DOMAČA NALOGA

pri predmetu Statika in Kinematika

1

Domača naloga zajema vaje iz področij:
osnove vektorskega računa, obremenitve,
reakcije in podpore konstrukcij

Študent: Boštjan Kreutz

Predavatelj: Igor Emri

Asistent: Robert Cvelbar

Ljubljana, 16.10.2001

Naloga 1.1:

Dana sta vektorja $\vec{a}$=(5,3,2) in $\vec{b}$=(-1,2,3). Določite: $\vec{a}+\vec{b}$, $\vec{a}$-2$\vec{b}$, $\vec{e}=\vec{a}\times\vec{b}$, $\vec{f}=\vec{a}\times 2\vec{b}$, $\vec{f}\cdot(\vec{a}+2\vec{b})$. Določite še kot φ, ki ga oklepata vektorja $\vec{e}$ in $\vec{f}$ ter kot δ, ki ga oklepata vektorja $\vec{a}$ in $\vec{b}$.

$$\vec{a}+\vec{b}=(5+(-1),3+2,2+3)=(4,5,5)$$

$$\vec{a}-2\vec{b}=(5+2,3+(-4),2+(-6))=(7,-1,-4)$$
$$2\vec{b}=2(-1,2,3)=(-2,4,6)$$

$$\vec{e}=\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 5 & 3 & 2\\ -1 & 2 & 3\end{vmatrix}=\begin{matrix}\vec{i}(3\cdot 3-2\cdot 2)+\\ \vec{j}(2\cdot(-1)-5\cdot 3)+=5\vec{i}-17\vec{j}+13\vec{k}=(5,-17,13)\\ \vec{k}(5\cdot 2-3\cdot(-1))\end{matrix}$$

$$\vec{f}=\vec{a}\times\overrightarrow{2b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 5 & 3 & 2\\ -2 & 4 & 6\end{vmatrix}=\begin{matrix}\vec{i}(3\cdot 6-4\cdot 2)+\\ \vec{j}(2\cdot(-2)-5\cdot 6)+=10\vec{i}-34\vec{j}+26\vec{k}=(10,-34,26)\\ \vec{k}(5\cdot 4-3\cdot(-2))\end{matrix}$$

$$\vec{f}\cdot(\vec{a}+2\vec{b})=10\cdot 3+(-34)\cdot 7+26\cdot 8=0$$
$$\vec{a}+2\vec{b}=(5+(-2),3+4,2+6)=(3,7,8)$$

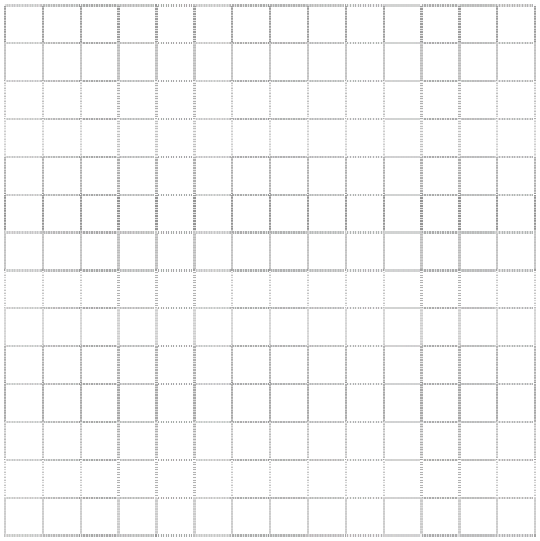
$$\cos\varphi=\frac{\vec{e}\cdot\vec{f}}{e\cdot f}=\frac{966}{\sqrt{483}\sqrt{1932}}\Rightarrow\varphi=0°$$
$$e=\sqrt{5^2+(-17)^2+13^2}=\sqrt{483}=21.98$$
$$f=\sqrt{10^2+(-34)^2+26^2}=\sqrt{1932}=43.96$$
$$\vec{e}\cdot\vec{f}=5\cdot 10+(-17)\cdot(-34)+13\cdot 26=966$$

$$\cos\delta=\frac{\vec{a}\cdot\vec{b}}{a\cdot b}=\frac{7}{\sqrt{38}\sqrt{14}}\Rightarrow\delta=72°18'42''$$
$$a=\sqrt{5^2+3^2+2^2}=\sqrt{38}=6.16$$
$$b=\sqrt{(-1)^2+2^2+3^2}=\sqrt{14}=3.74$$
$$\vec{a}\cdot\vec{b}=5\cdot(-1)+3\cdot 2+2\cdot 3=7$$

Naloga 1.2:

Iz $T_1(3,3,3)$ gre sila $\vec{P}$ velikosti 200N v smeri točke $T_2(0,2,3)$ in sila $\vec{Q}$ velikosti 400N v smeri točke $T_3(3,6,0)$. Izračunajte rezultanto in kote, ki jih rezultanta oklepa z osmi kartezijevega koordinatnega sistema.

Skica:



T_1 (3,3,3)
T_2 (0,2,3)
T_3 (3,6,0)

P = 200 N
Q = 400 N

$$d_p = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \sqrt{(-3)^2 + (-1)^2 + 0^2)} = \sqrt{10} = 3.16$$

$$\cos\alpha_p = \frac{x_2 - x_1}{d_p} = \frac{-3}{3.16} = -0.95 \;;\; P_x = P \cdot \cos\alpha_p = 200N \cdot (-0.95) = -190N$$

$$\cos\beta_p = \frac{y_2 - y_1}{d_p} = \frac{-1}{3.16} = -0.32 \;;\; P_y = P \cdot \cos\beta_p = 200N \cdot (-0.32) = -64N$$

$$\cos\gamma_p = \frac{z_2 - z_1}{d_p} = \frac{0}{3.16} = 0 \;;\; P_z = P \cdot \cos\gamma_p = 200N \cdot 0 = 0N$$

$$d_q = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2} = \sqrt{0^2 + 3^2 + (-3)^2)} = \sqrt{18} = 4.24$$

$$\cos\alpha_q = \frac{x_3 - x_1}{d_q} = \frac{0}{4.24} = 0 \;;\; Q_x = Q \cdot \cos\alpha_q = 400N \cdot 0 = 0N$$

$$\cos\beta_q = \frac{y_3 - y_1}{d_q} = \frac{3}{4.24} = 0.71 \;;\; Q_y = Q \cdot \cos\beta_q = 400N \cdot 0.71 = 284N$$

$$\cos\gamma_q = \frac{z_3 - z_1}{d_q} = \frac{-3}{4.24} = -0.71 \;;\; Q_z = Q \cdot \cos\gamma_q = 400N \cdot (-0.71) = -284N$$

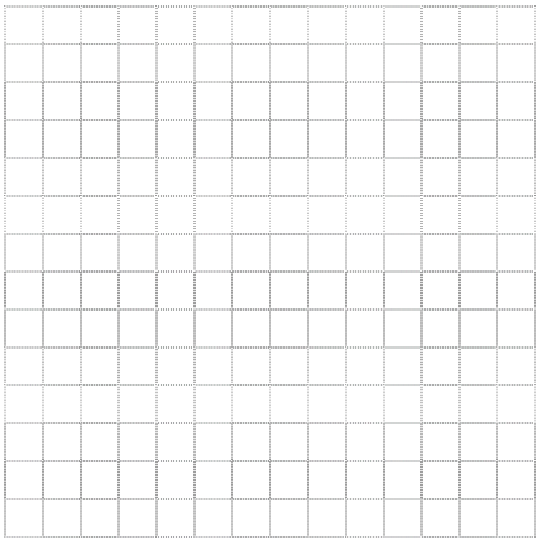
$$R_x = P_x + Q_x = -190N + 0N = -190N$$
$$R_y = P_y + Q_y = -64N + 284N = 220N$$
$$R_z = P_z + Q_z = 0N + (-284)N = -284N$$
$$R = \sqrt{R_x{}^2 + R_y{}^2 + R_z{}^2} = \sqrt{(-190)^2 + 220^2 + (-284)^2} = 406.39N$$

$$\cos\alpha_R = \frac{R_x}{R} = \frac{-190}{406.39} = -0.47 \Rightarrow \alpha_R = 117.87°$$
$$\cos\beta_R = \frac{R_y}{R} = \frac{220}{406.39} = 0.54 \Rightarrow \beta_R = 57.22°$$
$$\cos\gamma_R = \frac{R_z}{R} = \frac{-284}{406.39} = -0.70 \Rightarrow \gamma_R = 134.33°$$

Naloga 1.3:

V ravnini x-y je dan vektor $\vec{a}$=(3,4). Smernica *p* gre skozi točki (0,0) in (3,2), smernica *q* pa skozi točki (0,0) in (-3,2). Razstavite dani vektor v smeri smernic in zapišite enotska vektorja smernic. Nalogo rešite grafično in analitično.

Grafično:



Merilo: 0.5 cm = 1 enota

Analitično:

izračun kotov:

$$tg\alpha_p = \frac{2}{3} \Rightarrow \alpha_p = 33.69°, \alpha_q = -33.69°$$

$$tg\alpha_a = \frac{4}{3} \Rightarrow \alpha_a = 53.13°$$

nastavimo enačbo:

$$a_x = a_{px} - a_{qx} = 3 \qquad \left|\vec{a}\right| = \sqrt{3^2 + 4^2} = 5$$

$$\left|\vec{a}\right| \cdot \cos\alpha_a = \left|\overrightarrow{a_p}\right| \cdot \cos\alpha_p - \left|\overrightarrow{a_q}\right| \cdot \cos\alpha_q = 3 \Rightarrow \left|\overrightarrow{a_p}\right| = \frac{\left|\overrightarrow{a_q}\right| \cdot \cos\alpha_q + 3}{\cos\alpha_p}$$

$$\left|\vec{a}\right| \cdot \sin\alpha_a = \left|\overrightarrow{a_p}\right| \cdot \sin\alpha_p + \left|\overrightarrow{a_q}\right| \cdot \sin\alpha_q = 4$$

$$\left|\overrightarrow{a_p}\right| = \frac{3 + \left|\overrightarrow{a_q}\right| \cdot \cos\alpha_p}{\cos\alpha_q} = \frac{3 + 1.8 \cdot \cos(33.69°)}{\cos(33.69°)} = 5.4$$

$$\frac{3 \cdot \sin\alpha_p}{\cos\alpha_p} + \frac{\left|\overrightarrow{a_q}\right| \cdot \cos\alpha_q \cdot \sin\alpha_p}{\cos\alpha_p} + \left|\overrightarrow{a_q}\right| \cdot \sin\alpha_p = 4$$

$$\frac{\left|\overrightarrow{a_q}\right| \cdot \cos\alpha_q \cdot \sin\alpha_p}{\cos\alpha_p} + \left|\overrightarrow{a_q}\right| \cdot \sin\alpha_p = 2$$

$$\left|\overrightarrow{a_q}\right| \cdot \sin\alpha_p \left(\frac{\cos\alpha_q}{\cos\alpha_p} + 1\right) = 2$$

$$\left|\overrightarrow{a_q}\right| = \frac{1}{\sin\alpha_p} = 1.8$$

$$\left.\begin{aligned} a_{px} &= \left|\overrightarrow{a_p}\right| \cdot \cos\alpha_p = 5.4 \cdot \cos(33.69°) = 4.493 \\ a_{py} &= \left|\overrightarrow{a_p}\right| \cdot \sin\alpha_p = 5.4 \cdot \sin(33.69°) = 2.995 \end{aligned}\right\} \overrightarrow{a_p} = (4.493, 2.995)$$

$$\left.\begin{aligned} a_{qx} &= \left|\overrightarrow{a_q}\right| \cdot \cos(-\alpha_q) = 1.8 \cdot \cos(33.69°) = 1.498 \\ a_{qy} &= \left|\overrightarrow{a_q}\right| \cdot \sin\alpha_q = 1.8 \cdot \sin(33.69°) = 0.999 \end{aligned}\right\} \overrightarrow{a_q} = (1.498, 0.999)$$

Enotska vektorja smernic:

$$\cos\alpha_p = \frac{y}{1} \Rightarrow y = \cos\alpha_p = 0.832 \qquad \overrightarrow{e_p} = (0.555, 0.832)$$

$$\sin\alpha_p = \frac{x}{1} \Rightarrow x = \sin\alpha_p = 0.555 \qquad \overrightarrow{e_q} = (-0.555, 0.832)$$

Naloga 1.4:

Dan je vektor $\vec{a}$=(8, 6, -4). Določite vektor $\vec{b}$(7, y, z) tako, da bo pravokoten na $\vec{a}$ in bo oklepal z osjo *y* kot 60°.

