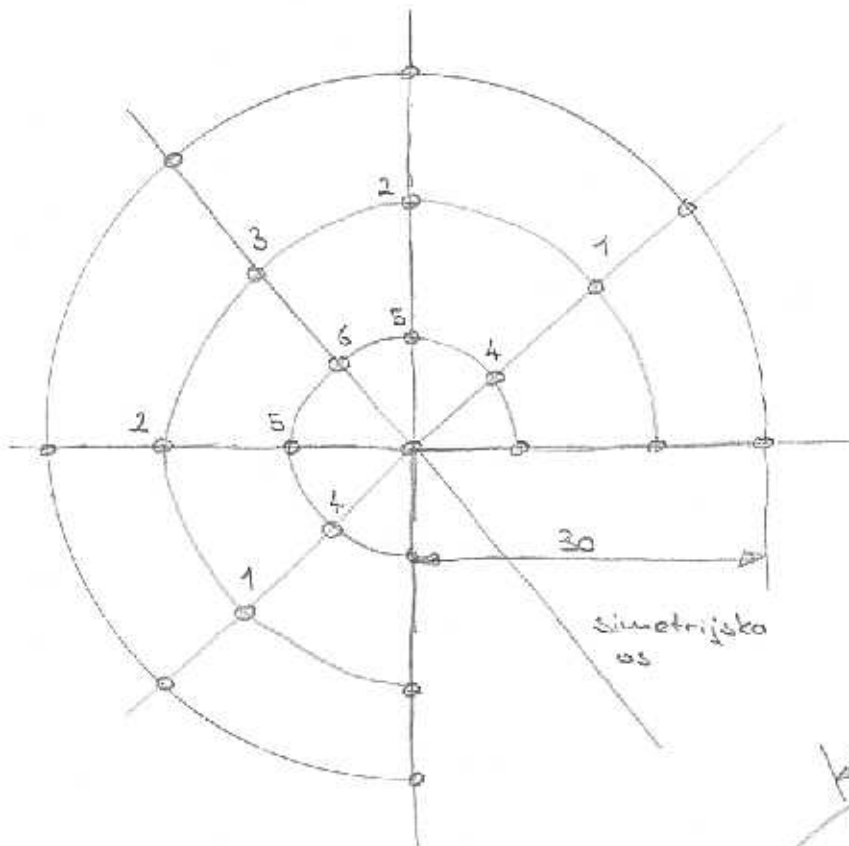
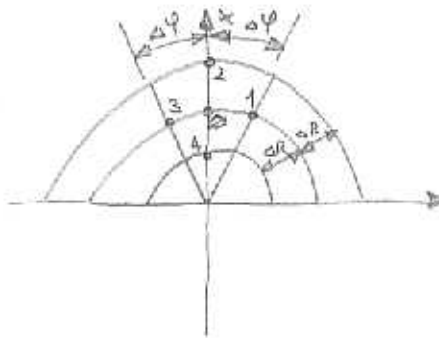


Imamo naslednji sistem:



$$\begin{aligned}\Delta R &= 0 \\ \Delta \varphi &= \pi/4 \\ H &= z\end{aligned}$$



$$\frac{1}{4a^2} + \frac{1}{r} \frac{1}{4r} + \frac{1}{r^2} \frac{1}{4\varphi^2} = 1$$

$$\left(\frac{\partial^2 U}{\partial r^2} \right)_{\text{os}} = \frac{U_2 - 2U_0 + U_4}{(\Delta R)^2}$$

$$\left(\frac{1}{r} \frac{\partial U}{\partial r} \right)_{\text{os}} = \frac{U_2 - U_4}{2\Delta R} \frac{1}{R_0}$$

$$\left(\frac{1}{r^2} \frac{\partial^2 U}{\partial \varphi^2} \right)_{\text{os}} = \frac{U_3 - 2U_0 + U_1}{(\Delta \varphi)^2} \frac{1}{R_0^2}$$

$$H = -4 \left[2U_1 + 2U_2 + U_3 + 2U_4 + 2U_5 + U_6 \right] h^2$$

• Točka 1:

$$\frac{-2U_1 + U_2}{a^2} + \frac{1}{2a} \frac{-U_4}{2a} + \frac{1}{4a^2} \frac{-2U_1 + U_2}{(\pi/4)^2} = 1$$

