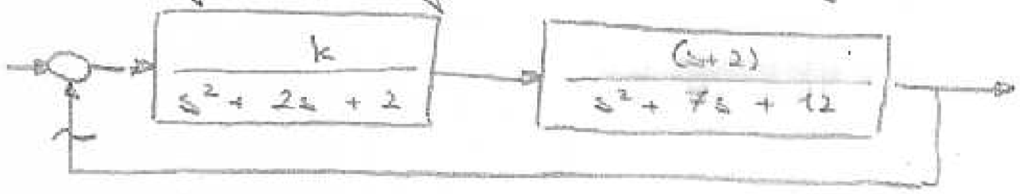
- Kolokvijska naloga korenske krivulje



Za razklenjen sistem je:

$$G_P(s) = \frac{k(s+2)}{(s^2+2s+2)(s^2+7s+12)}$$

$m = 1$

$n = 4$

$n - m = 3$

$n_1 = -2$

$$p_{1,2} = \frac{-2 \pm \sqrt{4-4\cdot 2}}{2} = \frac{-2 \pm 2j}{2} \qquad p_{3,4} = \frac{-7 \pm \sqrt{49-4\cdot 12}}{2} = \frac{-7 \pm 1}{2}$$

$$p_1 = -1-j \qquad p_2 = -1+j \qquad p_3 = -3 \qquad p_4 = -4$$

$$\sigma_r = \frac{\cancel{-1-j}\ \cancel{-1+j} - 3 - 4 + 2}{3} = -7/3 = -2{,}33$$

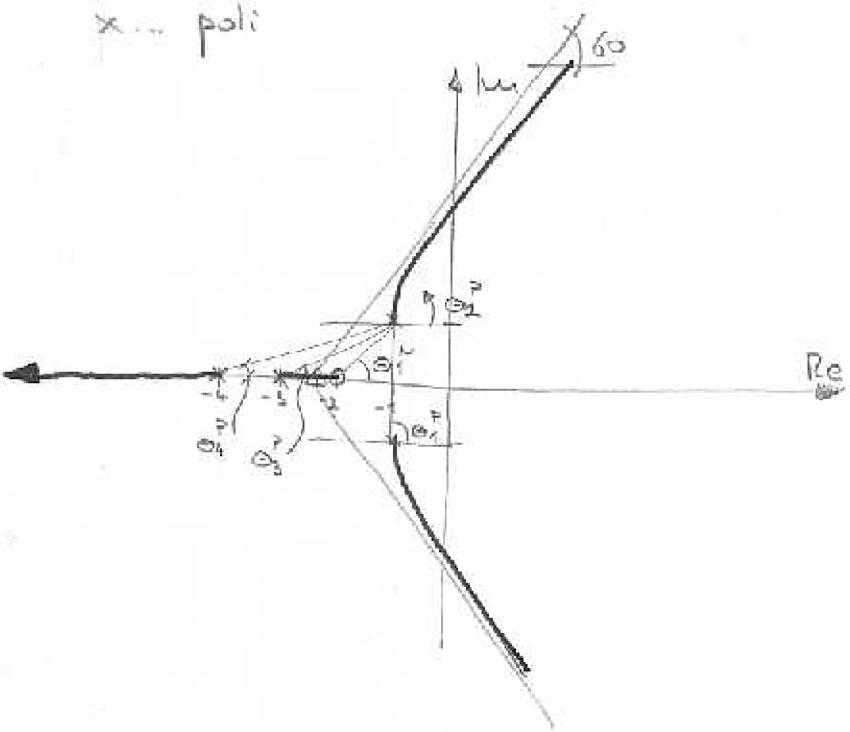
$$\alpha = \frac{(2k+1)\pi}{n-m} \quad ; \quad \alpha = \pi/3,\ 3\pi/3\ ;\ 5\pi/3 = 60°,\ 180°,\ 300°$$

Par stvari lahko takoj narišem:

o ... točka (izračunana) o(−2,33 ∡ za 60, 180, 300)

o ... ničle

x ... poli



$\Theta_1^p = 90°$

$\Theta_2^p = ?$

$\Theta_3^p = \mathrm{arctg}\, 0{,}5 = 26{,}6°$

$\Theta_4^p = \mathrm{arctg}\, 0{,}33 = 18{,}4°$

$\Theta_1^n = 45°$

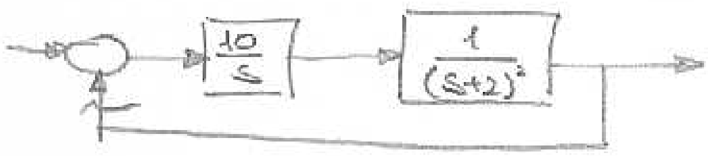
$$\sum \Theta_i^n - \sum \Theta_i^p = (2k+1)\pi$$

$45 - 90 - \Theta_2^p - 26{,}6° - 18{,}4° = 180 \quad ; k=0$

$\Theta_2^p = -270° = 90°$

Izstopni kot iz pola $-1+j$ je $90°$

• Nyquistov Diagram na kolokviju



Za razklenjen sistem je:

$G_P(s) = \frac{10}{s(s+2)^2}$; $s \to j\omega$, v imenovalu moramo dobiti j delno oziroma Im mora biti realen!