$\vec{a} = (8,6,-4)$
$\vec{b} = (7, y, z)$
$\varphi = 60°$

$\vec{a} \cdot \vec{b} = 0$
$8 \cdot 7 + 6y - 4z = 0$
$56 + 6y - 4z = 0$
$3136 + 36y^2 - 16z^2 = 0$

uvedemo novi vektor na osi y:

$\vec{c} = (0,3,0)$

$$\cos\varphi = \frac{\vec{b} \cdot \vec{c}}{b \cdot c} = \frac{3y}{3 \cdot \sqrt{49 + y^2 + z^2}} = \frac{1}{2}$$
$\vec{b} \cdot \vec{c} = 3y$
$c = \sqrt{3^2} = 3$
$b = \sqrt{49 + y^2 + z^2}$

$$\frac{y}{\sqrt{49 + y^2 + z^2}} = \frac{1}{2}$$
$$\sqrt{49 + y^2 + z^2} = 2y$$
$$49 + y^2 + z^2 = 4y^2 \Rightarrow y^2 = \frac{49 + z^2}{3}$$

$$3136 + \frac{1764 + 36z^2}{3} - 16z^2 = 0$$
$$\frac{1764}{3} + \frac{36z^2}{3} - 16z^2 = -3136$$
$$12z^2 - 16z^2 = -3724$$
$$-4z^2 = -3724$$
$z = \sqrt{931}$
$z_1 = 30.5$
$z_2 = -30.5$
$$y = \sqrt{\frac{49 + z^2}{3}}$$
$y_1 = 18.1$
$y_2 = -18.1$

Naloga 1.5:

Kdaj je skalarni produkt enak 0?

Skalarni produkt je enak 0, kadar sta vektorja pravokotna (kot med njima $\varphi = 90°$).

Naloga 1.6:

Kdaj je vektorski produkt enak 0?

Vektorski produkt je enak 0, kadar sta vektorja vzporedna.

Naloga 1.7:

Kaj podaja enotski vektor vektorja in kako ga določimo?

Enotski vektor podaja usmerjenost; določimo ga po enačbi

$$\left|\overrightarrow{e_A}\right| = \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Naloga 1.8:

Smer vektorja $\vec{A}$ določa enotski vektor $\overrightarrow{e_A}$ in ga izračunamo po enačbi

$$\overrightarrow{e_A} = \frac{\vec{A}}{A} = \frac{(Ax, Ay, Az)}{A}$$

DOMAČA NALOGA

pri predmetu Statika in Kinematika

2

Domača naloga zajema vaje iz področij:
Sistemi sil s skupnim prijemališčem: ravnotežje,
sestavljanje in razstavljanje sil v ravnini in prostoru

Študent: Boštjan Kreutz

Predavatelj: Igor Emri

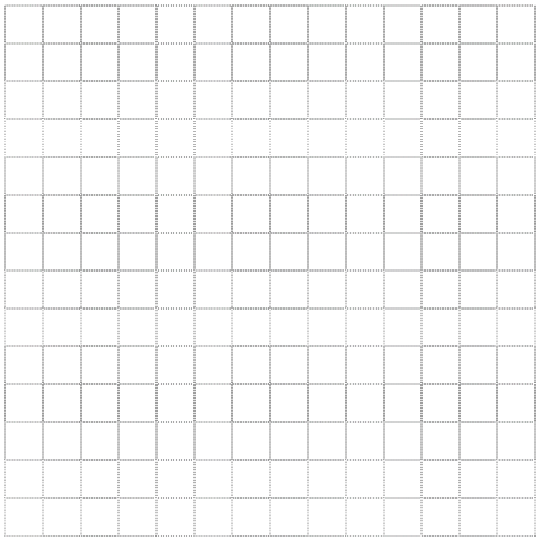
Asistent: Robert Cvelbar

Ljubljana, 23.10.2001

Naloga 2.1:

V ravnini je dan sistem sil s skupnim prijemališčem. Določite analitično in grafično rezultatno danih sil. F_1=5kN, α_1=30°, F_2=4kN, α_2=120°, F_3=7kN, α_3=210°

Grafično:



Merilo: 0.5 cm = 1 enota

Analitično:

$$R = \sqrt{Rx^2 + Ry^2}$$

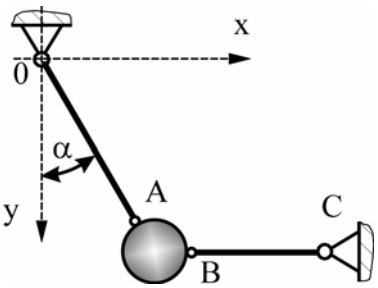
$$\sum F_{ix} = Rx = F_1 \cdot \cos 30° - F_2 \cdot \cos 60° - F_3 \cdot \cos 30° = 5kN \cdot \cos 30° - 4kN \cdot \cos 60° - 7kN \cdot \cos 30°$$
$$Rx = -4.6kN$$
$$\sum F_{iy} = Ry = F_1 \cdot \sin 30° + F_2 \cdot \sin 60° - F_3 \cdot \sin 30° = 5kN \cdot \sin 30° + 4kN \cdot \sin 60° - 7kN \cdot \sin 30°$$
$$Ry = 2.5kN$$

$$R = \sqrt{Rx^2 + Ry^2} = \sqrt{\left(-4.6kN\right)^2 + \left(2.5kN\right)^2} = 5.2kN$$

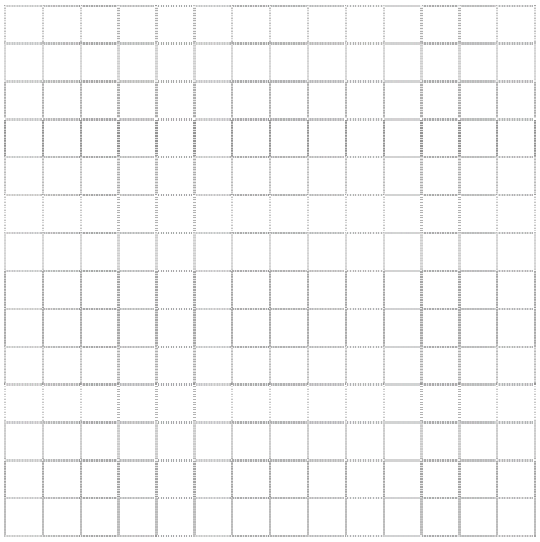
Naloga 2.2:

Krogla teže G=100N je obešena na vrvici OA in v točki B pritrjena na vrvico BC, ki je horizontalna. Krogla je v ravnotežju, ko je kot $\alpha = 30^{o.}$ Grafično in analitično določite sile v vrveh.

Skica:



Grafično:



Merilo: 0.5 cm = 10 N

Analitično: Skica:

$$\sum F_{ix} = 0; \quad C - A \cdot \sin\alpha = 0$$
$$\sum F_{iy} = 0; \quad -A \cdot \cos\alpha + G = 0 \quad \Rightarrow A \cdot \cos\alpha = G$$

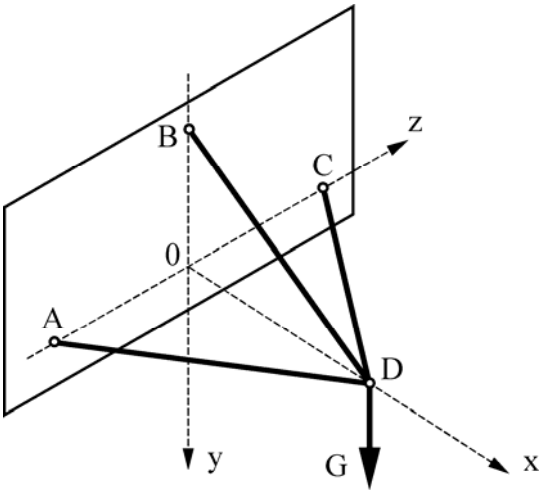
$$A = \frac{G}{\cos\alpha} = \frac{100N}{\cos 30°} = 115.47N$$
$$C = A \cdot \sin\alpha = 115.47N \cdot \sin 30° = 57.74N$$

Naloga 2.3:

Breme G = 1000N prenašajo trije drogovi, ki so pritrjeni na vertikalno steno. Določite sile v drogovih, če so koordinate točk:

A(0, 0, -3), B(0, -4, 0), C(0, 0, 3), D(5, 0, 0).

Skica:



Naloga 2.4:

Kdaj je ravninski sistem sil v ravnotežju?

Ravninski sistem je v ravnotežju, če zanj veljata naslednja pogoja:

$$\sum F_{ix} = 0 \qquad \sum F_{iy} = 0$$

Naloga 2.5:

Razdalja med dvema točkama na togem telesu se po delovanju sile ne spremeni.

(DA) NE

Naloga 2.6:

Če se telo giblje enakomerno pospešeno je vsota vseh sil, ki delujejo na telo enaka 0.

DA (NE)

Naloga 2.7:

Dva sistema sil sta enaka, če je rezultanta sil enaka.

(DA) NE

Naloga 2.8:

Dve sili sta v ravnotežju,…

…če imata enako velikost in sta nasprotno usmerjeni.

Naloga 2.9:

Če dve sili, ki sta enaki a nasprotno usmerjeni, dodamo sistemu sil, se sistem sil ne spremeni.

(DA) NE

Naloga 2.10:

Sili s katerima delujeta eno na drugo dve telesi sta enake po velikosti in leže na isti smernici, sta pa nasprotno usmerjeni.

(DA) NE

Naloga 2.11:

Smer reakcije v podpori kaže v smeri preprečitve gibanja telesa v podpori.

(DA) NE

Naloga 2.12:

Silo v ravnini lahko enolično razstavimo na

(2,) 3 ali 6 smeri.

Naloga 2.13:

Silo v prostoru lahko enolično razstavimo na

2, (3) ali 6 smeri.

Naloga 2.14:

Velikost rezultante je odvisna od vrstnega reda sestavljanja sil.

DA (NE)

Naloga 2.15:

Ko je sistem sil v ravnotežju je njihova rezultanta enaka 0.

DOMAČA NALOGA

pri predmetu Statika in Kinematika

3

Domača naloga zajema vaje iz področij:
Definicija momenta, ravnotežje obremenitev na togih telesih, sestavljanje in razstavljanje sil in momentov v ravnini in prostoru, dinama

Študent: Boštjan Kreutz

Predavatelj: Igor Emri

Asistent: Robert Cvelbar

Ljubljana, 29.10.2001

Naloga 3.1:

Dani prostorski sistem sil reducirajte v koordinatno izhodišče. Izračunajte dinamo in centralno os na kateri leži dinama.

