$e^{i\varphi}\cdot e^{i\varphi}=1$

$\vec r = r\cdot e^{i\varphi}$

HITROST

$$\vec v = \frac{d}{dt}(r\cdot e^{i\varphi}) = \dot r e^{i\varphi} + r i\dot\varphi e^{i\varphi}$$

POSPEŠEK

$$\vec a = \frac{dv}{dt} = \frac{d}{dt}\left(\dot r e^{i\varphi} + r i\dot\varphi e^{i\varphi}\right) =$$

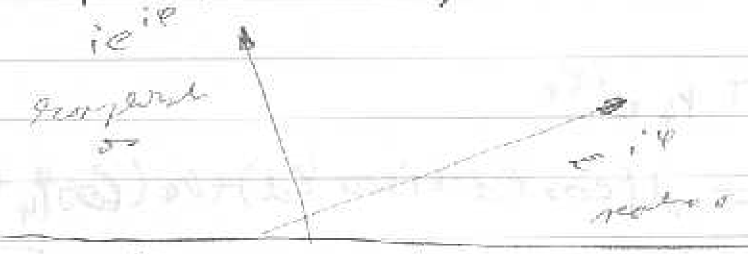
$$= \ddot r e^{i\varphi} + \dot r i\dot\varphi e^{i\varphi} + \dot r i\dot\varphi e^{i\varphi} + r i\ddot\varphi e^{i\varphi} + r i\dot\varphi i\dot\varphi =$$

$$= e^{i\varphi}(\ddot r - r\dot\varphi^2) + ie^{i\varphi}(2\dot r\dot\varphi + r\ddot\varphi)$$

$$\vec a = \vec a_r + \vec a_c$$

$$\vec a_r = (\ddot r - r\dot\varphi)e^{i\varphi}$$

$$\vec a_k = (2\dot r\dot\varphi + r\ddot\varphi)ie^{i\varphi}$$



REŠEVANJE NALOGE

Krajevni vektor B opisuje naslednja enačba

$$\vec r_B = r_2 e^{i\varphi_2} + r_3 e^{i\varphi_3} = \vec r_1 + r_4 e^{i\varphi_4}$$

- Diagonala d

$$\vec r_1 = r_2 e^{i\varphi_2} + d e^{i\varphi_d} \quad / \ \bar e^{i\varphi}$$

$$de^{i\varphi_d} = \vec r_1 - \vec r_2 e^{i\varphi_2}$$

$$(de^{i\varphi_d})(de^{-i\varphi_d}) = (\vec r_1 - r_2 e^{i\varphi_2})(\vec r_1 - r_2 e^{-i\varphi_2})$$

$$d = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\varphi_2}$$

3

• Iščemo φ_d

Enačbo napišemo posebej za realno in kompleksni del

$$r_2 e^{i\varphi_2} = r_1 - d e^{i\varphi_d}$$

$$\vec{r}_2 (\cos\varphi_2 + i\sin\varphi_2) = \vec{r}_1 - d(\cos\varphi_d + i\sin\varphi_d)$$

$$r_2 \cdot \cos\varphi_2 = \vec{r}_1 - d\cos\varphi_d \quad \text{realni del} \Rightarrow \cos\varphi_d$$

$$r_2 \cdot \sin\varphi_2 = 0 - d\cdot\sin\varphi_d \quad \text{kompleksni del} \Rightarrow \sin\varphi_d$$

$$\left.\begin{aligned} \cos\varphi_d &= -\frac{r_2\cos\varphi_2 - r_1}{d} \\ \sin\varphi_d &= -\frac{r_2 \sin\varphi_2}{r_2\cos\varphi_2 - \vec{r}_1} \end{aligned}\right\} \quad tg\,\varphi_d = \frac{r_2 \sin\varphi_2}{r_2 \cdot \cos\varphi_2 - \vec{r}_1}$$

