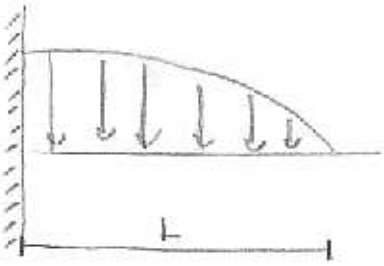
- $p(x) = p_0 \cos\left(\frac{\pi x}{2L}\right)$

$p_0 = 100\ N/m$

$L = 2.000\ mm$

$E = 210.000\ MPa$

$I_y = 2.140 \cdot 10^4\ mm^4$



I. Eksaktna rešitev

$$w_2''''(x) = \frac{1}{EI_y}\, p_2(x)$$

$$w_2'''(x) = \frac{1}{EI_y}\, p_0 \cos\left(\frac{\pi x}{2L}\right)$$

$$w_2''(x) = \frac{1}{EI_y}\, p_0\, \frac{2L}{\pi} \sin\left(\frac{\pi x}{2L}\right) + C_1$$

$$w_2'(x) = -\frac{2L\, p_0}{EI_y \pi}\, \frac{2L}{\pi} \cos\left(\frac{\pi x}{2L}\right) + C_1 x + C_2$$

$$w_2(x) = -\frac{4L^2 p_0}{EI_y \pi^2}\, \frac{2L}{\pi} \sin\left(\frac{\pi x}{2L}\right) + C_1 \frac{x^2}{2} + C_2 x + C_3$$

$$w_2(x) = \frac{8L^3 p_0}{EI_y \pi^3}\, \frac{2L}{\pi} \cos\left(\frac{\pi x}{2L}\right) + C_1 \frac{x^3}{6} + C_2 \frac{x^2}{2} + C_3 x + C_4$$

$$\Rightarrow w_2(x) = \frac{p_0}{EI_y} \left(\frac{2L}{\pi}\right)^4 \cos\left(\frac{\pi x}{2L}\right) + C_1 \frac{x^3}{6} + C_2 \frac{x^2}{2} + C_3 x + C_4$$

$\boxed{\text{RP:}}$

$w(x = \emptyset) = \emptyset$ 1.

$w'(x = \emptyset) = \emptyset$ 2.

$T(x = L) = \emptyset$ 3.

$M(x = L) = \emptyset$ 4.

1.) $w(x = \emptyset) = \emptyset$

Takoj lahko dobimo $C_4 = -\frac{p_0}{EI_y}\left(\frac{2L}{\pi}\right)^4$

2.) $w'(x = \emptyset) = \emptyset$

Dobimo $C_3 = \emptyset$

3.) $T(x = L) = \emptyset = -EI_y\, w_2'''(x = L)$

Dobimo iz:

$w_2'''(x) = -\frac{1}{EI_y}\, T_2(x) \rightarrow T_2(x) = -EI_y\, w_2'''(x)$

$$\emptyset = -EI_y \left[\frac{1}{EI_y}\, p_0\, \frac{2L}{\pi} \sin\left(\frac{\pi}{2}\right) + C_1\right]$$

$$\emptyset = -\frac{2L p_0}{\pi} - EI_y C_1$$

$$\Rightarrow C_1 = -\frac{2L p_0}{\pi EI_y}$$

4.) $M(x=L) = \emptyset = -EI_y\, w_2''(x=L)$

Dobimo iz

$$w_2''(x) = -\frac{1}{EI_y} M(x) \rightarrow M(x) = -EI_y\, w_2''(x)$$

$$\emptyset = \left[\cancel{\frac{p_0}{EI_y}\left(\frac{2L}{\pi}\right)^2 \cos\frac{\pi}{2}}^{\,\emptyset} - \frac{2L^2 p_0}{EI_y\,\pi} + C_2 \right] (\cdot EI_y)$$

$$\Rightarrow C_2 = +\frac{2L^2 p_0}{\pi EI_y}$$

Splošna rešitev:

$$w_2(x) = \frac{p_0}{EI_y}\left(\frac{2L}{\pi}\right)^4 \cos\left(\frac{\pi x}{2L}\right) - \frac{2Lp_0}{\pi EI_y}\frac{x^3}{6} + \frac{2L^2 p_0}{\pi EI_y}\frac{x^2}{2} + \emptyset - \frac{p_0}{EI_y}\left(\frac{2L}{\pi}\right)^4$$