$\vec{F}_1 = (-3,\ 2,\ 1)\text{kN};\quad \vec{r}_1 = (1,\ 1,\ 1)\text{m}$

$\vec{F}_2 = (1,\ 1,\ 1)\text{kN};\quad \vec{r}_2 = (-1,\ 2,\ -1)\text{m}$

$\vec{F}_3 = (5,\ 0,\ 0)\text{kN};\quad \vec{r}_3 = (2,\ -3,\ 1)\text{m}$

$\vec{F}_4 = (2,\ -8,\ 1)\text{kN};\quad \vec{r}_4 = (4,\ 2,\ 1)\text{m}$

$$\vec{R} = (5,-5,3)kN \qquad \overrightarrow{M_R} = \sum \vec{r_i} \times \overrightarrow{F_i} = \overrightarrow{r_R} \times \vec{R}$$

$$\overrightarrow{M_R} = \overrightarrow{F_1} \times \vec{r_1} + \overrightarrow{F_2} \times \vec{r_2} + \overrightarrow{F_3} \times \vec{r_3} + \overrightarrow{F_5} \times \vec{r_4} = -12\vec{i} + \vec{j} - 53\vec{k}$$

$$\overrightarrow{F_1} \times \vec{r_1} = \begin{matrix} \vec{i}(2\cdot 1 - 1\cdot 1) \\ +\ \vec{j}(1\cdot 1 - (-3)\cdot 1) \\ +\ \vec{k}((-3)\cdot 1 - 2\cdot 1) \end{matrix} = \vec{i} + 4\vec{j} - 5\vec{k}$$

$$\overrightarrow{F_2} \times \vec{r_2} = \begin{matrix} \vec{i}(1\cdot(-1) - 2\cdot 1) \\ +\ \vec{j}(1\cdot(-1) - 1\cdot(-1)) \\ +\ \vec{k}(1\cdot 2 - 1\cdot(-1)) \end{matrix} = -3\vec{i} + 3\vec{k}$$

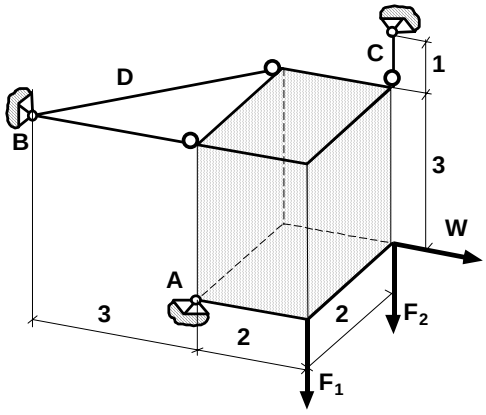
$$\overrightarrow{F_3} \times \vec{r_3} = \begin{matrix} \vec{i}(0\cdot 1 - (-3)\cdot 0) \\ +\ \vec{j}(0\cdot 2 - 5\cdot 1) \\ +\ \vec{k}(5\cdot(-3) - 0\cdot 2) \end{matrix} = -5\vec{j} - 15\vec{k}$$

$$\overrightarrow{F_4} \times \vec{r_4} = \begin{matrix} \vec{i}((-8)\cdot 1 - 2\cdot 1) \\ +\ \vec{j}(1\cdot 4 - 2\cdot 1) \\ +\ \vec{k}(2\cdot 2 - (-8)\cdot 4) \end{matrix} = -10\vec{i} + 2\vec{j} + 36\vec{k}$$

$$M_{Rx} = r_{RY}\cdot R_z - r_{Rz}\cdot R_y = -12$$
$$M_{Ry} = r_{Rz}\cdot R_x - r_{Rx}\cdot R_z = 1$$
$$M_{Rz} = r_{Rx}\cdot R_y - r_{Ry}\cdot R_x = -53$$

Naloga 3.2:

F_1=F_2=30kN, W=20kN. Določite sile v podporah A, B, C in D.



Izračun kotov:

$$tg\alpha = \frac{2}{3} \Rightarrow \alpha = 33.7°$$

$$\beta = 180° - 90° - \alpha = 56.3°$$

Ravnotežne enačbe:

$$\sum F_{ix} = 0: Ax + D \cdot \cos\beta = 0 \qquad \Rightarrow Ax = -Dx = -13.3kN$$

$$\sum F_{iy} = 0: Ay + W - B - D \cdot \sin\beta = 0$$

$$\sum F_{iz} = 0: Az - F_1 - F_2 + C = 0$$

$$\sum M_{i(x)} = 0: -3Ay + 2F_1 + 2F_2 - 3W - 2C = 0$$

$$\sum M_{i(y)} = 0: -3Ax + 2F_1 - 2Az = 0$$

$$\sum M_{i(z)} = 0: 2Ay - 2B = 0 \qquad \Rightarrow Ay = B$$

Računamo:

$$B - B - Dy = -W \Rightarrow Dy = W = 20kN$$

$$Dy = D \cdot \sin\beta \Rightarrow D = \frac{Dy}{\sin\beta} = \frac{20kN}{\sin 56.3°} = 24.0kN$$

$$Dx = D \cdot \cos\beta = 24.0kN \cdot \cos 56.3° = 13.3kN$$

$$Az = \frac{2F_1 - 3Ax}{2} = \frac{2 \cdot 30kN - 3 \cdot 13.3kN}{2} = 10.1kN$$

$$C = F_1 + F_2 - Az = 30kN + 30kN - 10.1kN = 50.0kN$$

$$Ay = \frac{2F_1 + 2F_2 - 3W - 2C}{3} = \frac{2 \cdot 30kN + 2 \cdot 30kN - 3 \cdot 20kN - 2 \cdot 50.0kN}{3} = -13.3kN$$

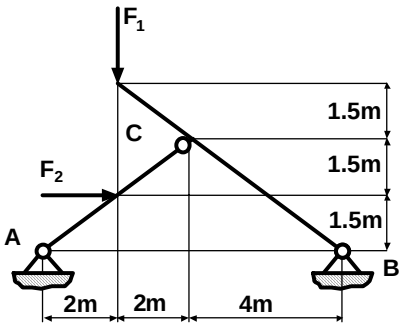
Izpis rešitev:

Ax = -13.3 kN	B = -13.3 kN
Ay = -13.3 kN	C = 50.0 kN
Az = 10.1 kN	D = 24.0 kN

Naloga 3.3:

F_1=40kN, F_2=20kN. Določite sile v podpornih točkah A, B in C.

Skica: Neznanke v podporah/prerez:



Ker je preveč neznank v sistemu sil, ga razrežemo na dva dela:

I.del: II.del:

$$\sum F_{ix} = 0 : Ax + F_2 - Cx = 0 \qquad \sum F_{ix} = 0 : Cx - Bx = 0$$
$$\sum F_{iy} = 0 : Ay + Cy = 0 \qquad \sum F_{iy} = 0 : By + Cy - F_1 = 0$$
$$\sum M_{i(A)} = 0 : -F_2 \cdot 1.5 - Cy \cdot 4 + Cx \cdot 3 = 0 \qquad \sum M_{i(B)} = 0 : F_1 \cdot 6 - Cx \cdot 3 - Cy \cdot 4 = 0$$

Ker sta obe momentni enačbi enaki 0, ju lahko izenačimo:

$$3Cx - 4Cy - 1.5F_2 = 6F_1 - 3Cx - 4Cy$$
$$3Cx + 3Cx - 4Cy + 4Cy = 6F_1 + 1.5F_2$$
$$6Cx = 6F_1 + 1.5F_2$$
$$Cx = \frac{6 \cdot 40kN + 1.5 \cdot 20kN}{6} = 45kN$$

Ostale neznanke izračunamo iz obeh ravnotežnih enačb:

$$Ax = Cx - F_2 = 45kN - 20kN = 25kN$$
$$Ay = -Cy = -26.25kN$$
$$Cy = \frac{3Cx - 1.5F_2}{4} = \frac{3 \cdot 45kN - 1.5 \cdot 20kN}{4} = 26.25kN$$
$$Bx = Cx = 45kN$$
$$By = F_1 - Cy = 40kN - 26.25kN = 13.75kN$$

Naloga 3.4:

Dvojico paralelnih sil lahko uravnotežimo s silo, ki je velika kot rezultanta teh sil in kaže v nasprotno smer.

DA (NE)

Naloga 3.5:

Dvojico sil lahko uravnotežimo z eno silo.

DA (NE)

Naloga 3.6:

Posledica dvojice sil je…

… čista rotacija (moment).

Naloga 3.7:

Če v neki točki telesa postavimo ravnotežni par sil se sistem sil ne spremeni.

(DA) NE

Naloga 3.8:

Kaj pravi Varignonov teorem?

Vsota momentov, ki jih povzročajo vse sile je enaka momentu, ki ga povzroči rezultanta vseh sil.

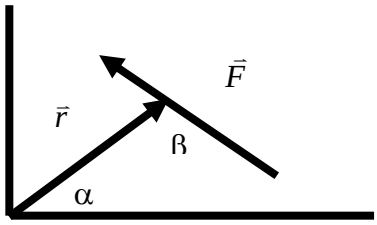
Naloga 3.9:

Moment okoli izhodišča koordinatnega sistema zaradi sile $\vec{F}$ je:

$M_0 = Fr$

$\vec{M}_0 = \vec{F} \times \vec{r}$

$(\vec{M}_0 = \vec{r} \times \vec{F})$



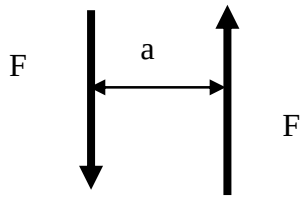
Naloga 3.10:

Moment dvojice sil na sliki je:

$M = F\,2a$

$M = Fa$

$M = 0.5Fa$



DOMAČA NALOGA

pri predmetu Statika in Kinematika

4

Domača naloga zajema vaje iz področij:
Težišča; masno središče, geometrijska središča in Guldinovi pravili

Študent: Boštjan Kreutz

Predavatelj: Igor Emri

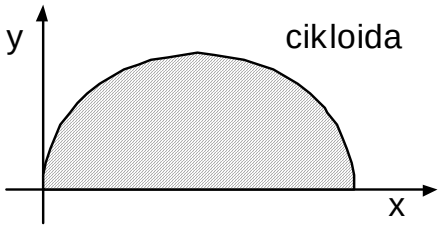
Asistent: Robert Cvelbar

Ljubljana, 12.11.2001

Naloga 4.1:

Določite težišče lika, ki ga tvorita os x in navadna cikloida.

x=a(t-sin t), y=a(1-cos t).



a… radij
t… kot zasuka krožnice

$$dA = y \cdot dx$$

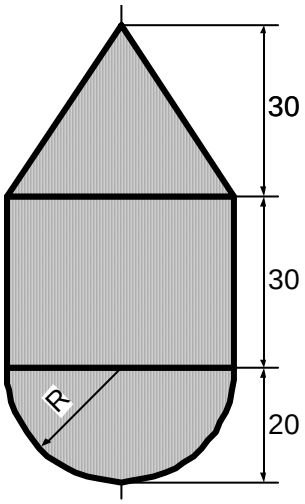
$$\tilde{y} = \frac{y}{2} \qquad dx = \frac{dx}{dt}dt = a(1-\cos t)dt$$

$x_T = \pi \cdot a$

$$y_T = \frac{\int_A \tilde{y} \cdot dA}{A} = \frac{\int_0^{2\pi a} \frac{y}{2} y \cdot dx}{\int_0^{2\pi a} y \cdot dx} = \frac{\int_0^{2\pi} \frac{a}{2}(1-\cos t)a(a-\cos t)a(a-\cos t)dt}{\int_0^{2\pi} a(1-\cos t)a(1-\cos t)dt}$$

$$y_T = \frac{\frac{a^3}{2}\int_0^{2\pi}(1-\cos t)^3 dt}{a^2\int_0^{2\pi}(1-\cos t)^2 dt} = \frac{a\int_0^{2\pi}(1-\cos t)^3 dt}{2\int_0^{2\pi}(1-\cos t)^2 dt}$$

Naloga 4.2:
Določite težišče ploščinskega lika.