Iščemo φ_3

$$r_3 e^{i\varphi_3} = d e^{i\varphi_d} + r_4 e^{i\varphi_4}$$

$$r_3 (\cos\varphi + i\sin\varphi_3) = d(\cos\varphi_d + i\sin\varphi_d) + r_4(\cos\varphi_4 + i\sin\varphi_4)$$

$$r_3 \cos\varphi_3 = d\cdot\cos\varphi_d + r_4\cdot\cos\varphi_4 \quad \text{realni del } \overline{AB} = AB$$

$$r_3 \sin\varphi_3 = d\sin\varphi_d + r_4\sin\varphi_4 \quad \text{kompleksni del } \overline{BD} = BD$$

$$r_3 \sin\varphi_3 - d\sin\varphi_d = r_4 \sin\varphi_4 \quad /^2$$

$$r_3^2 \sin^2\varphi_3 - 2 r_3 d \sin\varphi_3 \sin\varphi_d + d^2 \sin^2\varphi_d = r_4^2 \sin^2\varphi_4$$

$$r_3 \cos\varphi_3 - d\cos\varphi_d = r_4 \cos\varphi_4 \quad /^2$$

$$r_3^2 \cos^2\varphi_3 - 2 r_3 d\cos\varphi_3 \cdot \cos\varphi_d + d^2 \cos^2\varphi_d = r_4^2 \cos^2\varphi_4$$

Enačbi seštejemo

$$r_3^2 + d^2 - 2r_3 d \sin\varphi_3 \sin\varphi_d - 2r_3 d \cos\varphi_3 \cdot \cos\varphi_d = r_4^2$$

$$r_3^2 + d^2 - r_4^2 = 2r_3 d \cos(\varphi_3 - \varphi_d)$$

$$\cos(\varphi_3 - \varphi_d) = \frac{r_3^2 + d^2 - r_4^2}{2r_3 d}$$

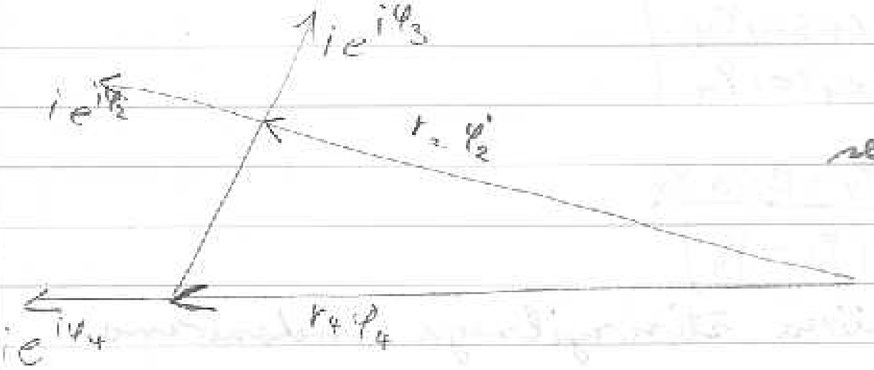
HITROSTNE RAZMERE ŠTIRIZGIBNEGA MEHANIZMA

$$\frac{d}{dt}\left(r e^{i\varphi}\right) = \dot r e^{i\varphi} + r i \dot\varphi e^{i\varphi}$$

Dobimo hitrost gibajoče točke glede na stalno referenčno točko.

Ker so elementi mehanizma toga telesa, je prvi odvod v radialni smeri 0.

$$r_2 \dot\varphi_2 i e^{i\varphi_2} + r_3 \dot\varphi_3 i e^{i\varphi} = r_3 \dot\varphi_4 i e^{i\varphi}$$



Trikotnik relativnih hitrosti

$$ie^{i\varphi} = -\sin\varphi + i\sin\varphi = i\cos\varphi - \sin\varphi$$

$$r_2\dot\varphi_2 i(\cos\varphi_2 + i\sin\varphi_2) + r_3\dot\varphi_3 i(\cos\varphi_3 + i\sin\varphi_3) = r_4\dot\varphi_4 i(\cos\varphi_4 + i\sin\varphi_4)$$

