{Dt} = \vec{\nabla}\cdot(\lambda\vec{\nabla}T) + I + \Phi$$

in enačbami stanja

$\rho = \rho(p,T)$; $c_p = c_p(p,T)$; $\eta = \eta(p,T)$; $\lambda = \lambda(p,T)$;

I..... viri toplote
λ.... toplotna prevodnost
Φ..... Rayleighova trošna funkcija

sestavljajo zaključen nelinearen sistem enačb z neznankami $(v_x, v_y, v_z, p, T, \rho, \eta, c_p, \lambda)$.
Za konstantno viskoznost $\eta = \eta_0$ se zapiše enačbe nekoliko poenostavijo. Zapišemo jih lahko v vektorski obliki:

$$\rho\frac{D\vec{v}}{Dt} = \rho\vec{f}_m - \vec{\nabla}p + \eta_0\Delta\vec{v} + \frac{1}{3}\eta_0\vec{\nabla}(\vec{\nabla}\cdot\vec{v})$$

Če upoštevamo še, da je tekočina nestisljiva $\rho = \rho_0$ se ta enačba ob upoštevanju $div\,\vec{v} = 0$ reducira v odvisnost:

vektorski zapis - $\frac{D\vec{v}}{Dt} = \vec{f}_m - \frac{1}{\rho_0}\vec{\nabla}p + \nu_0\Delta\vec{v}$

tenzorski zapis - $\frac{Dv_i}{Dt} = f_{mi} - \frac{1}{\rho_0}\frac{\partial p}{\partial x_i} + \nu_0\frac{\partial^2 v_i}{\partial x_j \partial x_j}$

ν_0 - kinematična viskoznost

rešitev je:

$$\frac{\partial v_x}{\partial t} + (\vec{v}\cdot\vec{\nabla})v_x = f_{mx} - \frac{1}{\rho_0}\frac{\partial p}{\partial x} + \nu_0\Delta v_x$$
$$\frac{\partial v_y}{\partial t} + (\vec{v}\cdot\vec{\nabla})v_y = f_{my} - \frac{1}{\rho_0}\frac{\partial p}{\partial y} + \nu_0\Delta v_y$$
$$\frac{\partial v_z}{\partial t} + (\vec{v}\cdot\vec{\nabla})v_z = f_{mz} - \frac{1}{\rho_0}\frac{\partial p}{\partial z} + \nu_0\Delta v_z$$

$$(\vec{v}\cdot\vec{\nabla}) = v_x\frac{\partial}{\partial x} + v_y\frac{\partial}{\partial y} + v_z\frac{\partial}{\partial z}$$
$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

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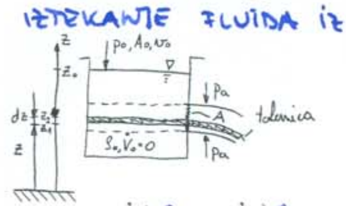
IZTEKANJE FLUIDA IZ STRANSKE VELIKE ODPRTINE



$$\frac{v^2}{2} + \frac{p}{\rho_0} + gz = \frac{v_0^2}{2} + \frac{p_0}{\rho_0} + g\cdot z_0 \qquad p = p_0$$

$$v = \sqrt{v_0^2 + 2g(z_0 - z)}$$

$d\dot{Q} = \rho_0 v\,dA = \rho_0 v\,b(z)\,dz$ $\qquad \dot{Q} = \rho_0\cdot\dot{V} = \rho_0\cdot v\cdot A$ $\qquad dA = b(z)\cdot dz$

$$\dot{Q} = \rho_0\int_{z_1}^{z_2} v(z)\,b(z)\,dz$$

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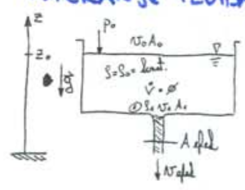
IZTEKANJE FLUIDA IZ DNA POSODE



$\dot{V}_{dej} = A_{ef}\cdot v_{ef}$
$A_{ef} = A_1\cdot\Psi$
$\varphi \approx 0{,}96$
$v_{ef} = \varphi\cdot v_1$

φ... koeficient notranjega trenja

Ψ... koeficient oblike iztočne rebe

Poznamo več vrst tega koeficienta:

$\Psi = 0{,}67$ $\qquad \Psi = 0{,}5$ $\qquad \Psi = 0{,}85 - 0{,}90$

Najprej poiščem prehod med v_0 in v_1:

$$\dot{V} = v_1\cdot A_{ef} = v_0\cdot A_0$$
$$v_0 = v_1\cdot\frac{A_{ef}}{A_0} = v_1\cdot\frac{\Psi\cdot A_1}{A_0}$$

Uporabim še ~~kontinuitetno~~ Bernoullijevo enačbo:

$$\frac{v^2}{2} + \frac{p}{\rho_0} + g\cdot z = C$$

$$\frac{v_0^2}{2} + \frac{p_0}{\rho_0} + g\cdot z_0 = \frac{v_1^2}{2} + \frac{p_1}{\rho_0} + g\cdot z_1$$

$$\frac{v_0^2}{2} - \frac{v_1^2}{2} = \frac{p_0 - p_1}{\rho_0} + g(z_0 - z_1)$$

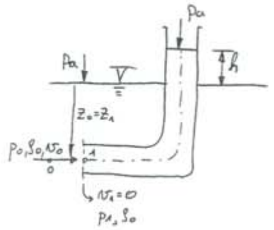
$$\frac{1}{2}(v_0^2 - v_1^2) = \frac{1}{2}v_1^2\left[1 - \left(\frac{\Psi\cdot A_1}{A_0}\right)^2\right] \longrightarrow v_1 = \sqrt{\frac{2}{1 - \left(\frac{\Psi\cdot A_1}{A_0}\right)^2}\cdot\left[\frac{p_0 - p_1}{\rho_0} + g(z_0 - z_1)\right]}$$

$$\dot{V}_{dej} = v_{ef}\cdot A_{ef} = \varphi\cdot v_1\cdot\Psi\cdot A_1 = \varphi\cdot\Psi\cdot v_1\cdot A_1 = \mu\cdot v_1\cdot A_1$$

μ... iztočno število

MERJENJE HITROSTI FLUIDOV

Najpogostejša naprava za merjenje zastojnega tlaka in s tem hitrosti kapljevin je PITOTOVA CEV.



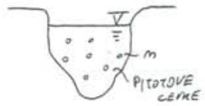
$\rho = \rho_0 = konst.$

Uporabim Bernoullijevo načelo:

$$\frac{v_0^2}{2} + \frac{p_0}{\rho_0} + g z_0 = \frac{v_1^2}{2} + \frac{p_1}{\rho_0} + g(z_0 + h)$$

$$v_0 = \sqrt{\frac{2}{\rho_0}(p_1 - p_0) + 2 \cdot g \cdot h} \qquad / \ p_1 = p_0$$

$$\boxed{v_0 = \sqrt{2 \cdot g \cdot h}}$$



$$\bar{v} = \frac{\sum v_i}{m}$$

$$\dot{V} = v \cdot A$$

Hitrost v opazovani točki 0 je sorazmerna razliki gladin h v Pitotovi cevi. Pri visokih tlakih, oz. ko je delovni medij plin, uporabimo za merjenje hitrosti diferenčni U manometer.