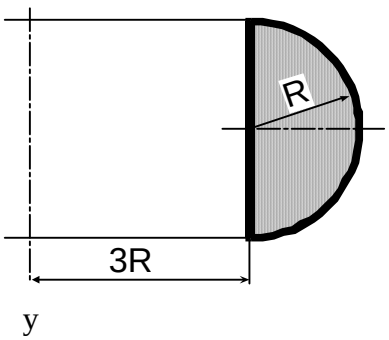
$$x_T = \frac{x_1A_1 + x_2A_2 + x_3A_3}{A_1 + A_2 + A_3} = 0$$

$$y_T = \frac{y_1A_1 + y_2A_2 + y_3A_3}{A_1 + A_2 + A_3} = \frac{36000 + 42000 + 7232.0}{600 + 1200 + 628.32} = 35.1$$

	x_{Ti}	y_{Ti}	A_i	$x_{Ti} \cdot A_i$	$y_{Ti} \cdot A_i$
	*0*	*60*	*600*	*0*	*36000*
	*0*	*35*	*1200*	*0*	*42000*
	*0*	$20 - \frac{4R}{2\pi} =$ *11.51*	*628.32*	*0*	*7232.0*

Naloga 4.3:

Določite površino in volumen rotacijskega telesa. R=5dm, če rotira polkrog okoli osi y-y.



$$A_T = 2\pi \cdot x_{T\check{C}} \cdot O = 2\pi \cdot \frac{\sum x_{Ti} \cdot l_i}{l} \cdot l = 2\pi \cdot (3R \cdot 2R + (3R + \frac{2R}{\pi})\pi R$$

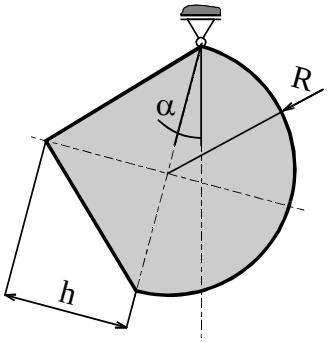
$$A_T = 2737.08 dm^2$$

$$V_T = 2\pi \cdot x_{TA} \cdot A = 2\pi \cdot \frac{\sum x_{Ti} \cdot A_i}{A} \cdot A = 2\pi \cdot (3R + \frac{4R}{3\pi}) \cdot \frac{\pi R^2}{2}$$

$$V_T = 4224.70 dm^3$$

Naloga 4.4:

Določite kot α za katerega se lik odmakne, če je polmer R = 20cm in višina trikotnika h=20cm.



$$x_T = \frac{-\frac{R}{3}R^2 + \frac{\pi R^2}{2}\cdot\frac{4R}{3\pi}}{\frac{\pi R^2}{2} + R^2} = \frac{-\frac{20}{3}400 + \frac{\pi\cdot 400}{2}\cdot\frac{4\cdot 20}{3\pi}}{\frac{\pi\cdot 400}{2} + 400} cm = 2.59cm$$

$$\tan\alpha = \frac{2.59cm}{2cm} \Rightarrow \alpha = 7.39°$$

Naloga 4.5:

Ali je osnova za izračun težišča ravnotežje sil? DA (NE)

Naloga 4.6:

Ali je težišče telesa vedno na telesu? DA (NE)

DOMAČA NALOGA

pri predmetu Statika in Kinematika

5

Domača naloga zajema vaje iz področij:
Statika konstrukcijskih elementov: definicije elementov in njihovih osnovnih lastnosti, določitev notranjih veličin.

Študent: Boštjan Kreutz

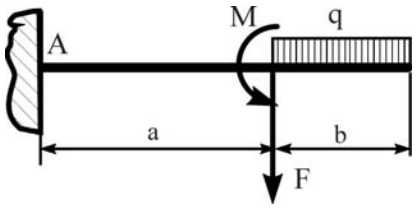
Predavatelj: Igor Emri

Asistent: Robert Cvelbar

Ljubljana, 19.11.2001

Naloga 5.1:

Določite N, T, M diagrame in določite mesto ter velikost M_{max}, če je M=2kNm, q=4kN/m, F=2kN, a=3m in b=2m.



1. SKICA IN DIAGRAM N, T, M:

2. REAKCIJE V PODPORAH:

$$\sum F_{ix} = 0 : -Ax = 0$$
$$\sum F_{iy} = 0 : -Ay - F - Q = 0$$
$$\sum M_{i(A)} = 0 : -M_A + M - a \cdot F - Q(a + \frac{b}{2}) = 0$$

$$Q = q \cdot b = 4 \cdot 2kN = 8kN$$
$$Ax = 0$$
$$Ay = F + Q = (2 + 8)kN = 10kN$$
$$M_A = M - aF - Q(a + \frac{b}{2}) =$$
$$= (2 - 3 \cdot 2 - 8 \cdot 4)kNm = -36kNm$$

3. I.POLJE:

$$\sum F_{iX} = 0 : N - Ax = 0 \Rightarrow N = Ax = 0$$
$$\sum F_{iY} = 0 : T - Ay = 0 \Rightarrow T = Ay = 10kN$$
$$\sum Mi = 0 : M - M_A - xT = 0$$

$$M = M_A + xT = \begin{cases} x = 0m; M = -36kNm \\ x = 1m; M = -26kNm \\ x = 2m; M = -16kNm \\ x = 3m; M = -6kNm \end{cases}$$

4. II.POLJE:

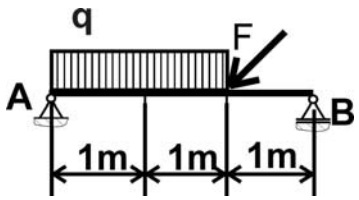
$$\sum F_{iX} = 0 : N = 0$$
$$\sum F_{iY} = 0 : T - x \cdot q = 0$$
$$\sum Mi = 0 : M + q\frac{x^2}{2} = 0$$

$$T = q \cdot x = \begin{cases} x = 0m; T = 0kN \\ x = 1m; T = 4kN \\ x = 2m; T = 8kN \end{cases}$$

$$M = -q\frac{x^2}{2} = \begin{cases} x = 0m; M = 0kNm \\ x = 1m; M = -2kNm \\ x = 2m; M = -8kNm \end{cases}$$

Naloga 5.2:

Določite N, T, M diagrame in določite mesto ter velikost $M_{max,}$ če je, q=4kN/m in F=2kN.



1. SKICA IN DIAGRAM N, T, M:

2. REAKCIJE V PODPORAH:

$$\sum F_{ix} = 0 : Ax - F \cdot \cos 45° = 0$$
$$\sum F_{iy} = 0 : \; Ay + By - Q - F \cdot \sin 45° = 0$$
$$\sum M_{i(A)} = 0 : 1By + 1Q - 2Ay = 0$$

$$Q = q \cdot 2 = 4 \cdot 2kN = 8kN$$
$$Ax = F \cdot \cos 45° = 2kN \cdot \cos 45° = 1.41kN$$
$$Ay = \frac{2Q + F \cdot \sin 45°}{3} = \frac{2 \cdot 8 + 2 \cdot \sin 45°}{3} kN = 5.81kN$$
$$By = 2Ay - Q = (2 \cdot 5.81 - 8)kN = 3.62kN$$

3. I.POLJE:

$$\sum F_{iX} = 0 : N + Ax = 0 \Rightarrow N = -Ax = -1.41kN$$
$$\sum F_{iY} = 0 : T + qx - Ay = 0$$
$$\sum Mi = 0 : M - xAy + q\frac{x^2}{2} = 0$$

$$T = Ay - q \cdot x = \begin{cases} x = 0m; T = 5.81kN \\ x = 1m; T = 1.81kN \\ x = 2m; T = -2.19kN \end{cases}$$

$$M = xAy - q\frac{x^2}{2} = \begin{cases} x = 0m; M = 0kNm \\ x = 1m; M = 3.81kNm \\ x = 2m; M = 3.62kNm \end{cases}$$

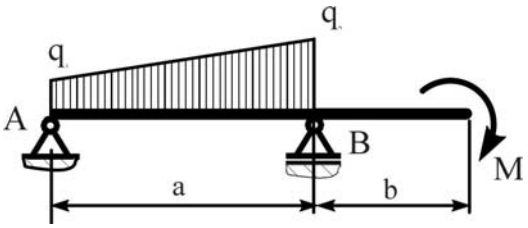
4. II.POLJE:

$$\sum F_{iX} = 0 : N = 0$$
$$\sum F_{iY} = 0 : T + By = 0 \Rightarrow T = -By = -3.62kN$$
$$\sum Mi = 0 : M - xBy = 0$$

$$M = xBy = \begin{cases} x = 0m; M = 0kNm \\ x = 1m; M = 3.62kNm \end{cases}$$

Naloga 5.3:

Določite N, T, M diagrame in določite mesto ter velikost M_{max} če je M_1=2kNm, q_A=2kN/m, q_B=4kN/m, a=3m in b=2m.



1. SKICA IN DIAGRAM N, T, M:

2. REAKCIJE V PODPORAH:

$$\sum F_{ix} = 0 : Ax = 0$$

$$\sum F_{iy} = 0 : \; Ay + B - \frac{q_A + q_B}{2} 3 = 0$$

$$\sum M_{i(A)} = 0 : 3B - M_1 - Q_{[\,]} \frac{3}{2} - 2Q_{\Delta} = 0$$

$$B = \frac{M_1 + Q_{[\,]} \frac{3}{2} + 2Q_{\Delta}}{3} = 5.67kN$$

$$Q_{[\,]} = q_A \cdot 3 = 2 \cdot 3kN = 6kN$$

$$Q_{\Delta} = \frac{3(q_B - q_A)}{2} = \frac{2 \cdot 3}{2} kN = 3kN$$

$$Ay = \frac{q_Q + q_b}{2} 3 - B = \frac{2+4}{2} 3 - 5.67 = 3.33kN$$

3. I.POLJE:

$$q(x) = kx + n$$

$$n = q_A = 2kN / m$$

$$k = \frac{q_B - q_A}{3} = \frac{2}{3}$$

$$q(x) = \frac{2}{3} x + 2$$

$$\sum F_{iX} = 0 : Ax + N = 0 \qquad \Rightarrow N = -Ax = 0kN$$

$$\sum F_{iY} = 0 : T - Ay + x \cdot q_A + \frac{q(x) - q_A}{2} \cdot \frac{x^2}{3} = 0$$

$$\sum Mi = 0 : M - xAy + \frac{x^2}{2} \cdot q_A + \frac{q(x) - q_A}{2} \cdot \frac{x^2}{3} = 0$$

$$T = Ay - q_A \cdot x + \frac{q(x) - q_A}{2} x = \begin{cases} x = 0m; T = 3.33kN \\ x = 1.5m; T = 1.08kN \\ x = 3m; T = 0.33kN \end{cases}$$

$$M = xAy - \frac{x^2}{2} \cdot q_A - \frac{q(x) - q_A}{2} \cdot \frac{x^2}{3} = \begin{cases} x = 0m; M = 0kNm \\ x = 1.5m; M = 1.50kNm \\ x = 3m; M = -2.01kNm \end{cases}$$

4. II.POLJE:

$$\sum F_{iX} = 0 : N = 0$$

$$\sum F_{iY} = 0 : T = 0$$

$$\sum Mi = 0 : M + M_1 = 0$$

$$M = -M_1 = -2kNm$$

Naloga 5.4:

Ko se notranja prečna sila spreminja po paraboli ima notranji upogibni moment obliko linearne funkcije.

DA (NE)

Naloga 5.5:

Vsiljeni moment lahko predstavimo z dvojico sil.

(DA) NE

Naloga 5.6:

Do preskoka v diagramu notranjega upogibnega momenta pride ko…

… JE PREČNA SILA ENAKA NIČ (0) ALI MENJA PREDZNAK.

Naloga 5.7:

Polje na nosilcu je …

… JE OBMOČJE NOSILCA, NA KATEREM NI NEZVEZNE SPREMEMBE OBREMENITVE ALI GEOMETRIJE.

Naloga 5.8:

Določite elemente konstrukcije na sliki!