$-r_2\dot\varphi_2 \sin\varphi_2 - r_3\dot\varphi_3 \sin\varphi_3 = -r_4\dot\varphi_4 \sin\varphi_4$ imaginarni del

$r_2\dot\varphi_2 \cos\varphi_2 + r_3\dot\varphi_3 \cos\varphi_3 = r_4\dot\varphi_4 \cos\varphi_4$ realni del

$$\dot\varphi_3 = \frac{\begin{vmatrix} r_4\sin\varphi_4 & -r_2\sin\varphi_2 \\ -r_4\cos\varphi_4 & r_2\cos\varphi_2 \end{vmatrix}\dot\varphi_2}{\begin{vmatrix} r_4\sin\varphi_4 & -r_3\sin\varphi_3 \\ -r_4\cos\varphi_4 & r_3\cos\varphi_3 \end{vmatrix}}$$

$$\dot\varphi_3 = \dot\varphi_2 \frac{r_4 r_2}{r_3 r_4} \frac{\sin(\varphi_4 - \varphi_2)}{\sin(\varphi_4 - \varphi_3)}$$

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Pospeškovne razmere štirizgibnega mehanizma

$$r_2\ddot{\varphi}_2\left(ie^{i\varphi_2}\right)+r_2\dot{\varphi}_2^2\left(-e^{i\varphi_3}\right)+r_3\ddot{\varphi}_3\left(ie^{i\varphi_3}\right)+$$
$$+r_3\dot{\varphi}_3^2\left(-e^{i\varphi_3}\right)=r_4\ddot{\varphi}_4\left(ie^{i\varphi_4}\right)+r_4\dot{\varphi}_4^2\left(-e^{i\varphi_4}\right)$$

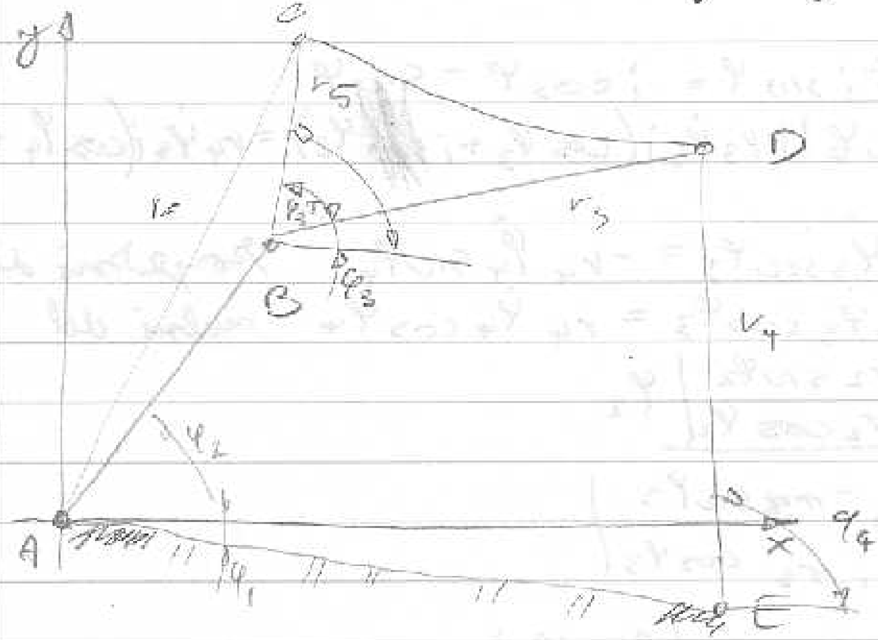
$$\ddot{\varphi}_3\left(-r_3\sin\varphi_3\right)+\ddot{\varphi}_4\left(r_4\sin\varphi_4\right)=-r_2\ddot{\varphi}_2\sin\varphi_2+r_2\dot{\varphi}_2^2\cos\varphi_2+$$
$$+r_3\dot{\varphi}_3^2\cos\varphi_3-r_4\dot{\varphi}_4^2\cos\varphi_4$$ imaginarni del „A"