Sedaj pomik pri $x = L$!

$$w_2(L) = \overset{\text{zaradi } \cos\frac{\pi}{2}}{\emptyset} - \frac{L^4 p_0}{3\pi EI_y} + \frac{L^4 p_0}{\pi EI_y} - \frac{16 p_0 L^4}{\pi^4 EI_y} = \frac{-\pi^3 L^4 p_0 + 3\pi^3 L^4 p_0 - 48 p_0 L^4}{3\pi^4 EI_y} = \frac{2\pi^3 L^4 p_0 - 48 L^4 p_0}{3\pi^4 EI_y}$$

$$\boxed{w_2(L) = \frac{L^4 p_0 (2\pi^3 - 48)}{3\pi^4 EI_y}}$$

$$\boxed{w_2(L) = 0{,}017 \text{ mm}}$$

Drugo metoda reševanja s Taylorjem

$p(x) = p_0 \cos\left(\frac{\pi x}{2L}\right)$; $L = 2000$ mm ; $E = 210.000$ MPa ; $I_y = 2140 \cdot 10^4$ mm^4, $p_0 = 100$ N/m

- $\cos\left(\frac{\pi x}{2L}\right) = 1 + \frac{0}{1} - \frac{1}{2!}\left(\frac{\pi x}{2L}\right)^2 + \frac{1}{4!}\left(\frac{\pi x}{2L}\right)^4 - \frac{1}{6!}\left(\frac{\pi x}{2L}\right)^6$

$$p(x) = p_0\left[1 - \frac{1}{2!}x^2\left(\frac{\pi}{2L}\right)^2 + \frac{1}{4!}x^4\left(\frac{\pi}{2L}\right)^4 - \frac{1}{6!}x^6\left(\frac{\pi}{2L}\right)^6\right]$$

$$p(L) = p_0\left[1 - \frac{1}{2}\frac{\pi^2}{4} + \frac{1}{24}\frac{\pi^4}{16} - \frac{1}{720}\frac{\pi^6}{64}\right]$$

$$p(L) = p_0\left[1 - \frac{\pi^2}{8} + \frac{\pi^4}{384} - \frac{\pi^6}{46080}\right]$$

- $EI_y\, w^{IV}(x) = p(x) = p_0 \cos\left(\frac{\pi x}{2L}\right)$

$$w^{IV}(x) = \frac{1}{EI_y}\, p_0 \cos\left(\frac{\pi x}{2L}\right)$$

$$w(x) = \sum_{k=0}^{N=6} c_k x^k = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + C_5 x^5 + C_6 x^6$$

- R.P.:

1.) $w(x=0) = 0$

Takoj sledi $\boxed{C_0 = 0}$

2.) $w'(x=0) = 0$

$$w'(x) = C_1 + 2C_2 x + 3C_3 x^2 + 4C_4 x^2 + 5C_5 x^4 + 6C_6 x^5$$

Dobimo $\boxed{C_1 = 0}$

3.) $T(x=L) = 0$; oziroma $0 = -EI_y w'''(x=L)$

$$0 = -EI_y\left[6C_3 + 24C_4 L + 60C_5 L^2 + 120C_6 L^2\right]$$

4.) $M(x=L) = 0$; oziroma $0 = -EI_y w''(x=L)$

$$0 = -EI_y\left[2C_2 + 6C_3 L + 12C_4 L^2 + 20C_5 L^3 + 30C_6 L^4\right]$$

5.)