• Točka 2:

$$\frac{-2U_2 + U_6}{a^2} + \frac{1}{2a} \frac{-U_6}{2a} + \frac{1}{4a^2} \frac{U_1 - 2U_2 + U_3}{(\pi/4)^2} = 1$$

• Točka 3:

$$\frac{-2U_3 + U_6}{a^2} + \frac{1}{2a} \frac{-U_6}{2a} + \frac{1}{4a^2} \frac{U_2 - 2U_3 + U_4}{(\pi/4)^2} = 1$$

• Točka 4:

$$\frac{U_1 - 2U_4}{a^2} + \frac{1}{0} \frac{U_4}{2a} + \frac{1}{a^2} \frac{-2U_4 + U_5}{(\pi/4)^2} = 1$$

• Točka 5:

$$\frac{U_2 - 2U_5}{a^2} + \frac{1}{0} \frac{U_5}{2a} + \frac{1}{a^2} \frac{U_4 - 2U_5 + U_6}{(\pi/4)^2} = 1$$

• Točka 6:

$$\frac{U_3 - 2U_6}{a^2} + \frac{1}{0} \frac{U_6}{2a} + \frac{1}{a^2} \frac{U_5 - 2U_6 + U_5}{(\pi/4)^2} = 1$$

Dobivamo sistem jednačina:

$$-8U_1 + 3U_4 + (\pi/4)^2 (-2U_4 + U_5) = 4a^2$$

$$-8U_2 + 3U_5 + (\pi/4)^2 (U_4 - 2U_5 + U_6) = 4a^2$$

$$-8U_3 + 3U_6 + (\pi/4)^2 (U_5 - 2U_6 + U_5) = 4a^2$$

$$3U_1 - 4U_4 + 2(\pi/4)^2 (-2U_4 + U_5) = 2a^2$$

$$3U_2 - 4U_5 + 2(\pi/4)^2 (U_4 - 2U_5 + U_6) = 2a^2$$

$$3U_3 - 4U_6 + 2(\pi/4)^2 (U_5 - 2U_6 + U_5) = 2a^2$$

Rešimo sistem jednačina i dobivamo:

```
In[1]:= eqns = [-8 U1 + 3 U4 + ((e / 4) ^ -2) * (-2 U1 + U2) = 4 a ^ 2,
-8 U2 + 3 U5 + ((e / 4) ^ -2) * (U1 - 2 U2 + U3) = 4 a ^ 2,
-8 U3 + 3 U6 + ((e / 4) ^ -2) * (U2 - 2 U3 + U2) = 4 a ^ 2,
3 U1 - 4 U4 + 2 * ((e / 4) ^ -2) * (-2 U4 + U5) = 2 a ^ 2,
3 U2 - 4 U5 + 2 * ((e / 4) ^ -2) * (U4 - 2 U5 + U6) = 2 a ^ 2,
3 U3 - 4 U6 + 2 * ((e / 4) ^ -2) * (U5 - 2 U6 + U5) = 2 a ^ 2]
```

```
Out[1]= {-8 U1 + 3 U4 +  $\frac{16(U2 - 2 U1)}{e^2} = 4 a^2$ , -8 U2 + 3 U5 +  $\frac{16(U1 - 2 U2 + U3)}{e^2} = 4 a^2$ ,
 $\frac{16(2 U2 - 2 U3)}{e^2} - 8 U3 + 3 U6 = 4 a^2$ , 3 U1 - 4 U4 +  $\frac{32(U5 - 2 U4)}{e^2} = 2 a^2$ ,
3 U2 - 4 U5 +  $\frac{32(U4 - 2 U5 + U6)}{e^2} = 2 a^2$ , 3 U3 - 4 U6 +  $\frac{32(2 U5 - 2 U6)}{e^2} = 2 a^2$ }
```

```
In[3]:= Solve[eqns, {U1, U2, U3, U4, U5, U6}] // Simplify
```

```
Out[3]= {{U6 ->  $-\frac{4 a^2 e^2 (3703 e^{10} + 309120 e^8 + 7934976 e^6 + 61087744 e^4 + 142344192 e^2 + 37748736)}{(23 e^4 + 640 e^2 + 2048)(529 e^8 + 29440 e^6 + 267264 e^4 + 655360 e^2 + 262144)}$ ,
U3 ->  $-\frac{2 a^2 e^2 (5819 e^{10} + 485760 e^8 + 12469248 e^6 + 105316352 e^4 + 324796416 e^2 + 301989888)}{(23 e^4 + 640 e^2 + 2048)(529 e^8 + 29440 e^6 + 267264 e^4 + 655360 e^2 + 262144)}$ ,
U2 ->  $-\frac{2 a^2 e^2 (253 e^6 + 14080 e^4 + 104960 e^2 + 131072)}{529 e^8 + 29440 e^6 + 267264 e^4 + 655360 e^2 + 262144}$ ,
U1 ->  $-\frac{2 a^2 e^2 (5819 e^{10} + 438656 e^8 + 9588736 e^6 + 73367552 e^4 + 205783040 e^2 + 167772160)}{(23 e^4 + 640 e^2 + 2048)(529 e^8 + 29440 e^6 + 267264 e^4 + 655360 e^2 + 262144)}$ ,
U4 ->  $-\frac{4 a^2 e^2 (3703 e^{10} + 261832 e^8 + 5138432 e^6 + 34975744 e^4 + 77332480 e^2 + 20971520)}{(23 e^4 + 640 e^2 + 2048)(529 e^8 + 29440 e^6 + 267264 e^4 + 655360 e^2 + 262144)}$ ,
U5 ->  $-\frac{4 a^2 e^2 (161 e^6 + 8960 e^4 + 56320 e^2 + 16384)}{529 e^8 + 29440 e^6 + 267264 e^4 + 655360 e^2 + 262144}}$ }}
```

```
In[21]:- eqns = {-8 U1 + 3 U4 + ((e / 4) ^ -2) * (-2 U1 + U2) = 4 a ^ 2,
  -8 U2 + 3 U5 + ((e / 4) ^ -2) * (U1 - 2 U2 + U3) = 4 a ^ 2,
  -8 U3 + 3 U6 + ((e / 4) ^ -2) * (U2 - 2 U3 + U2) = 4 a ^ 2,
  3 U1 - 4 U4 + 2 * ((e / 4) ^ -2) * (-2 U4 + U5) = 2 a ^ 2,
  3 U2 - 4 U5 + 2 * ((e / 4) ^ -2) * (U4 - 2 U5 + U6) = 2 a ^ 2,
  3 U3 - 4 U6 + 2 * ((e / 4) ^ -2) * (U5 - 2 U6 + U5) = 2 a ^ 2}
```

```
Out[21]:- {-8 U1 + 1.62114 (U2 - 2 U1) + 3 U4 = 4 a^2, -8 U2 + 1.62114 (U1 - 2 U2 + U3) + 3 U5 = 4 a^2,
  1.62114 (2 U2 - 2 U3) - 8 U3 + 3 U6 = 4 a^2, 3 U1 - 4 U4 + 3.24228 (U5 - 2 U4) = 2 a^2,
  3 U2 - 4 U5 + 3.24228 (U4 - 2 U5 + U6) = 2 a^2, 3 U3 + 3.24228 (2 U5 - 2 U6) - 4 U6 = 2 a^2}
```

```
In[42]:- e = π // N
  h = a
```

```
Out[42]:- 3.14159
```

```
Out[43]:- a
```

```
In[22]:- Solve[eqns, {U1, U2, U3, U4, U5, U6}] // simplify
```

```
Out[22]:- {{U1 -> -0.655423 a^2, U2 -> -0.830864 a^2,
  U3 -> -0.870522 a^2, U4 -> -0.673834 a^2, U5 -> -0.955679 a^2, U6 -> -1.03092 a^2}}
```

Dobimo naslednje rešitve sistema :

```
In[35]:- U1 = -0.6554229805700649 a^2
  U2 = -0.8308641928582077 a^2
  U3 = -0.8705217609411773 a^2
  U4 = -0.6738336595951954 a^2
  U5 = -0.955679231889284 a^2
  U6 = -1.0309183143875988 a^2
```

```
Out[35]:- -0.655423 a^2
```

```
Out[36]:- -0.830864 a^2
```

```
Out[37]:- -0.870522 a^2
```

```
Out[38]:- -0.673834 a^2
```

```
Out[39]:- -0.955679 a^2
```

```
Out[40]:- -1.03092 a^2
```

Iskan torzijski moment je :

```
In[44]:- It = -4 (2 U1 + 2 U2 + U3 + 2 U4 + 2 U5 + U6) h^2
```

```
Out[44]:- 32.5322 a^4
```