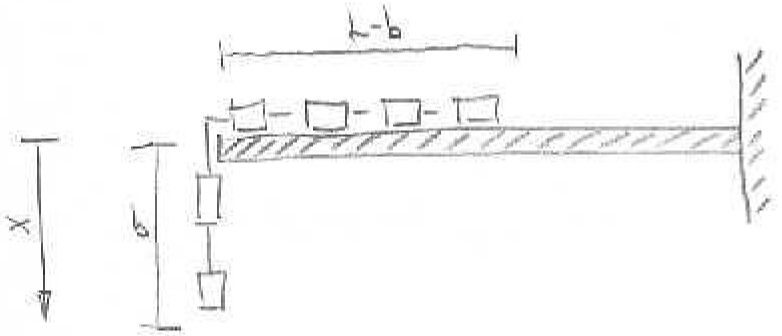
Domača naloga



$L = 4m$
$\rho = 5\ kg/m$
$\mu_{stat} = 0{,}2$
$\mu_{din} = 0{,}1$

A.) Določi $b_{zač}$, da bo veriga drsela.

$$\boxed{F = ma}$$

$$F = ma = \rho (L - b_{zač})\, a(t)$$

$$\cancel{\rho}\, b_{zač}\, \cancel{g} = \cancel{\rho} (L - b_{zač})\, \cancel{g}\, \mu_{stat}$$

$$b_{zač} - L\mu_s + b_{zač}\mu_s = 0$$

$$b_{zač}(1 + \mu_s) = L\mu_s$$

$$b_{zač} = \frac{L\mu_s}{1+\mu_s} = \frac{8}{12} = \underline{\underline{\frac{2}{3}\ m}}$$

B) Določi v kolikšnem času tk bo veriga od upoštevanja
trenja padla s police; $b = 3m$

- $F_{ek} = F - F_{tr}$

$$\boxed{F_{ek} = ma}$$
$$\boxed{F - F_{tr} = ma}$$

- $F = \rho g\, x(t)$
- $F_{tr} = \rho g\, \mu_{din} (L - x(t))$
- $ma = \rho L\, a(t)$

t_0 $$\rho g\, x(t) - \rho g\, \mu_{din} (L - x(t)) = \rho L\, a(t)$$

Zp:

$t=\emptyset$	$t=t_k$	
$x(t=\emptyset)=b$	$x(t_k)=L$	$b\to\emptyset\ ;\ t_k\to\infty$
$v(t=\emptyset)=\emptyset$	$v(t_k)>0$	$b\to L\ ;\ t_k\to\emptyset$
$a(t=\emptyset)>0$	$a(t_k)=g$	

$$x(t)\cancel{\rho}g-(L-x(t))\cancel{\rho}g\,\mu_{din}=\cancel{\rho}L\,a(t)$$

$$x(t)g-Lg\mu d+x(t)g\mu d=La(t)$$

$$\boxed{x(t)(g+g\mu d)=La(t)+Lg\mu d}$$

$$x(t=0)=b\ ,\ a(t=0)>0$$

$$\to\ b(g+g\mu d)=La(t)+Lg\mu d$$

$$\boxed{a(t)=\frac{b(g+g\mu d)-Lg\mu d}{L}}$$

$$a(t)=\frac{dv}{dt}\frac{dx}{dx}=\frac{dv}{dx}v$$

$$\cancel{(g+g\mu d)\int_b^{x(t)}x(t)=L\int a(t)+\int Lg\mu d}$$

$$x(t)(g+g\mu d)=L\frac{dv}{dx}v+Lg\mu d\quad /\cdot dx$$

$$\int_b^{x(t)}x(t)(g+g\mu d)dx-\int_b^{x(t)}Lg\mu d\,dx=\int_0^{v(t)}L\,dv\,v$$

$$(g+g\mu d)(x(t)^2-b^2)\frac{1}{2}-Lg\mu d(x(t)-b)=L\frac{1}{2}v(t)^2\quad /\cdot\frac{2}{L}$$

$$(g+g\mu d)(x(t)^2-b^2)\frac{1}{L}-2g\mu d(x(t)-b)=v(t)^2$$

$$v(x)=\sqrt{(g+g\mu d)(x^2-b^2)\frac{1}{L}-2g\mu d(x-b)}$$

$$\frac{dx}{dt}=\sqrt{\frac{g}{L}(1+\mu d)(x^2-b^2)-2g\mu d(x-b)}$$

$$\int_b^L \frac{dx}{\sqrt{\frac{g}{L}(1+\mu d)(x^2-b^2) - 2g\mu d(x-b)}} = \int_0^{tk} dt$$

$$A = \frac{g}{L}(1+\mu d) \quad ; \quad B = 2g\mu d \quad ;$$

$$\int_b^L \frac{dx}{\sqrt{A(x^2-b^2) - B(x-b)}} = tk$$

Priročnik:

$$\mathcal{X} = ax^2 + bx + c$$

$$\int \frac{dx}{\sqrt{\mathcal{X}}} = \frac{1}{\sqrt{a}} \ln\left(2\sqrt{a\mathcal{X}} + 2ax + b\right) + C \qquad \text{za } a > 0$$

$$tk = \int_b^L \frac{dx}{\sqrt{Ax^2 - Bx - Ab^2 + Bb}} =$$

$$= \left[\frac{1}{\sqrt{A}} \ln\left(2\sqrt{A(Ax^2 - Bx - Ab^2 + Bb)} + 2Ax - B\right)\right]\Bigg|_b^L$$

$$= \frac{1}{\sqrt{A}} \ln\left[\frac{2\sqrt{A(AL^2 - BL - Ab^2 + Bb)} + 2AL - B}{2\sqrt{A(\cancel{Ab^2} - \cancel{Bb} - \cancel{Ab^2} + \cancel{Bb})} + 2Ab - B}\right]$$

$$A = 2{,}697 \; ; \; B = 1{,}962 \; ; \; g = 9{,}81 \; ; \; b = 3$$

$$tk = \frac{1}{\sqrt{2{,}697}} \ln \frac{2\sqrt{2{,}697(2{,}697 \cdot 4^2 - 1{,}962 \cdot 4 - 2{,}697 \cdot 3^2 + 1{,}962 \cdot 3)} + 2 \cdot 2{,}697 \cdot 4 - 1{,}962}{2 \cdot 2{,}697 \cdot 3 - 1{,}962}$$

$$\boxed{tk = 0{,}51\,s}$$