$$w^{IV}(x = L/3) = \frac{1}{EI_y}\, p(x = L/3)$$

$$w^{IV}(x) = 24C_4 + 120C_5 x + 360C_6 x^2$$

$$\Rightarrow 24C_4 + 120C_5 \frac{L}{3} + 360C_6 \frac{L^2}{9} = \frac{1}{EI_y}\, p_0\left[1 - \frac{1}{2}\frac{L^2}{9}\frac{\pi^2}{4L^2} + \frac{1}{24}\frac{L^4}{81}\frac{\pi^4}{16L^4} - \frac{1}{720}\frac{L^6}{729}\frac{\pi^6}{64L^6}\right]$$

$$24C_4 + 40C_5 L + 40C_6 L^2 = \frac{p_0}{EI_y}\left[1 - \frac{\pi^2}{72} + \frac{\pi^4}{31104} - \frac{\pi^6}{720 \cdot 729 \cdot 64}\right]$$

6) $w^{IV}(x=\frac{2L}{3}) = \frac{1}{EI_y}\, p(x=\frac{2L}{3})$

$w^{IV}(x) = 24 C_4 + 120 C_5 x + 360 C_6 x^2$

$$\Rightarrow 24 C_4 + 120 C_5 \frac{2L}{3} + 360 C_6 \frac{4L^2}{9} = \frac{1}{EI_y} p_0 \left[1 - \frac{\pi^2}{18} + \frac{\pi^4}{1944} - \frac{\pi^6}{720 \cdot 729}\right]$$

$$24 C_4 + 80 C_5 L + 160 C_6 L^2 = \frac{p_0}{EI_y}\left[1 - \frac{\pi^2}{18} + \frac{\pi^4}{1944} - \frac{\pi^6}{720 \cdot 729}\right]$$

7.) $w^{IV}(x = L/2) = \frac{1}{EI_y}\, p(x = L/2)$

$w^{IV}(x) = 24 C_4 + 120 C_5 x + 360 C_6 x^2$

$$\Rightarrow 24 C_4 + 120 C_5 \frac{L}{2} + 360 C_6 \cdot \frac{L^2}{4} = \frac{p_0}{EI_y}\left[1 - \frac{1}{2}\frac{L^2}{4}\frac{\pi^2}{4L^2} + \frac{1}{24}\frac{L^4}{16}\frac{\pi^4}{16L^4} - \frac{1}{720}\frac{L^6}{64}\frac{\pi^6}{64L^6}\right]$$

$$24 C_4 + 60 C_5 L + 90 C_6 L^2 = \frac{p_0}{EI_y}\left[1 - \frac{\pi^2}{32} + \frac{\pi^4}{6144} - \frac{\pi^6}{720 \cdot 64 \cdot 64}\right]$$

Sistem enačb:

$$24 C_4 + 40 C_5 L + 40 C_6 L^2 = \frac{p_0}{EI_y}\left[1 - \frac{\pi^2}{72} + \frac{\pi^4}{31104} - \frac{\pi^6}{720 \cdot 729 \cdot 64}\right]$$

$$24 C_4 + 80 C_5 L + 160 C_6 L^2 = \frac{p_0}{EI_y}\left[1 - \frac{\pi^2}{18} + \frac{\pi^4}{1944} - \frac{\pi^6}{720 \cdot 729}\right]$$

$$24 C_4 + 60 C_5 L + 90 C_6 L^2 = \frac{p_0}{EI_y}\left[1 - \frac{\pi^2}{32} + \frac{\pi^4}{6144} - \frac{\pi^6}{720 \cdot 64 \cdot 64}\right]$$

* Mathematica

- Izračunam še:

$$C_3 = (-1)(4 C_4 L + 10 C_5 L^2 + 20 C_6 L^3)$$

$$C_2 = (-1)(3 C_3 L + 6 C_4 L^2 + 10 C_5 L^3 + 15 C_6 L^4)$$

* Vsi rezultati v Excelu

Dobimo:

$$w(L) = C_0 + C_1 L + C_2 L^2 + C_3 L^3 + C_4 L^4 + C_5 L^5 + C_6 L^6$$

$$\boxed{w(L) = 0{,}0164 \text{ mm}}$$

```
In[1]:= eqns = {24 * C4 + 40 * C5 * L + 40 * C6 * L * L == a,
          24 * C4 + 80 * C5 * L + 160 * C6 * L * L == b, 24 * C4 + 60 * C5 * L + 90 * C6 * L * L == d}
```