$$\ddot{\varphi}_3\left(r_3\cos\varphi_3\right)+\ddot{\varphi}_4\left(-r_4\cos\varphi_4\right)=-r_2\ddot{\varphi}_2\cos\varphi_2+$$
$$+r_2\dot{\varphi}_2^2\sin\varphi_2+r_3\dot{\varphi}_3^2\sin\varphi_3-r_4\dot{\varphi}_4^2\sin\varphi_4$$ realni del „B"

$$\ddot{\varphi}_3=\frac{\begin{vmatrix} A & r_4\sin\varphi_4 \\ B & -r_4\cos\varphi_4 \end{vmatrix}}{\begin{vmatrix} -r_3\sin\varphi_3 & r_4\sin\varphi_4 \\ r_3\cos\varphi_3 & -r_4\cos\varphi_4 \end{vmatrix}}$$

$$\ddot{\varphi}_3=-\frac{1}{r_3}\cdot\frac{A\cos\varphi_4+B\sin\varphi_4}{\sin\left(\varphi_3-\varphi_4\right)}$$

Dinamična analiza štirizgibnega mehanizma



Podano: $\varphi_1, \varphi_2, r_1, r_2, r_3, r_4, r_9, r_8 = r_2 + r_5$
Določimo $\varphi_3, \varphi_4, \varphi_5 = \varphi_3 + \varphi_{3T}$

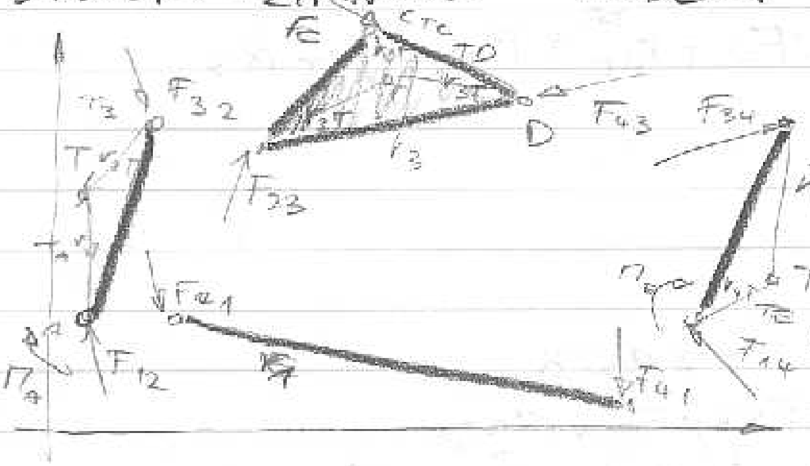
NEWTON – EULERJEVA METODA

$\sum_j F_j = m_i a_i$ ravnotežje sil

$\sum_j r_{ij}^{\,j i} \times F_j + \sum M_k = J_{iT}\,\alpha_i$ ravnotežje momentov

Ravnotežna pogoja se nanašata na težišče telesa.

DISKRITIZIRANJE MODELA



Ročica 2

Obremenitve: $M_A, F_{12}, F_{32}, F_{g2}$

Vektorska oblika

$F_{12} + F_{32} + F_{g2} = m_2\, a_{2TX}$

$F_{32} = -F_{23}$ $\quad F_{g2} = m_2 g$

Momentna enačba

$r_{2T}^{TA} \times F_{12} + r_{2T}^{TB} \times F_{32} + M_A = J_{2T}\,\alpha_2$

Skalarna oblika

$F_{12X} - F_{32X} = m_2\, a_{2TX}$

$F_{12y} - F_{23y} - m_2 g = m_2 a_{2Ty}$

$$\left(r_{2Tx}^{TA} F_{12y} - r_{2Ty}^{TA} F_{12x}\right) - \left(r_{2Tx}^{TB} F_{23y} - r_{2Ty}^{TB} F_{23x}\right) + M_A = J_{2T} \cdot \ddot{\varphi}_2$$