1 – nosilec

2 – palica

3 – nosilec

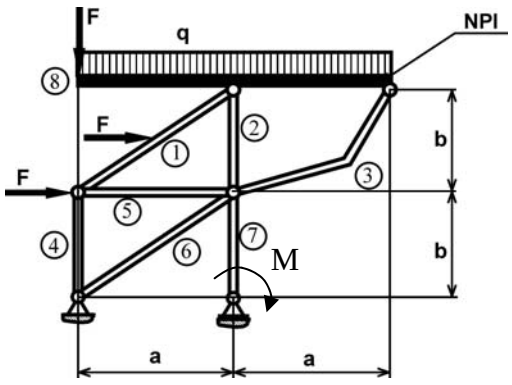
4 – palica

5 – palica

6 – palica

7 – nosilec

8 - nosilec



DOMAČA NALOGA

pri predmetu Statika in Kinematika

6

Domača naloga zajema vaje iz področij:
Ravni in lomljeni nosilci v ravnini
in prostoru

Študent: Boštjan Kreutz

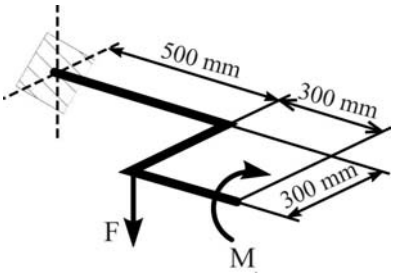
Predavatelj: Igor Emri

Asistent: Robert Cvelbar

Ljubljana, 04.12.2001

Naloga 6.1:

Določite N, T, M diagrame in mesto ter velikost M_{max}, če je F=2kN in M= 1kNm.



$$Ax = 0$$
$$Ay = 0$$
$$Az = F = 2kN$$
$$M_{Ax} = 0.3F - M = -0.4kNm$$
$$M_{Ay} = 0.5F = 1kNm$$
$$M_{Az} = 0$$

I.

$$N = 0$$
$$Ty = 0$$
$$Tz = 0$$
$$Mx = -M = -1kNm$$
$$My = 0$$
$$Mz = 0$$

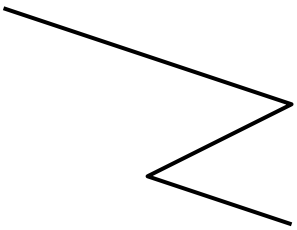
II.

$$N = 0$$
$$Ty = -F = -2kN$$
$$Tz = 0$$
$$Mx = 0$$
$$My = 0$$
$$Mz = M - xF$$

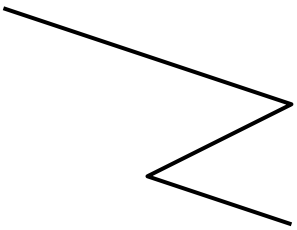
III.

$$N = 0$$
$$Ty = -F = -2kN$$
$$Tz = 0$$
$$Mx = 0.3F - M = -0.4kNm$$
$$My = 0$$
$$Mz = -xF$$

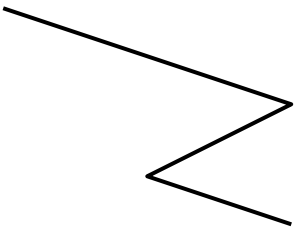
N



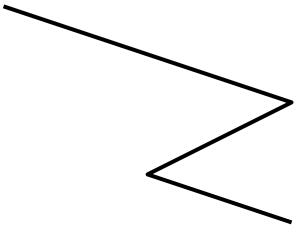
Ty



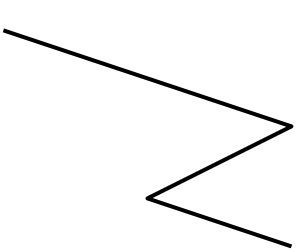
Tz



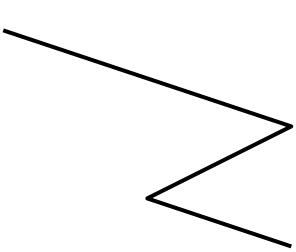
Mx



My

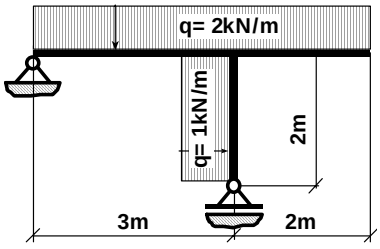


Mz



Naloga 6.2:

Določite N, T, M diagrame in mesto ter velikost $M_{max.}$



$$Ax = Q_2 = 2kN$$
$$Ay = Q_1 - By = 2.33kN$$
$$1Q_2 - 2.5Q_1 + 3By = 0$$
$$By = \frac{25 \cdot 2}{3} = 7.67kN$$

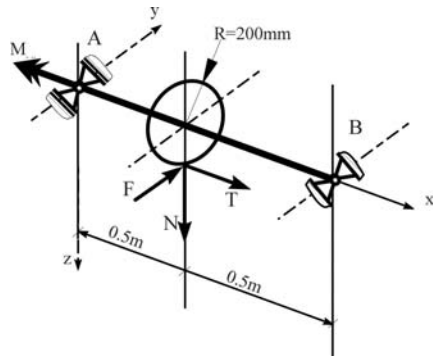
$$N = 0$$
$$T = xq_1 = \begin{cases} x = 0,\ T = 0 \\ x = 1,\ T = 2kN \\ x = 2,\ T = 4kN \end{cases}$$
$$M = -q_q \cdot \frac{x^2}{2} = \begin{cases} x = 0, M = 0 \\ x = 1, M = -1kNm \\ x = 2, M = -4kNm \end{cases}$$

$$N = -By = -7.67kN$$
$$T = -xq_2 = \begin{cases} x = 0,\ T = 0 \\ x = 1,\ T = -1kN \\ x = 2,\ T = -2kN \end{cases}$$
$$M = q_2 \cdot \frac{x^2}{2} = \begin{cases} x = 0, M = 0 \\ x = 1, M = 0.5kNm \\ x = 2, M = 2kNm \end{cases}$$

$$N = Ax = 2kN$$
$$T = Ay - xq_1 = \begin{cases} x = 0, T = 2.33kN \\ x = 1, T = 0.33kN \\ x = 2, T = -1.67kN \\ x = 3, T = -3.67kN \end{cases}$$
$$M = xAx - q_1 \cdot \frac{x^2}{2} = \begin{cases} x = 0, M = 0 \\ x = 1, M = 1.33kNm \\ x = 2, M = 0.66kNm \\ x = 3, M = -2.01kNm \end{cases}$$

Naloga 6.3:

Določite N, T, M diagrame in mesto ter velikost $M_{max,}$ če je F=10kN, T=2kN in N=5kN.



$$M = 0.2F = 2kN$$
$$Bx = -T = -2kN$$
$$Ay = -By - F = -15kN$$
$$Az = N - Bz = 2.9kN$$
$$By = -0.5F = -5kN$$
$$Bz = 0.5N - 0.2T = 2.1kN$$

Naloga 6.4:

Pri lomljenem nosilcu, kjer je kot 90 stopinj prečna sila preide v osno sila. (DA) NE

Naloga 6.5:

Kdaj pride pri diagramih notranjih veličin do preskoka velikosti notranjega momenta in kdaj do preskoka velikosti notranjega prečne sile?

Pri diagramih notranjih veličin pride do preskoka notranjega momenta, kadar na sistem deluje točkovna obremenitev ali zunanji moment.

Naloga 6.6:

Če je obremenitev znotraj enega polja kontinuirana in se linearno spreminja (narašča) je potek:

- notranjega momenta krivulja 3. reda
- notranje prečne sile krivulja 2. reda

Naloga 6.7:

Pri prostorskem nosilcu imamo za izračun reakcij v splošnem na voljo 6 enačb.

DOMAČA NALOGA

pri predmetu Statika in Kinematika

7

Domača naloga zajema vaje iz področij:
Ukrivljeni nosilci v ravnini in prostoru

Študent: Boštjan Kreutz

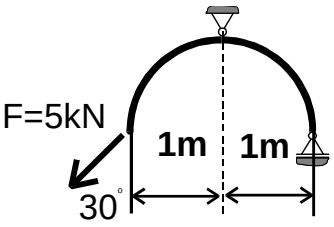
Predavatelj: Igor Emri

Asistent: Robert Cvelbar

Ljubljana, 11.12.2001

Naloga 7.1:

Določite N, T, M diagrame in določite mesto ter velikost M_{max}.



$$By = F(\cos 30° - \sin 30°) = 1.83kN$$
$$Ay = F \cdot \sin 30° + By = 6.8kN$$
$$Ax = 2.5kN$$

I.polje:

$$\sum F_{iX} = 0 : N - Fx \sin\varphi - Fy \cos\varphi = 0$$
$$\sum F_{iY} = 0 : T + Fy \sin\varphi - Fx \cos\varphi = 0$$
$$\sum M_{i0} = 0 : M + Fy \cdot R(1 - \cos\varphi) - Fx \cdot R \cdot \sin\varphi = 0$$

$$N = Fx \sin\varphi + Fy \cos\varphi = F \sin 30° \sin\varphi + F \cos 30° \cos\varphi =$$
$$\begin{cases} \varphi = 0° \;\; N = 4.33kN \\ \varphi = 30° \;\; N = 5kN \\ \varphi = 60° \;\; N = 4.33kN \\ \varphi = 90° \;\; N = 2.5kN \end{cases}$$

$$T = Fx \cos\varphi - Fy \sin\varphi = F \sin 30° \cos\varphi - F \cos 30° \sin\varphi =$$
$$\begin{cases} \varphi = 0° \;\; T = 2.5kN \\ \varphi = 30° \;\; T = 0kN \\ \varphi = 60° \;\; T = -2.5kN \\ \varphi = 90° \;\; T = -4.33kN \end{cases}$$

$$M = F \sin 30° \cdot 1m \cdot \sin\varphi - F \cos 30° \cdot 1m(1 - \cos\varphi) =$$
$$\begin{cases} \varphi = 0° \;\; M = 0kNm \\ \varphi = 30° \;\; M = 0.67kNm \\ \varphi = 60° \;\; M = 0kNm \\ \varphi = 90° \;\; M = -1.83kNm \end{cases}$$

II.polje:

$$\sum F_{iX} = 0 : N - By\cos\varphi = 0$$
$$\sum F_{iY} = 0 : T = By\sin\varphi$$
$$\sum M_{i0} = 0 : M + By \cdot R(1-\cos\varphi) = 0$$

$N = By\cos\varphi =$

$$\begin{cases} \varphi = 0° \;\; N = 1.8kN \\ \varphi = 30° \;\; N = 1.56kN \\ \varphi = 60° \;\; N = 0.9kN \\ \varphi = 90° \;\; N = 0kN \end{cases}$$

$T = By\sin\varphi =$

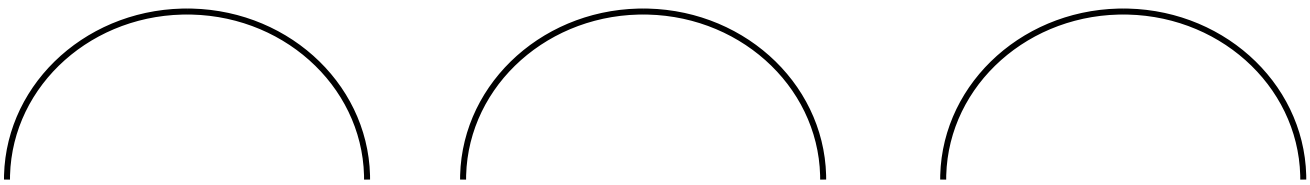
$$\begin{cases} \varphi = 0° \;\; T = 0kN \\ \varphi = 30° \;\; T = 0.9kN \\ \varphi = 60° \;\; T = 1.56kN \\ \varphi = 90° \;\; T = 1.8kN \end{cases}$$

$M = -By \cdot R(1-\cos\varphi) =$

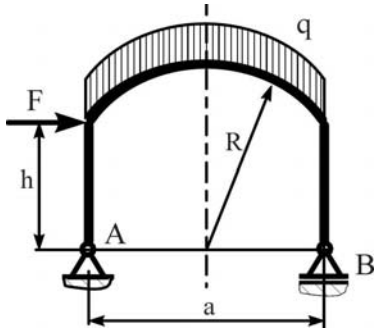
$$\begin{cases} \varphi = 0° \;\; M = 0kNm \\ \varphi = 30° \;\; M = -0.25kNm \\ \varphi = 60° \;\; M = -0.92kNm \\ \varphi = 90° \;\; M = -1.83kNm \end{cases}$$

Diagrami notranjih sil in momentov:

N: **T:** **M:**



Naloga 7.2:
Določite N, T, M diagrame in določite mesto ter velikost $M_{max,}$ če je F=10kN, q=2kN/m, R=8m, a=3m.



$$h = \sqrt{R^2 - \frac{a^2}{4}} = 7{,}86m$$

$$\cos\varphi_1 = \frac{a}{2R} \Rightarrow \varphi_1 = 79.2°$$

$$Q = 3q = 3 \cdot 2kN = 6kN$$
$$Ax = F = 10kN$$
$$Ay = Q - By = 6kN - 29.2kN = -23.2kN$$
$$By = \frac{1{,}5m \cdot Q + hF}{3} = \frac{1{,}5m \cdot 6kN + 7{,}86m \cdot 10kN}{3} = 29.2kN$$

