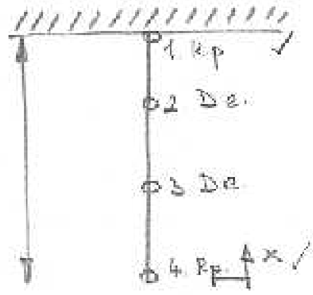
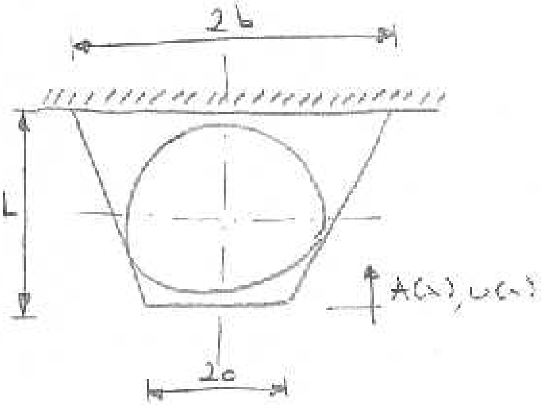
Imamo naslednji sistem:





$h = L/3$

$u_1, u_2, u_3, u_4 = ?$

$n(x) = -A(x)\,\rho_0$

$$A(x) = \pi\left[\frac{b-a}{L}x + a\right]^2$$

- Aproksimacija odvodov na osnovi centralnih razlik

$$\frac{du_0}{dx} \approx \frac{u_1 - u_{-1}}{2h}$$

$$\frac{d^2u_0}{dx^2} \approx \frac{v_1 - 2v_0 + v_{-1}}{h^2}$$

- Aproksimacija odvodov na osnovi desnih in levih razlik

$$\frac{dv_0}{dx} \approx \mp\frac{1}{2h}(3v_0 - 4v_{\pm1} + v_{\pm2})$$ +... leva shema, −... desna shema

$$\frac{d^2v_0}{dx^2} \approx \frac{1}{h^2}(2v_0 - 5v_{\pm1} + 4v_{\pm2} - v_{\pm3})$$

- Diferencialna enačba

$$\frac{d}{dx}\left[EA(x)\frac{du}{dx}\right] = -n(x)$$

$$E\left[\frac{d}{dx}A(x)\right]\frac{du}{dx} + EA(x)\frac{d^2v}{dx^2} = -n(x)$$

- Robni pogoji:
- $u(x=L) = \varnothing$
- $N(x=0) = \varnothing$

$$\frac{du}{dx}(x=0) = \varnothing$$

- Rešujemo sistem:
  - $u(x=L) = \varnothing$
    $\rightarrow u_1 = \varnothing$
  - $\frac{du}{dx}(x=0) = \varnothing$ Desna shema

$$\varnothing = \frac{-3u_4 + 4u_3 - u_2}{2(L/3)}$$

$$-3u_4 + 4u_3 - u_2 = \varnothing$$

- Diferencialna enačba za točko 2

$$E\left[\frac{d}{dx}(A(x))\right]\frac{du}{dx} + EA(x)\frac{d^2u}{dx^2} = -w(x)$$

$$E\left[\frac{d}{dx}A(x=L/3)\right]\left[\frac{u_1-u_3}{2(L/3)}\right] + EA(x=L/3)\left[\frac{u_1-2u_2+u_3}{(L/3)^2}\right] = -w(x=L/3)$$

$$\frac{dA(x)}{dx} = \pi 2\left[\frac{b-a}{L}x + a\right]\cdot\left(\frac{b-a}{L}\right) = 2\pi\left(\left(\frac{b-a}{L}\right)^2 x + \frac{b-a}{L}a\right)$$

$$\frac{d}{dx}A(x=L/3) = 2\pi\left[\frac{(b-a)^2}{L^2}\frac{L}{3} + \frac{b-a}{L}a\right] = 2\pi\left[\frac{(b-a)^2}{3L} + \frac{a(b-a)}{L}\right] = 2\pi\frac{b^2-2ab+a^2+3ab-3a^2}{3L}$$

$$\frac{d}{dx}A(x=L/3) = 2\pi\frac{b^2+ab-2a^2}{3L} = \frac{2\pi}{3L}(b+2a)(b-a)$$

$$E\frac{2\pi}{3L}(b+2a)(b-a)\frac{(u_1-u_3)3}{2L} + E\pi\left[\frac{b+2a}{3}\right]^2\frac{(u_1-2u_2+u_3)9}{L^2} = \left[\frac{b+2a}{3}\right]^2\pi\rho_0$$