ROČICA 3

Uvrednitve: $F_{23}, F_{43}, F_C, F_{g3}$

Vektorska oblika

$F_{23} + F_{43} + F_C + F_{g3} = m_3 a_{3T}$

$r_{3T}^{TB} \times F_{23} + r_{3T}^{TC} \times F_C + r_{3T}^{TD} \times F_{43} = J_{3T} \alpha_3$

$F_{43} = -F_{34}$

$F_{3g} = m_3 g$

Skalarna oblika

$F_{23x} - F_{Cx} + F_{34x} = m_3 a_{3Tx}$

$F_{23y} - F_{Cy} + m_3 g - F_{34y} = m_3 a_{3Ty}$

$\left(r_{3Tx}^{TD} F_{34y} - r_{3Ty}^{TD} F_{34x}\right) = J_{3T} \ddot{\varphi}_3$

ROČICA 4

Uvrednitve $M_E, F_{14}, F_{34}, F_{g4}$

Vektorska oblika

$F_{14} + F_{34} + F_{g4} = m_4 a_{4T}$

$r_{4T}^{TE} \times F_{14} + r_{4T}^{TD} \times F_{34} + M_E = J_{4T} \alpha_4$

$F_{g4} = m_4 g \qquad F_{14} = -F_{41}$

Skalarna oblika

$-F_{41x} + F_{34x} = m_4 a_{4Tx}$

$-F_{41y} + F_{34y} - m_4 g = m_4 a_{4Ty}$

$$-\left(r_{4Tx}^{TE} F_{41y} - r_{4Ty}^{TE} F_{41x}\right) + \left(r_{4Tx}^{TD} F_{34y} - r_{4Ty}^{TD} F_{34x}\right) + M_E = J_{4T} \ddot{\varphi}_4$$

$$
\begin{bmatrix}
1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\
-r^{TA}_{2Ty} & r^{TA}_{2Tx} & r^{TB}_{2Ty} & -r^{TB}_{2Tx} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -r^{TC}_{3Ty} & r^{TC}_{3Tx} & r^{TD}_{3Ty} & -r^{TD}_{3Tx} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -r^{TD}_{4Ty} & r^{TD}_{4Tx} & r^{TE}_{4Ty} & r^{TE}_{4Tx} & 0
\end{bmatrix}
\begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{23x} \\ F_{23y} \\ F_{34x} \\ F_{34y} \\ F_{41x} \\ F_{41y} \\ M_E \end{bmatrix}
=
\begin{bmatrix}
m_2 a_{2Tx} \\
m_2 (a_{2Ty} + g) \\
m_3 a_{3Tx} - F_{3x} \\
m_3 (a_{3Ty} + g) - F_{cy} \\
m_4 a_{4Tx} \\
m_4 (a_{4Ty} + g) \\
J_{2T} \ddot{\varphi}_2 - M_A \\
J_{3T} \ddot{\varphi}_3 - (-r^{TC}_{3Ty} F_{cx}) - r^{TC}_{3Tx} F_{cy} \\
J_{4T} \ddot{\varphi}_4
\end{bmatrix}
$$

URAVNOTEŽENJE ROČIČNIH MEHANIZMOV

1.) Pri rotacijskem gibanju telesa ali sistema teles lahko v primeru ko se simetrijska os ne ujema s trenutno rotacijsko osjo pride do pojava notranjih sil, ki predstavljajo dodatne obremenitve, ki jih skušamo minimizirat

2.) Pri ravninskem gibanju pa se notranje sile pojavijo če težišče telesa ne sovpada z ~~rotacijsko~~ osjo rotacije saj se v tem primeru pojavi radialna ~~pospešek~~ sila pospeška težišča.

3.) Če so rotacijska telesa povezana v nek ~~sist~~ sistem rotirajočih teles potem pa vsako telo, katerega težišče ne sovpada z rotacijsko osjo rotacijskih teles spet prispeva k nastanku teh sil.