I. polje

$$N = -Ay = 23.2kN$$
$$T = Ax = 10kN$$
$$M = xAx = \begin{cases} x = 0m; M = 0kNm \\ x = 3.93m; M = 39.3kNm \\ x = 7.86m; M = 78.6kNm \end{cases}$$

II. polje

$$N = -By = -29.2kN$$
$$T = 0kN$$
$$M = 0kNm$$

III. polje

φ = 79.2°

φ = 84.6°

φ = 90.0°

φ = 95.4°

φ = 100.8°

$$N = Q \cdot \sin(90 - \varphi) - F \cdot \cos(90 - \varphi) = R \cdot \sin(\varphi - \varphi_1) \cdot q \cdot \sin(90 - \varphi) - F \cdot \cos(90 - \varphi) =$$

$$\begin{cases} \varphi = 79.2° & N = -9.82kN \\ \varphi = 84.6° & N = -10.10kN \\ \varphi = 90.0° & N = -10kN \\ \varphi = 95.4° & N = -9.54kN \\ \varphi = 100.8° & N = -8.72kN \end{cases}$$

$$T = -Q \cdot \cos(90 - \varphi) - F \cdot \sin(90 - \varphi) = -R \cdot \sin(\varphi - \varphi_1) \cdot q \cdot \cos(90 - \varphi) - F \cdot \sin(90 - \varphi) =$$

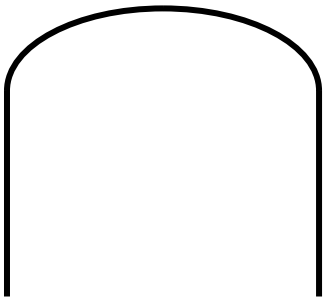
$$\begin{cases} \varphi = 79.2° & T = -1.87kN \\ \varphi = 84.6° & T = -2.44kN \\ \varphi = 90.0° & T = -3.00kN \\ \varphi = 95.4° & T = -3.50kN \\ \varphi = 100.8° & T = -3.91kN \end{cases}$$

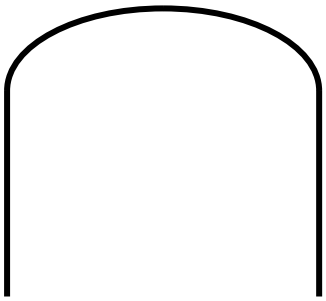
$$M = -Q \cdot \frac{R \cdot \sin(\varphi - \varphi_1)}{2} + AyR\sin(\varphi - \varphi_1) + Ax(h + R(1 - \cos(\varphi - \varphi_1))) - F \cdot R(1 - \cos(\varphi - \varphi_1)) =$$

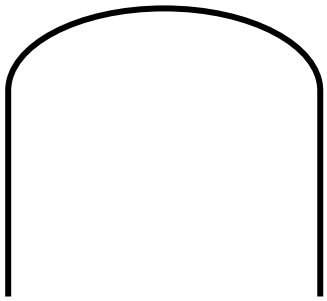
$$\begin{cases} \varphi = 79.2° & M = 78.6kNm \\ \varphi = 84.6° & M = 60.6kNm \\ \varphi = 90.0° & M = 41.6kNm \\ \varphi = 95.4° & M = 21.8kNm \\ \varphi = 100.8° & M = 1.6kNm \end{cases}$$

Diagrami notranjih sil in momentov:

N: **T:** **M:**

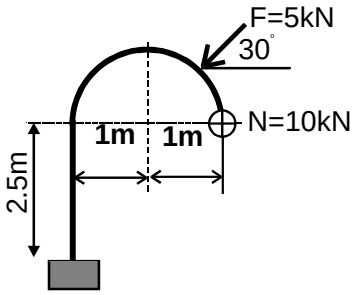




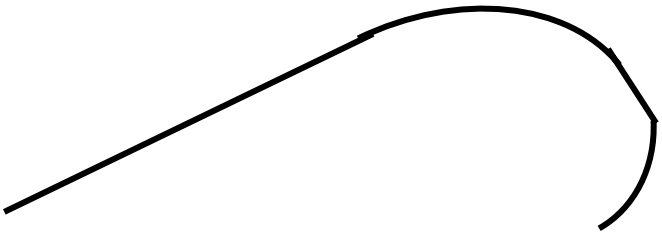


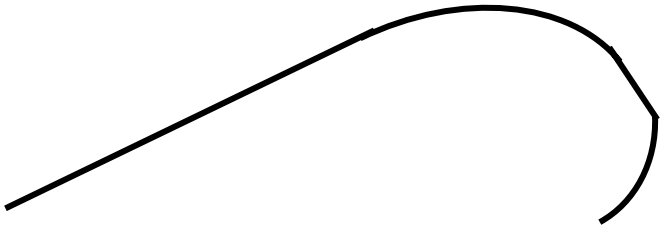
Naloga 7.3:

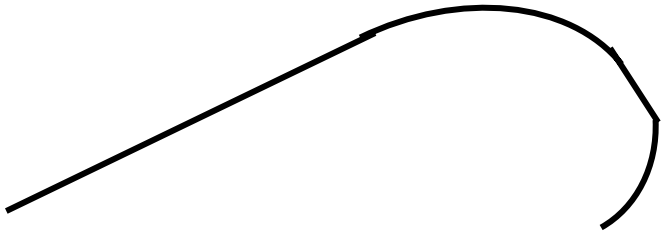
Določite N, T, M diagrame in določite mesto ter velikost M_{max}.

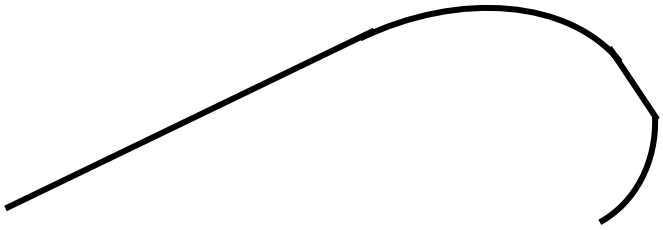


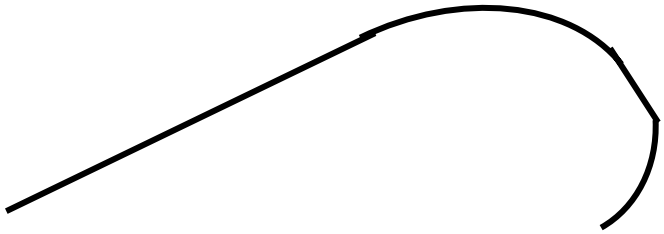
Diagrami notranjih sil in momentov:

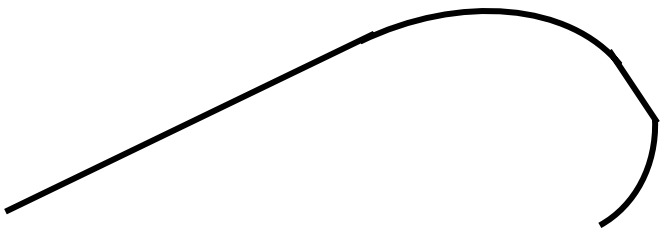












DOMAČA NALOGA

pri predmetu Statika in Kinematika

8

Domača naloga zajema vaje iz področij:
Paličja in mešani sistemi

Študent: Boštjan Kreutz

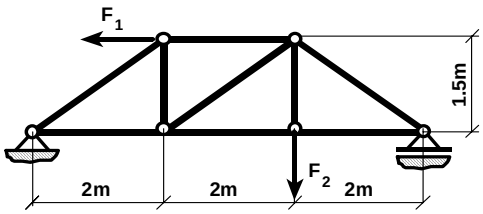
Predavatelj: Igor Emri

Asistent: Robert Cvelbar

Ljubljana, 18.12.2001

Naloga 8.1:

F_1=5kN, F_2=10kN. Analitično določite sile v palicah.



$$Ax = F_1 = 5kN$$

$$6By = -1{,}5F_1 + 4F_2 \Rightarrow By = \frac{4F_2 - 1{,}5F_1}{6} = 5{,}4kN$$

$$Ay = F_2 - By = 4{,}6kN$$

$$tg\alpha = \frac{1{,}5}{2} \Rightarrow \alpha = 36{,}9°$$

$$\sum F_{iX} = 0 : Ax + N_1 \cos\alpha + N_2 = 0$$
$$\sum F_{iY} = 0 : Ay + N_1 \sin\alpha = 0$$

$$N_1 = -\frac{Ax}{\sin\alpha} = -7{,}66kN$$
$$N_2 = -Ax - N_1 \cos\alpha = 1{,}13kN$$

$$\sum F_{iX} = 0 : -N_4 - N_5 \cos\alpha + N_2 = 0$$
$$\sum F_{iY} = 0 : By + N_5 \sin\alpha = 0$$

$$N_5 = -\frac{By}{\sin\alpha} = -9{,}00kN$$
$$N_4 = -N_5 \cos\alpha = 7{,}28kN$$

$$\sum F_{iX} = 0 : N_6 - F_1 - N_1 \cos\alpha = 0$$
$$\sum F_{iY} = 0 : -N_7 - N_1 \sin\alpha = 0$$

$$N_7 = -N_1 \sin\alpha = 4{,}60kN$$
$$N_6 = F_1 + N_1 \cos\alpha = -1{,}13kN$$

$$\sum F_{iX} = 0 : N_5 \cos\alpha - N_8 \cos\alpha - N_6 = 0$$
$$\sum F_{iY} = 0 : -N_9 - N_5 \sin\alpha - N_8 \sin\alpha = 0$$

$$N_8 = \frac{N_5 \cos\alpha - N_6}{\cos\alpha} = -7{,}59kN$$
$$N_2 = -N_5 \sin\alpha - N_8 \sin\alpha = 9{,}96kN$$

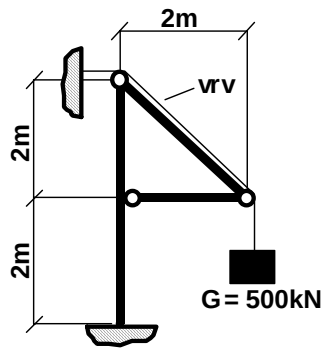
$$\sum F_{iX} = 0 : N_4 - N_3 = 0$$
$$\sum F_{iY} = 0 : N_9 - F_2 = 0$$

$$N_3 = N_4 = 7{,}28kN$$
$$N_9 = F_2 \cong 10kN$$

IZPIS REZULTATOV:

N	[kN]
1	-7,66
2	+1,13
3	+7,28
4	+7,28
5	-9,00
6	-1,13
7	+4,60
8	-7,59
9	+9,96

Naloga 8.2:
Rešite mešani sistem - narišite NTM diagrame za nosilec.



$$tg\alpha = \frac{2}{2} = 1$$
$$\alpha = 45°$$

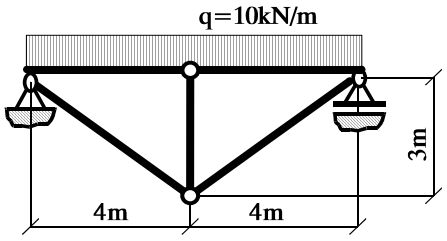
$$\sum F_{iX} = 0 : -N_1 \cos\alpha - G\cos\alpha = 0$$
$$\sum F_{iY} = 0 : G\sin\alpha - G + N_2 \sin\alpha = 0$$

$$N_2 = \frac{G - G\sin\alpha}{\sin\alpha} = 207{,}12kN$$
$$N_2 = -N_2 \cos\alpha - G\cos\alpha = -500{,}01kN$$

N	T	M

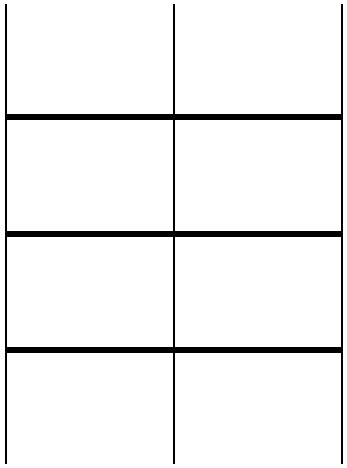
Naloga 8.3:

Rešite mešani sistem narišite NTM diagrame za nosilec.



$$Ax = 0$$
$$Ay + By = 10 \cdot 8 = 80kN$$
$$8By = q \cdot 8 \cdot 4 = 320kN$$
$$By = 40kN$$
$$Ay = 80 - By = 40kN$$

$$tg\alpha = \frac{3}{4}$$
$$\alpha = 36{,}9°$$



I. II.

III.