Out[1]= $\{24\,C4 + 40\,C5\,L + 40\,C6\,L^2 == a,\ 24\,C4 + 80\,C5\,L + 160\,C6\,L^2 == b,\ 24\,C4 + 60\,C5\,L + 90\,C6\,L^2 == d\}$

```
In[20]:= a = (po / (EI)) * (1 - (Pi^2) / 72 + (Pi^4) / 31104 - (Pi^6) / (720 * 729 * 64))
         b = (po / (EI)) * (1 - (Pi^2) / 18 + (Pi^4) / 1944 - (Pi^6) / (720 * 729))
         d = (po / (EI)) * (1 - (Pi^2) / 32 + (Pi^4) / 6144 - (Pi^6) / (720 * 64 * 64))
```

Out[20]= $\frac{\left(1 - \frac{\pi^2}{72} + \frac{\pi^4}{31104} - \frac{\pi^6}{33592320}\right) po}{EI}$

Out[21]= $\frac{\left(1 - \frac{\pi^2}{18} + \frac{\pi^4}{1944} - \frac{\pi^6}{526080}\right) po}{EI}$

Out[22]= $\frac{\left(1 - \frac{\pi^2}{32} + \frac{\pi^4}{6144} - \frac{\pi^6}{2949120}\right) po}{EI}$

```
In[23]:= Solve[eqns, {C4, C5, C6}]
```

Out[23]= $\left\{\left\{C4 \to -\frac{-5971968\,po - 2592\,\pi^4\,po + 19\,\pi^6\,po}{143327232\,EI},\ C5 \to \frac{7\,(-4320\,\pi^4\,po + 29\,\pi^6\,po)}{1433272320\,EI\,L},\ C6 \to -\frac{7464960\,\pi^2\,po - 237600\,\pi^4\,po + 1351\,\pi^6\,po}{21499084800\,EI\,L^2}\right\}\right\}$

In[92]:=
```
eqns = {24 * C4 + 40 * C5 * L + 40 * C6 * L * L == a,
  24 * C4 + 80 * C5 * L + 160 * C6 * L * L == b, 24 * C4 + 60 * C5 * L + 90 * C6 * L * L == d}
```

Out[92]= $\{24\,C4 + 80000\,C5 + 160000000\,C6 == 1.92707\times10^{-14},$
$24\,C4 + 160000\,C5 + 640000000\,C6 == 1.11252\times10^{-14},$
$24\,C4 + 120000\,C5 + 360000000\,C6 == 1.57344\times10^{-14}\}$

In[93]:=
```
a = (po / (P * M)) * (1 - (Pi^2) / 72 + (Pi^4) / 31104 - (Pi^6) / (720 * 729 * 64))
b = (po / (P * M)) * (1 - (Pi^2) / 18 + (Pi^4) / 1944 - (Pi^6) / (720 * 729))
d = (po / (P * M)) * (1 - (Pi^2) / 32 + (Pi^4) / 6144 - (Pi^6) / (720 * 64 * 64))
```

Out[93]= 1.92707×10^{-14}

Out[94]= 1.11252×10^{-14}

Out[95]= 1.57344×10^{-14}

In[87]:= `L = 2000`

Out[87]= 2000

In[88]:= `po = 0.1`

Out[88]= 0.1

In[89]:= `P = 210000`

Out[89]= 210000

In[90]:= `M = 21400000`

Out[90]= 21400000

In[96]:= `Solve[eqns, {C4, C5, C6}]`

Out[96]= $\{\{C4 \to 9.63525\times10^{-16},\ C5 \to -2.1351\times10^{-20},\ C6 \to -1.34114\times10^{-23}\}\}$

Ko rešimo sistem enačb dobimo naslednje konstante:

L	2000
C0	0
C1	0
C2	1,00522E-08
C3	-4,70834E-12
C4	9,63525E-16
C5	-2,1351E-20
C6	-1,34114E-23

Sledi:

Rešitev	0,016417078 mm