Domača naloga

$p_z = ax^2 + bx + c$

1. $p_z(x=0) = 0$
2. $p_z(x=L) = p_0$
3. $p_z(x=2L) = 0$

1. $0 = 0 \cdot 0 + b \cdot 0 + c$
   $\Rightarrow c = 0$
2. $p_0 = a L^2 + bL + 0$
   $p_0 = aL^2 + bL$
3. $0 = a\,4L^2 + 2Lb + 0$
   $-a\,4L^2 = 2Lb$
   $b = \frac{-a\,4L^2}{2L}$
   $b = -2aL$

$c = 0$
$b = -2aL$
$a = \frac{-p_0}{L^2}$

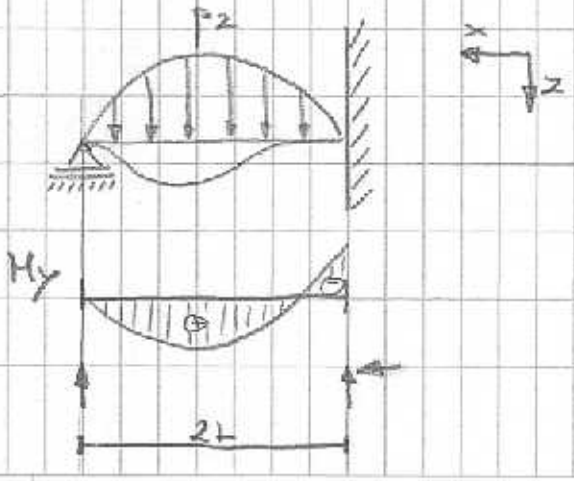
$\rightarrow$ $b = \frac{2p_0}{L}$

2. $p_0 = aL^2 + (-2aL)L$
   $p_0 = aL^2 - 2aL^2$
   $p_0 = aL^2(1-2) = -aL^2$
   $a = \frac{-p_0}{L^2}$

$p_z = ax^2 + bx + c$

$p_z = -\frac{p_0}{L^2}x^2 + \frac{2p_0}{L}x + 0$

enačba obremenitve



$$w_2''(x) = -\frac{1}{EI_y} M_y(x) \quad / d/dx \quad ; \quad EI_y = \text{konst}$$

$$w_2'''(x) = -\frac{1}{EI_y} \frac{dM_y(x)}{dx} = -\frac{1}{EI_y} T_z(x) \quad / d/dx$$

$$w_2^{IV}(x) = +\frac{1}{EI_y} p_z(x) \quad ; \quad p_z = a x^2 + b x \;,\; p_z = -\frac{p_0}{L^2} x^2 - 2aL x$$

to to bar uzel do bo krajše

$$w_2''(x) = \frac{1}{EI_y} (a x^2 + b x) \quad ; \quad a = -\frac{p_0}{L^2} \;,\; b = -2aL$$

$$w_2'''(x) = \frac{1}{EI_y} \left(a \frac{x^3}{3} + b \frac{x^2}{2} \right) + C_1$$

$$w_2''(x) = \frac{1}{EI_y} \left(a \frac{x^4}{3 \cdot 4} + b \frac{x^3}{2 \cdot 3} \right) + C_1 x + C_2$$

$$w_2'(x) = \frac{1}{EI_y} \left(a \frac{x^5}{3 \cdot 4 \cdot 5} + b \frac{x^4}{2 \cdot 3 \cdot 4} \right) + C_1 \frac{x^2}{2} + C_2 x + C_3$$

$$w_2(x) = \frac{1}{EI_y} \left(a \cdot \frac{x^6}{3 \cdot 4 \cdot 5 \cdot 6} + b \frac{x^5}{2 \cdot 3 \cdot 4 \cdot 5} \right) + C_1 \frac{x^3}{6} + C_2 \frac{x^2}{2} + C_3 x + C_4$$

Robni pogoji:

1. $w_2(x = 0) = 0$
2. $w_2(x = 2L) = 0$
3. $w_2'(x = 0) = 0$
4. $M(x = 2L) = 0$; $w_2''(x = 2L) = 0$