$$A(x=L/3) = \pi\left[\frac{b-a}{L}\frac{L}{3} + a\right]^2 = \pi\left[\frac{b-a}{3} + \frac{3a}{3}\right]^2 = \pi\left[\frac{b+2a}{3}\right]^2$$

$$-w(x=L/3) = A(x=L/3)\rho_0 = \pi\rho_0\left[\frac{b+2a}{3}\right]^2$$

$$\frac{E\pi}{L^2}(b+2a)(b-a)(u_1-u_3) + \frac{E\pi}{L^2}(b+2a)^2(u_1-2u_2+u_3) = \frac{\pi\rho_0}{9}(b+2a)^2 \quad /\cdot\frac{L^2}{E}$$

$$(b-a)(u_1-u_3) + (b+2a)(u_1-2u_2+u_3) = \frac{\rho_0 L^2}{9E}(b+2a) \quad /(b+2a)$$

$$\frac{b-a}{b+2a}(\cancel{u_1}^{=0}-u_3) + (\cancel{u_1}^{=0}-2u_2+u_3) = \frac{\rho_0 L^2}{9E}$$

$$-\frac{b-a}{b+2a}u_3 - 2u_2 + u_3 = \frac{\rho_0 L^2}{9E}$$

$$u_3\left(1-\frac{b-a}{b+2a}\right) - 2u_2 = \frac{\rho_0 L^2}{9E} \rightarrow u_3\left(\frac{b+2a-b+a}{b+2a}\right) - 2u_2 = \frac{\rho_0 L^2}{9E}$$

$$\boxed{u_3\frac{3a}{b+2a} - 2u_2 = \frac{\rho_0 L^2}{9E}}$$

- Diferencialna enačba za točko 3.

$$E\left[\frac{d}{dx}A(x=L/3)\right]\left[\frac{u_2-u_4}{2(L/3)}\right] + EA(x=L/3)\left[\frac{u_2-2u_3+u_4}{(L/3)^2}\right] = -w(x=L/3)$$

Analogno lahko takoj dobimo

$$\frac{b-a}{b+2a}(u_2-u_4) + (u_2-2u_3+u_4) = \frac{\rho_0 L^2}{9E}$$

$$u_2\left[\frac{b-a}{b+2a} + \frac{b+2a}{b+2a}\right] + u_4\left[\frac{\cancel{b}+2a}{b+2a} - \frac{\cancel{b}+a}{b+2a}\right] - 2u_3 = \frac{\rho_0 L^2}{9E}$$

$$\boxed{u_2\left[\frac{2b+a}{b+2a}\right]+u_4\left[\frac{3a}{b+2a}\right]-2u_3=\frac{\rho_0 L^2}{9E}}$$

- Nadaljujemo:

$$u_2 = 4u_3 - 3u_4$$

$$u_3\frac{3a}{b+2a} - 8u_3 + 6u_4 = \frac{\rho_0 L^2}{9E}$$

$$u_3\left[\frac{3a}{b+2a} - 8\right] - \frac{\rho_0 L^2}{9E} = -6u_4$$

$$u_3\left(\frac{8b+13a}{b+2a}\right) + \frac{\rho_0 L^2}{9E} = 6u_4 \longrightarrow \boxed{u_4 = \tfrac{1}{6}\,u_3\left(\frac{8b+13a}{b+2a}\right) + \frac{\rho_0 L^2}{54E}}$$

$$u_2 = 4u_3 - \left[\tfrac{1}{2}\,u_3\left(\frac{8b+13a}{b+2a}\right) + \frac{\rho_0 L^2}{18E}\right]$$

$$u_2 = u_3\left[4 - \tfrac{1}{2}\,\frac{8b+13a}{b+2a}\right] - \frac{\rho_0 L^2}{18E} = u_3\left[\frac{3a}{2(b+2a)}\right] - \frac{\rho_0 L^2}{18E}$$

$$\boxed{u_2 = u_3\left[\frac{3a}{2(b+2a)}\right] - \frac{\rho_0 L^2}{18E}}$$

$$u_3\frac{3a(2b+a)}{2(b+2a)(b+2a)} - \frac{\rho_0 L^2}{18E}\frac{(2b+a)}{(b+2a)} + \tfrac{1}{6}\,u_3\frac{(8b+13a)\,3a}{(b+2a)(b+2a)} + \frac{\rho_0 L^2}{54E}\frac{3a}{(b+2a)} - 2u_3 = \frac{\rho_0 L^2}{9E}$$