I.

$$M_{iC} = 0 : -4Ay + 4N_2 \sin\alpha + q \cdot 4 \cdot 2 = 0 \Rightarrow N_2 = \frac{4Ay - 4 \cdot 2 \cdot q}{4\sin\alpha} = 33{,}3kN$$

II.

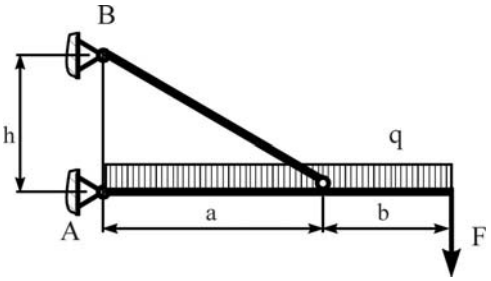
$$M_{iC} = 0 : -4By - 4N_3 \sin\alpha - q \cdot 4 \cdot 2 = 0 \Rightarrow N_3 = \frac{4By - 4 \cdot 2 \cdot q}{4\sin\alpha} = 33{,}3kN$$

III.

$$\sum F_{iX} = 0 : N_3 \cos\alpha = N_2 \cos\alpha \Rightarrow N_2 = N_3 = 33{,}3kN$$
$$\sum F_{iY} = 0 : N_1 + N_3 \sin\alpha + N_2 \sin\alpha = 0 \Rightarrow N_1 = -\sin\alpha \cdot (N_2 + N_3) = -40{,}0kN$$

Naloga 8.4:

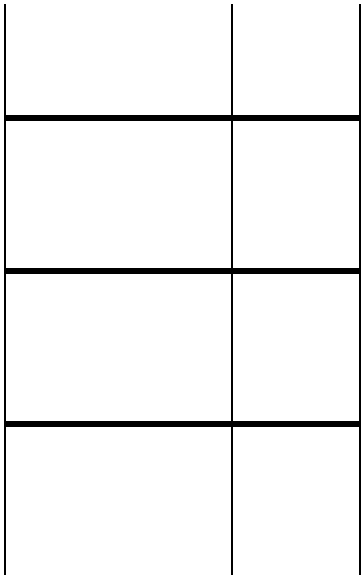
F=5kN, q=2kN/m, a=4m, h=3m in b=2m. Rešite mešani sistem – narišite NTM diagrame za nosilec.



$$\sum F_{iX} = 0 : Ax - By = 0$$
$$\sum F_{iY} = 0 : Ay + By - 6q - F = 0$$
$$\sum M_{i(B)} = 0 : 3Ax - q \cdot 6 \cdot 3 - 6F = 0$$

$$tg\alpha = \frac{3}{4}$$
$$\alpha = 36{,}9 \circ$$

$$Ax = \frac{6F + 6q \cdot 3}{3} = 22kN$$
$$By = Ax = 22kN$$



$$\sum F_{iX} = 0 : Ax - N_1 \cos\alpha = 0$$
$$\sum F_{iY} = 0 : Ay - F + N_1 \sin\alpha - 6q = 0$$

$$N_1 = \frac{Ax}{\cos\alpha} = 27{,}5kN$$
$$Ay = F + 6q - N_1 \cos\alpha = 0{,}5kN$$

Naloga 8.5:
Kako pridemo do enačbe za statično določenost ravninskega paličja?

Število neznank mora biti enako številu enačb →

št.enačb = 2x št.vozlišč
št.neznank = št.palic + reakcije v podporah

Naloga 8.6:

Kako pridemo do enačbe za preveranje statične določenosti 3D paličja?

Število neznank mora biti enako številu enačb →

št.enačb = 3x št.vozlišč
št.neznank = št.palic + reakcije v podporah

Naloga 8.7:

Kakšne predpostavke veljajo pri obravnavanju paličja?

- napetost zaradi dveh sil je mnogo večja, kot napetost zaradi momenta ali prečne sile
- palice morajo biti členkasto pritrjene
- prečnih sil ne sme biti
- palične konstrukcije so lahko obremenjene samo v členkih (vozliščih)
- sistem mora biti statično določen

DOMAČA NALOGA

pri predmetu Statika in Kinematika

9

Domača naloga zajema vaje iz področij:
Vrvi: točkovno in zvezno obremenjene

Študent: Boštjan Kreutz

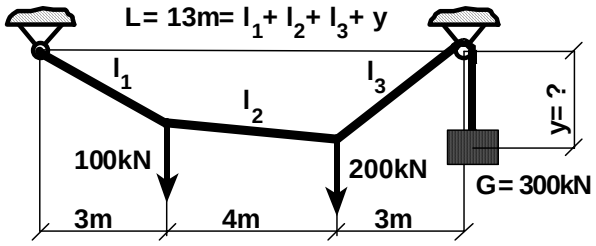
Predavatelj: Igor Emri

Asistent: Robert Cvelbar

Ljubljana, 08.01.2002

Naloga 9.1:

Določite sile v podporah, S_{MAX}, y kjer je dolžina vrvi L=13m=l1+l2+l3+y.



Reakcije v podporah:

$$\sum M_{i(B)} = 0 : -10Ay + 7F_1 + 3F_2 = 0 \qquad \Rightarrow Ay = \frac{7F_1 + 3F_2}{10} = 130kN$$

$$\sum Fix = 0 : Bx - Ax = 0 \qquad \Rightarrow Bx = Ax = H$$

$$\sum Fiy = 0 : Ay + By - F_1 - F_2 - G = 0 \qquad \Rightarrow By = F_1 + F_2 + G - Ay = 470kN$$

Režemo:

$$G = N_3 = 300kN$$

$$By = G + G\sin\alpha_3 \Rightarrow \alpha_3 = 34{,}5°$$

$$Bx = G\cos\alpha_3 = H = 247{,}2kN$$

$$\cos\alpha_3 = \frac{3}{l_3} \Rightarrow l_3 = \frac{3}{\cos\alpha_3} = 3{,}64m$$

$$N_1 = \sqrt{Ax^2 + Ay^2} = 279{,}33kN$$

$$Ax = N_1\cos\alpha_1 \Rightarrow \alpha_1 = 27{,}75°$$

$$\cos\alpha_1 = \frac{3}{l_1} \Rightarrow l_1 = \frac{3}{\cos\alpha_1} = 3{,}39m$$

$$N_1\sin\alpha_1 = F_1 + N_2\sin\alpha_2$$

$$N_1\cos\alpha_1 = N_2\cos\alpha_2 \qquad \Rightarrow N_2 = \frac{N_1\cos\alpha_1}{\cos\alpha_2} = 246{,}33kN$$

$$\Downarrow$$

$$tg\alpha_2 = \frac{N_1\sin\alpha_1 - F_1}{N_1\cos\alpha_1} \Rightarrow \alpha_2 = 6{,}9°$$

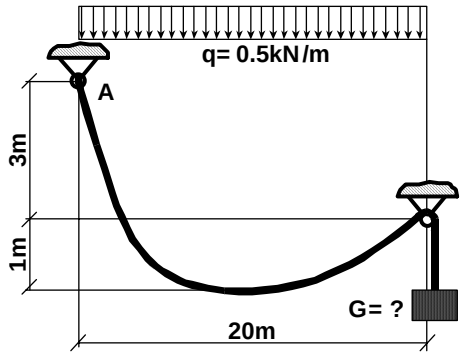
$$\cos\alpha_2 = \frac{4}{l_2} \Rightarrow l_2 = \frac{4}{\cos\alpha_2} = 4{,}03m$$

$$y = 13 - l_1 - l_2 - l_3$$

$$y = 1{,}94m$$

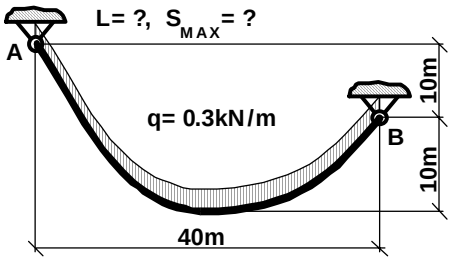
$$S_{\max} = G = 300kN$$

Naloga 9.2:
Določite sile v podporah, G in L_{AB}.



Naloga 9.3:

Določite sile v podporah, S_{MAX} in L.



Naloga 9.4:

Od česa je odvisno kdaj bomo za reševanje vrvi vzeli parabolično in kdaj hiperbolično rešitev? Na kaj je pri reševanju potrebno paziti?

Od povesa (pri velikem povesu vzamemo hiperbolično, pri manjšem pa parabolično rešitev). Pri reševanju je potrebno upoštevati vse predpostavke, ki smo veljajo za vrvi.

Naloga 9.5:

Kakšne predpostavke veljajo pri definiciji vrvi v mehaniki?

- vrvi so idealno gibke,
- prenašajo samo natezne obremenitve,
- predpostavimo, da je vrv toga.

DOMAČA NALOGA

pri predmetu Statika in Kinematika

10

Domača naloga zajema vaje iz področij:
Trenje: drsno trenje, kotalno trenje, trenje gibkih elementov

Študent: Boštjan Kreutz

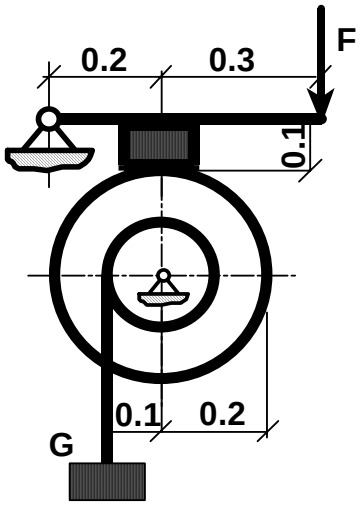
Predavatelj: Igor Emri

Asistent: Robert Cvelbar

Ljubljana, 01.02.2002

Naloga 10.1:

Na zavoro pritiskamo s silo F tako, da breme G ne gre navzdol. Če je koeficient trenja μ_0=0.3 in teža bremena G=3kN, določite potrebno velikost sile F.



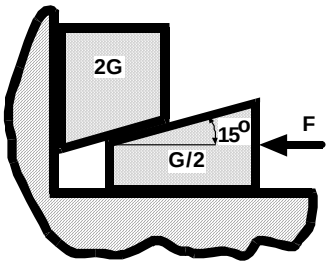
$\mu_0 = 0,3$
G = 3 kN

$$0,5m \cdot F = 0,2m \cdot N + 0,1m \cdot F_t$$
$$F = \frac{0,2m \cdot N + 0,1m \cdot F_t}{0,5m} = 2,3kN$$

$$0,2m \cdot F_t = 0,1m \cdot G$$
$$F_t = \frac{0,1m \cdot G}{0,2m} = 1,5kN$$
$$F_t = N \cdot \mu_0 \Rightarrow N = \frac{F_t}{\mu_0} = 5kN$$

Naloga 10.2:

Dvigamo breme 2G. Določite minimalno silo F. μ_0=0.1, α=15°.



$$\sum F_{iX} = 0 : N_1 - F_{t2}\cos 15 - N_2 \sin 15 = 0$$
$$\sum F_{iY} = 0 : N_2 \cos 15 - F_{t1} - 2G - F_{t2}\sin 15 = 0$$
$$\sum F_{iX} = 0 : F_{t3} + F_{t2}\cos 15 + N_2 \sin 15 - F = 0$$
$$\sum F_{iY} = 0 : N_3 + F_{t2}\sin 15 - N_2 \cos 15 - \frac{G}{2} = 0$$

$$\boxed{F_{ti} = \mu \cdot N_i}$$

$$N_1 - N_2(\mu\cos 15 + \sin 15) = 0$$
$$N_2(\cos 15 + \mu \sin 15) - N_1\mu - 2G = 0$$
$$N_3\mu + N_2(\mu\cos 15 + \sin 15) - F = 0$$
$$N_3 + N_2(\mu \sin 15 - \cos 15) - \frac{G}{2} = 0$$