Težimo k temu da se skupno težišče ne premika (statično ravnotežni pogoj)

Za dva pogoja - vsota vseh sil mora biti 0 $\sum F_i = 0$

- vsota vseh momentov mora biti 0 $\sum_{i=1} M_i = 0$

(Dinamični ravnotežni pogoj)

Lega težišča rotirajočih se teles

$$r_{2T} = r_{2T}^{TA} e^{i(\varphi_2 + \varphi_{2T})}$$

$$r_{3T} = r_2 e^{i\varphi_2} + r_{3T}^{TB} e^{i(\varphi_3 + \varphi_{3T})}$$

$$r_{4T} = r_1 e^{i\varphi_1} + r_{4T}^{TE} e^{i(\varphi_4 + \varphi_{4T})}$$

Skupna masa štirizgibnega mehanizma

$$m_T = \sum_{i=2}^{4} m_i = m_2 + m_3 + m_4$$

Celotni masni moment okoli izhodišča nepomičnega koordinatnega sistema:

$$m_T r_T = \sum_{i=2}^{4} m_i r_{iT} = m_2 r_{2T} = m_2 r_{2T} + m_3 r_{3T} + m_4 r_{4T}$$

$$r_T = \frac{\sum_{i=2}^{4} m_i r_{iT}}{\sum_{i=2}^{4} m_i}$$

Lege težišča rotirajočih ročic vstavimo v enačbo celotnega masnega momenta tako, da ločimo časovno neodvisne koeficiente od časovno odvisnih kotov ročic

$$m_T r_T = (m_4 r_1 e^{i\varphi_1}) + (m_2 r_{2T}^{TA} e^{i\varphi_{2T}} + m_3 r_2) e^{i\varphi_2} + (m_3 r_{3T}^{TB} e^{i\varphi_{3T}}) e^{i\varphi_3} + (m_4 r_{4T}^{TE} e^{i\varphi_{4T}}) e^{i\varphi_4}$$

Iz razširjene vektorske zanke

$$r_2 e^{i\varphi_2} + r_3 e^{i\varphi_3} - r_4 e^{i\varphi_4} - r_1 e^{i\varphi_1} = 0$$

določimo φ_3 in ga eliminiramo iz enačbe *

$$m_T r_T = \left(m_2 r_{2T}^{TB} e^{i\varphi_{2T}} + m_2 r_2 - m_3 r_{3T}^{TB} \frac{r_2}{r_3} e^{i\varphi_{3T}} \right) e^{i\varphi_2} +$$
$$\left(m_4 r_4^{TB} e^{i\varphi_{4T}} + m_3 r_{3T}^{TB} \frac{r_4}{r_3} e^{i\varphi_{3T}} \right) e^{i\varphi_4} +$$
$$+ m_4 r_1 e^{i\varphi_1} + m_3 r_{3T}^{TB} \frac{r_1}{r_3} e^{i\varphi_{3T}} e^{i\varphi_1}$$

Če želimo, da bo r_T konstanten morata biti člena v oklepajih enaka nič.

$$m_2 r_{2T}^{TA} e^{i\varphi_{2T}} = m_3 \left(r_2 - r_{3T}^{TB} \frac{r_2}{r_3} e^{i\varphi_{3T}} \right)$$
$$m_4 r_{4T}^{TB} e^{i\varphi_{4T}} = m_3 r_3^{TB} \frac{r_4}{r_3} e^{i\varphi_{3T}}$$

Običajno privzamemo m_3 in r_{3T}^{TB} in določimo produkta $m_2 r_{2T}^{TA}$ in $m_4 r_{4T}^{TB}$ ki zagotavljata samo rotacijsko gibanje.

V uravnoteženem štiričlenem mehanizmu s konstantno lego težišča se pojavi notranji obremenilni moment

$$M_H = -M_4 + r_1 \times F_{41}.$$