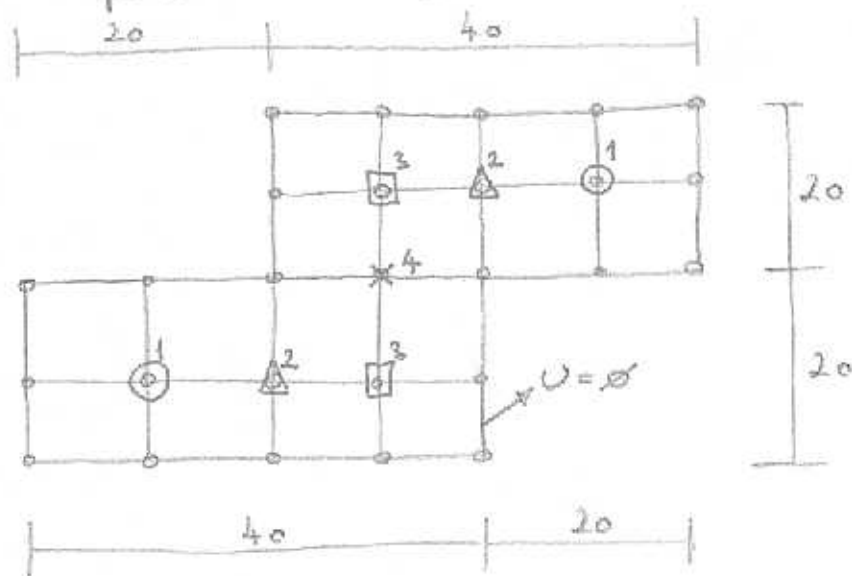


Imamo podan naslednji sistem



$$h=0$$

$$I_{\text{tor}} = ?$$

Vse točke na robu imajo vrednost napetosti U enako σ , zato nam ostane samo 7 notranjih točk. Od tega so tri simetrične in ena nesimetrična, tako da imamo za izračunati 4 točke.

$$I_{\text{tor}} = -4 \int_{\Omega} u d\Omega \approx -4 \sum_{k=1}^N u_k \omega_k$$

$$\Delta U = 1$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 1$$

$$(\Delta U)_p = \frac{u_1 + u_2 + u_3 + u_4 - 4u_0}{h^2}$$

• Točka 1.

$$\frac{u_2 - 4u_1}{h^2} = 1$$

• Točka 2.

$$\frac{u_3 + u_1 - 4u_2}{h^2} = 1$$

• Točka 3.

$$\frac{u_4 + u_2 - 4u_3}{h^2} = 1$$

• Točka 4.

$$\frac{u_2 + u_3 - 4u_4}{h^2} = 1$$

Rešimo sistem:

$$u_2 - 4u_1 = h^2 \checkmark$$

$$u_3 + u_1 - 4u_2 = h^2 \checkmark$$

$$u_4 + u_2 - 4u_3 = h^2 \checkmark$$

$$u_3 + u_3 - 4u_4 = h^2 \checkmark$$

$$- u_2 = h^2 + 4u_1$$

$$- u_3 = h^2 + 4u_2 - u_1 = h^2 + 4h^2 + 16u_1 - u_1 = 5h^2 + 15u_1$$

$$- 2u_3 = h^2 + 4u_4$$

$$u_3 = \frac{1}{2}h^2 + 2u_4$$

$$5h^2 + 15u_1 = \frac{1}{2}h^2 + 2u_4$$

$$\frac{9}{2}h^2 + 15u_1 = 2u_4$$

$$u_4 = \frac{9}{4}h^2 + \frac{15}{2}u_1$$

$$- u_4 + u_2 - 4u_3 = h^2$$

$$\frac{9}{4}h^2 + \frac{15}{2}u_1 + h^2 + 4u_1 - 20h^2 - 60u_1 = h^2$$

$$u_1 \left[\frac{15}{2} + 4 - 60 \right] = h^2 \left[20 - \frac{9}{4} \right]$$

$$-\frac{47}{2} u_1 = \frac{71}{4} h^2 \rightarrow \boxed{u_1 = -\frac{71 h^2}{194}}$$

$$u_2 = h^2 - \frac{4 \cdot 71 h^2}{194} \rightarrow u_2 = -\frac{90 h^2}{194} \Rightarrow \boxed{u_2 = -\frac{45 h^2}{97}}$$

$$u_3 = 5h^2 - 15u_1 = 5h^2 - \frac{15 \cdot 71 h^2}{194} \Rightarrow \boxed{u_3 = -\frac{95 h^2}{194}}$$

$$u_4 = \frac{9}{4}h^2 + \frac{15}{2}u_1 = \frac{9}{4}h^2 - \frac{15}{2} \cdot \frac{71}{194} h^2 = -\frac{192 h^2}{4 \cdot 97} \Rightarrow \boxed{u_4 = -\frac{48 h^2}{97}}$$

$$I_{\text{tor}} = -4 \left[2u_1 h^2 + 2u_2 h^2 + 2u_3 h^2 + u_4 \right] =$$

$$= -4 h^4 \left[-\frac{2 \cdot 71}{194} - \frac{2 \cdot 45 \cdot 2}{194} - \frac{2 \cdot 95}{194} - \frac{2 \cdot 48}{194} \right]$$

$$I_{\text{tor}} = \frac{4 \cdot 608}{194} h^4$$

$$\boxed{I_{\text{tor}} = \frac{1216}{97} h^4}$$