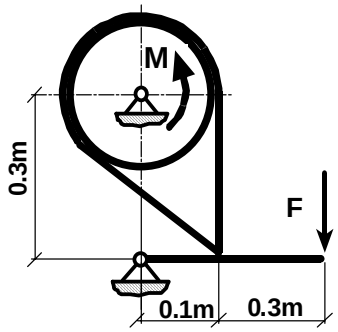
$$N_1 = N_2(\mu\cos 15 + \sin 15)$$
$$N_2(\cos 15 + \mu \sin 15) - N_2(\mu\cos 15 + \sin 15)\mu - 2G = 0$$
$$N_2 = \frac{2G}{(\cos 15 - \mu \sin 15) - (\mu\cos 15 + \sin 15)\mu} = 2{,}1G$$

$$N_3 = \frac{G}{2} - N_2(\mu \sin 15 + \cos 15)$$
$$N_3 = \frac{G}{2} - 2{,}1G(\mu \sin 15 + \cos 15) = 2{,}5G$$

$$F = N_3 + N_2(\mu\cos 15 + \sin 15) = 3{,}25G$$

Naloga 10.3:

Na zavoro pritiskamo s silo F tako, da se boben, ki je gnan z momentom M=200Nm ne vrti. Koeficient tranja med bobnom in trakom je μ_0=0.25. Določite potrebno velikost sile F.



$$tg\alpha = \frac{0,1}{0,3} \Rightarrow \alpha = 18,4° \qquad \hat{\alpha} = 0,32$$

$$S_2 = S_1 \cdot e^{\mu\hat{\alpha}}$$

$$\sum M_i = 0: \; S_1 R + M = S_2 R = S_1 \cdot e^{\mu\hat{\alpha}} R$$

$$M = S_1 R(e^{\mu\hat{\alpha}} - 1)$$

$$S_1 = \frac{M}{R(e^{\mu\hat{\alpha}} - 1)} = 24kN$$

$$S_2 = S_1 + \frac{M}{R} = 26kN$$

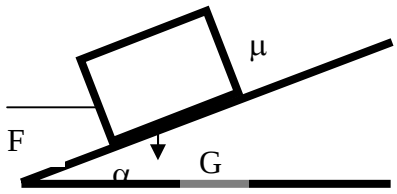
$$0,4F = 0,1S_2 - 0,1S_1 \cdot \sin(90 - 2\alpha)$$

$$\Downarrow$$

$$F = \frac{0,1S_2 - 0,1S_1 \cdot \sin(90 - 2\alpha)}{0,4m} = 1,7kN$$

Naloga 10.4:

Od česa je za dani primer odvisna sila trenja? Izrazite jo!



$$\sum F_{iT} = 0: \; F\cos\alpha - Ft - G\sin\alpha = 0$$
$$\sum F_{iN} = 0: \; N = G\cos\alpha + F\sin\alpha$$

$$Ft = \mu(G\cos\alpha + F\sin\alpha)$$

Sila trenja je odvisna od koeficienta trenja μ, sile teže G, sile F ter kota naklona α.

Naloga 10.5:

Ali pri kotalnem trenju upoštevamo drsno trenje. Če ga, zakaj?

Pri kotalnem trenju ne upoštevamo drsnega trenja.

Naloga 10.6:

Sila trenja deluje v smeri gibanja: DA (NE)

Naloga 10.7:

Od česa je odvisno kotalno trenje?

Kolo (kolut, krogla, valj), obremenjeno s silo (npr. težo) G, se kotali po ravnem kotališču zaradi nanj delujoče kotalne sile F oz. kotalnega momenta M. Prijemališče (proti središču kolesa usmerjene) odporne sile R je pomaknjeno za krak *f* pred kolo.

Naloga 10.8:

Od česa je odvisno trenje vrvi na kolutu? Izrazite velikost sile na enem koncu vrvi, če poznate velikost sile v drugem koncu vrvi.

Trenje na vrvenici je odvisno od sile F, koeficienta trenja μ ter kota objema α.

$$F_{t\max} = F - F_0 = \frac{F(e^{\mu\bar{\alpha}} - 1)}{e^{\mu\bar{\alpha}}} \Rightarrow F_0 = F - \frac{F(e^{\mu\bar{\alpha}} - 1)}{e^{\mu\bar{\alpha}}}$$

DOMAČA NALOGA

pri predmetu Statika in Kinematika

11

Domača naloga zajema vaje iz področij:
Kinematika točke: koordinatni sistemi, opisovanje in delitev gibanj, hitrost in pospešek, ravninsko gibanje točke, polarne koordinate, kroženje točke, harmonično gibanje, premočrtno gibanje točke

Študent: Boštjan Kreutz

Predavatelj: Igor Emri

Asistent: Robert Cvelbar

Ljubljana, 07.01.2002

Naloga 11.1:

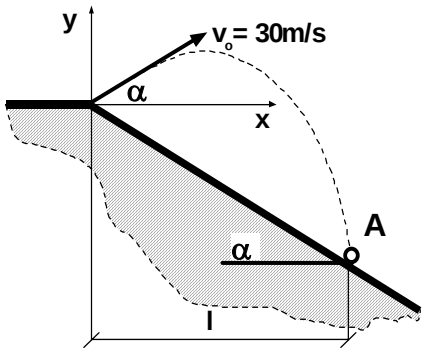
V naslednjih nalogah določite $\vec{v}$, $\vec{a}$, a_n, a_t, in tir

a . $x=5\cos t$ *(m),* $y=3-5\sin t$ *(m)*

b. $\rho=0.3t$ *(m),* $\varphi=0.5\pi t$

Naloga 11.2:

Določite *l, vektorja hitrosti in pospeška, a_n in a_t* v točki A, če je $\alpha=30^o$.



Naloga 11.3:

a=-kv, k=0.1 (s^{-1}) t=0: v_0=50m/s, s=0. Določite čas v katerem je hitrost 20m/s in pot, ki jo opravi točka v tem času.

Naloga 11.4:

Opišite fizikalni pomen normalnega pospeška.

Naloga 11.5:

Opišite fizikalni pomen tangencialnega pospeška.

Naloga 11.6:

Opišite fizikalni pomen radialnega pospeška.

Naloga 11.7:

Če hitrost točke konstantno narašča, potem je pospešek_____

Naloga 11.8:

Kako pridemo do v, a, a_t, a_n, ρ, če poznate x(t) in y(t)?

DOMAČA NALOGA

pri predmetu Statika in Kinematika

12

Domača naloga zajema vaje iz področij:
Kinematika togega telesa: prostostne stopnje togega telesa in osnovni premiki, trenutno gibalno stanje in popis vrste gibanj, splošno gibanje togega telesa

Študent: Boštjan Kreutz

Predavatelj: Igor Emri

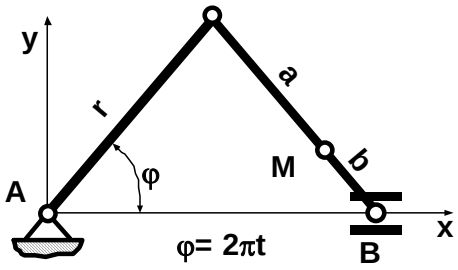
Asistent: Robert Cvelbar

Ljubljana, 14.01.2002

Naloga 12.1:

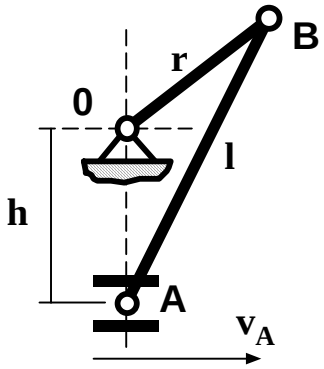
r=30 cm, a=10 cm, b=20 cm. Določite tir, po katerem se giblje točka M ter

$\vec{v}$, $\vec{a}$, a_n, a_t, in tir točke M.



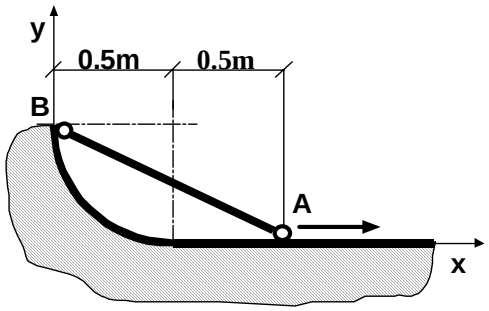
Naloga 12.2:

v_A=3m/s, r=0.5m, h=0.4m, l=0.8m. Določite v_B, a_B in ω_{0B}.



Naloga 12.3:

$x_A=(1+0.5t^2)$ m. Določite ω_{AB} in α_{AB} za čas t=0.



DOMAČA NALOGA

pri predmetu Statika in Kinematika

13

Domača naloga zajema vaje iz področij:
Sistemsko gibanje, relativno gibanje, absolutno in sestavljeno gibanje
ravninsko gibanje togega telesa, gibanje telesa okoli stalne točke

Študent: Boštjan Kreutz

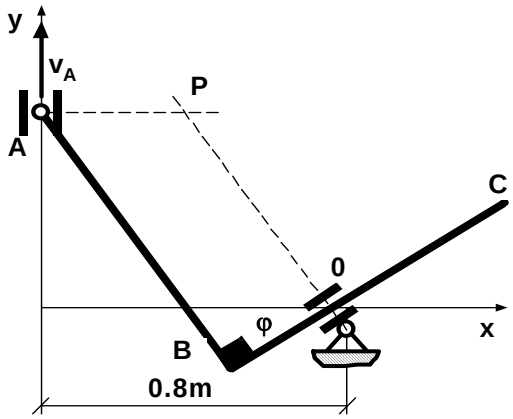
Predavatelj: Igor Emri

Asistent: Robert Cvelbar

Ljubljana, 14.01.2002

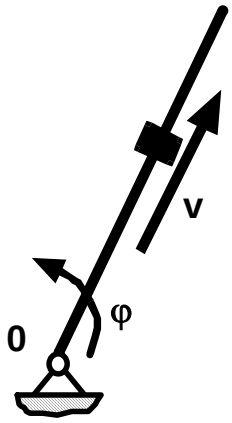
Naloga 13.1:

Palice AB in BC sta v B togo spojeni pod kotom $90^{o.}$ AB=BC=1m, v_A=2m/s, $\varphi=20^o$. Določite pol hitrosti in hitrosti točk B,C in 0.



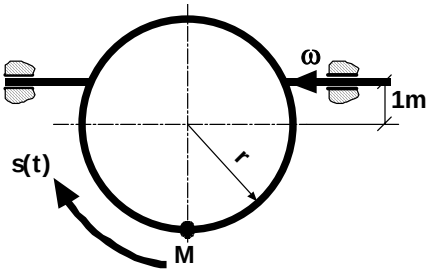
Naloga 13.2:

Drog prične gibanje iz horizontale z $\varphi=2t^2$, hkrati pa drsnik prične gibanje po drogu iz točke 0 po zakonu $\vec{v} = 5t\,[m/s]$. Določite absolutno hitrost in pospešek drsnika po času t=2s.



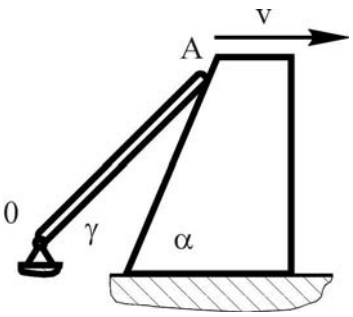
Naloga 13.3:

Določite absolutno hitrost in pospešek točke M po 1s gibanja. M prične gibanje iz narisane lege po zakonu s(t), poleg tega se plošča še vrti z ω. r=3m, $s(t)=(3\pi/8)(t^3+t^2)$, $\omega=2s^{-1}$.



Naloga 13.4:

Klada, katere strmina je pod kotom $\alpha = 75^o$ se giba s konstantno hitrostjo *v=5cm/s*. V danem trenutku je drog 0A=30cm pod kotom $\gamma = 50^o$. Določite hitrost gibanja točke A na drogu in kotno hitrost ter kotni pospešek palice.



Naloga 13.5:

Opišite fizikalni pomen Coriollisovega pospeška.

Naloga 13.6:

Naskicirajte s(t) in a(t), če je podana v(t)!