1. $w_2(x = 0) = 0$

$0 = C_4$

$C_4 = 0$

3. $w_2'(x = 0) = 0$

$0 = C_3$

$C_3 = 0$

2. $w_2(x=2L) = 0$

$$0 = \frac{1}{EI_y}\left(-\frac{(2L)^6}{360}\cdot\frac{p_0}{L^2} + \frac{(2L)^5}{120}\cdot\frac{2p_0}{L}\right) + C_1\frac{(2L)^3}{6} + C_2\frac{(2L)^2}{2} =$$

$$= \frac{1}{EI_y}\left(-\frac{64L^4p_0}{360} + \frac{32L^4p_0}{60}\right) + C_1\frac{8}{6}L^3 + C_2\frac{4}{2}L^2 = 0$$

$$= \frac{1}{EI_y}\left(\frac{128}{360}L^4p_0\right) + \frac{4}{2}L^2C_1 + 2L^2C_2 = 0 \quad /:2L^2$$

$$= \frac{1}{EI_y}\frac{8}{45}L^2p_0 + \frac{2}{2}LC_1 + C_2 = 0$$

Sledi: $\boxed{C_2 = -\frac{1}{EI_y}\frac{8}{45}L^2p_0 - \frac{2}{2}LC_1}$

4. $w_2'(x=2L) = 0$

$$0 = \frac{1}{EI_y}\left(-\frac{p_0}{L^2}\cdot\frac{32L^5}{60} + \frac{2p_0}{L}\cdot\frac{16L^4}{24}\right) + C_1 2L + C_2 =$$

$$= \frac{1}{EI_y}\left(\right.$$

$$0 = \frac{1}{EI_y}\left(-\frac{p_0}{L^2}\frac{16L^4}{12} + \frac{2p_0}{L}\frac{8L^2}{6}\right) + C_1 2L + C_2 =$$

$$= \frac{1}{EI_y}\left(-\frac{4p_0L^2}{3} + \frac{8p_0L^2}{3}\right) + C_1 2L - \frac{1}{EI_y}\frac{8}{45}L^2p_0 - \frac{2}{3}LC_1 = 0$$

$$= \frac{4p_0L^2}{3EI_y} + C_1\left[2L - \frac{2}{3}L\right] - \frac{8p_0L^2}{45EI_y} = 0$$

$$C_1 = \frac{-52p_0L^2}{45EI_y}\cdot\frac{3}{4L} \longrightarrow \boxed{C_1 = -\frac{13p_0L}{15EI_y}}$$

Rešitev dobimo, če vstavimo konstante v prvotno enačbo:

$$w_2(x) = \frac{1}{EI_y}\left(-\frac{p_0}{L^2}\frac{x^6}{360} + \frac{2p_0}{L}\frac{x^5}{120}\right) + \frac{x^3}{6}\cdot\left(-\frac{13\,p_0 L}{13\,EI_y}\right) + \frac{x^2}{2}\cdot\left(-\frac{8\,p_0 L^2}{45\,EI_y} + \frac{2L}{3}\cdot\frac{13\,p_0 L}{15\,EI_y}\right)$$

$$w_2(x) = \frac{1}{EI_y}\left(-\frac{x^6 p_0}{360 L^2} + \frac{x^5 p_0}{60 L}\right) - \frac{13 p_0 L x^3}{90\,EI_y} + \frac{x^2}{2}\frac{18 p_0 L^2}{45\,EI_y}$$

$$w_2(x) = -\frac{1}{EI_y}\frac{x^6 p_0}{360 L^2} + \frac{x^5 p_0}{60 EI_y L} - \frac{13\,p_0 L x^3}{90\,EI_y} + \frac{p_0 L^2 x^2}{5\,EI_y}$$

$$w_2(x) = \frac{1}{5}\frac{p_0 x^2}{EI_y}\left[-\frac{x^4}{72 L^2} + \frac{x^3}{12 L} - \frac{13 L x}{18} + L^2\right]$$

... rešitev

Ali res drži; $w_2(x=2L) = 0$?

$$w_2(x=2L) = \frac{p_0 4L^2}{5 EI_y}\left[-\frac{16 L^4}{72 L^2} + \frac{8 L^3}{12 L} - \frac{26 L^2}{18} + L^2\right] =$$

$$= \frac{p_0 4L^2}{5 EI_y}\left[-\frac{16}{72}L^2 + \frac{48}{72}L^2 - \frac{104}{72}L^2 + \frac{72}{72}L^2\right] \quad (\to 0)$$

$w_2(x=2L) = 0$ ✓✓