$$u_3\left[\frac{3a(2b+a)}{2(b+2a)(b+2a)} + \frac{(8b+13a)\,a}{2(b+2a)(b+2a)} - \frac{4(b+2a)(b+2a)}{2(b+2a)(b+2a)}\right] = \frac{\rho_0 L^2}{9E} + \frac{\rho_0 L^2}{18E}\frac{(2b+a)}{(b+2a)} - \frac{\rho_0 L^2}{54E}\frac{3a}{b+2a}$$

$$u_3\left[\frac{6ab + \cancel{3a^2} + 8ab + \cancel{13a^2} - 4b^2 - 16ab - \cancel{16a^2}}{2(b+2a)(b+2a)}\right] = \frac{\rho_0 L^2}{18E\,(b+2a)}\left[\begin{array}{l} 2(b+2a) + (2b+a) - a \\ \to 2b + 4a + 2b + \cancel{a} - \cancel{a} \\ \to 4a + 4b \end{array}\right]$$

$$u_3\left[\frac{-4b^2 - 2ab}{2(b+2a)^2}\right] = \frac{\rho_0 L^2}{18E}\frac{4(a+b)}{(b+2a)}$$

$$u_3\left[\frac{-2b(2b+a)}{2(b+2a)^2}\right] = \frac{\rho_0 L^2}{9E}\frac{(a+b)\,2}{(b+2a)} \longrightarrow u_3 = -\frac{\rho_0 L^2 (a+b)\,\cancel{2}\cdot 2(b+2a)^{\cancel{2}}}{9E\,\cancel{(b+2a)}\cdot\cancel{2}b\,(2b+a)}$$

$$\boxed{u_3 = -\frac{2\rho_0 L^2 (a+b)(b+2a)}{9E\,b\,(2b+a)}}$$

$$u_2 = u_3\left[\frac{3a}{2(b+2a)}\right] - \frac{g_0L^2}{18E} =$$

$$u_2 = -\frac{\cancel{2}g_0L^2(a+b)\cancel{(b+2a)}\cancel{3}a}{\cancel{3}9E\,b(2b+a)\,\cancel{2}\cancel{(b+2a)}} - \frac{g_0L^2}{18E}$$

$$u_2 = -\frac{g_0L^2(a+b)a}{3E\,b(2b+a)} - \frac{g_0L^2\,b(2b+a)}{18E\,b(2b+a)} = -\frac{g_0L^2}{18E\,b(2b+a)}\left[6a^2+6ba+2b^2+ab\right]$$

$$\boxed{u_2 = -\frac{g_0L^2(6a^2+7ab+2b^2)}{18E\,b(2b+a)}}$$

$$u_4 = 1/3\,u_3\left(\frac{8b+13a}{b+2a}\right) + \frac{g_0L^2}{54E} =$$

$$u_4 = -\frac{2g_0L^2(a+b)\cancel{(b+2a)}(8b+13a)}{\cancel{6}\cdot 9E\,b(2b+a)\,3\cancel{(b+2a)}} + \frac{g_0L^2}{54E}$$

$$u_4 = -\frac{g_0L^2(a+b)(8b+13a)}{27E\,b(2b+a)} + \frac{g_0L^2\,b(2b+a)}{54E\,b(2b+a)}$$

$$u_4 = -\frac{g_0L^2}{54E\,b(2b+a)}\left[2(a+b)(8b+13a) - b(2b+a)\right]$$

$$\to 2(8ab+13a^2+8b^2+13ab)-2b^2-ab$$

$$\to 16ab+26a^2+16b^2+26ab-2b^2-ab$$

$$\boxed{u_4 = -\frac{g_0L^2(26a^2+41ab+14b^2)}{54E\,b(2b+a)}}$$

- Zapišemo:

$$\boxed{\begin{aligned}
u_1 &= \varnothing \\
u_2 &= -\frac{g_0L^2(6a^2+7ab+2b^2)}{18E\,b(2b+a)} \\
u_3 &= -\frac{2g_0L^2(a+b)(b+2a)}{9E\,b(2b+a)} \\
u_4 &= -\frac{g_0L^2(26a^2+41ab+14b^2)}{54E\,b(2b+a)}
\end{aligned}}$$