$G_P(j\omega) = \frac{10 \quad (j\omega - 2)^2 (j\omega)}{(j\omega)^2 (j\omega+2)^2 (j\omega-2)^2}$

$G_P(j\omega) = \frac{10 \ j\omega (j\omega - 2)^2}{-\omega^2 (-\omega^2 - 4)^2} = \frac{-10j\omega^2 + 40\omega + 40j}{-\omega(-\omega^2-4)^2}$

Re: $\frac{40\omega}{-\omega(-\omega^2-4)^2} = \frac{-40}{(-\omega^2-4)^2}$

Im: $\frac{-10\omega^2 + 40}{-\omega(-\omega^2-4)^2}$

$m = 0$ st $Q(s)$
$n = 3$ st $p(s)$ za $G_P(s) = \frac{Q(s)}{p(s)}$
$r = 1$ red. integ.

$n - m = l = 3$ v kvadrantu 3!

$\alpha = l\,\pi/2 = 3/2\,\pi = 270°$

$\beta = r\,\pi/2 = \pi/2 = 90°$

Im = 0

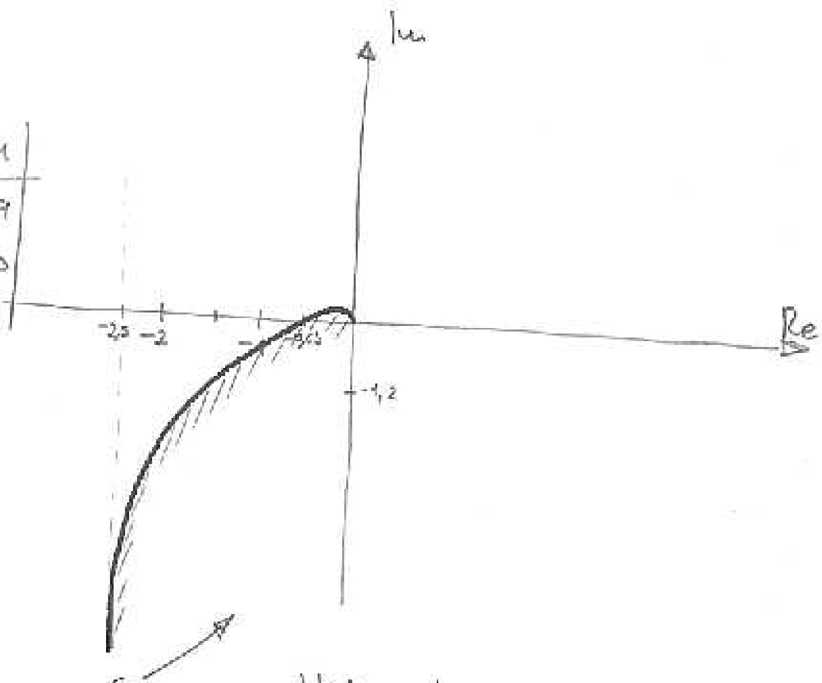
ko $-10\omega^2 + 40 = 0$

$\omega^2 = 4$

$\omega = \pm 2$

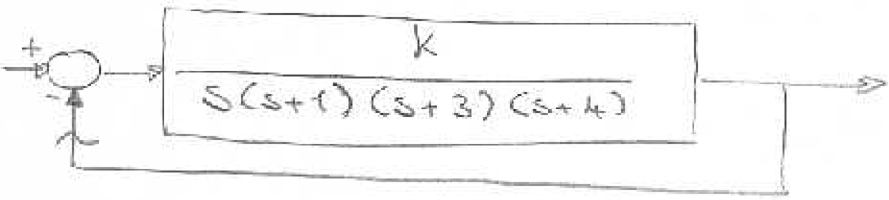
Tabela vrednosti:

ω	± 2	0	1	10	0,01
Re	-0,625	-2,5	-1,6	-0,004	-2,499
Im	0	0	-1,2	0,009	-250



Sistem je stabilen, ker ne pokriva -1.

Se ne približa osi



Za razklenjen sistem:

$m = \emptyset$

$n = 4$

$n - m = 4$

n //

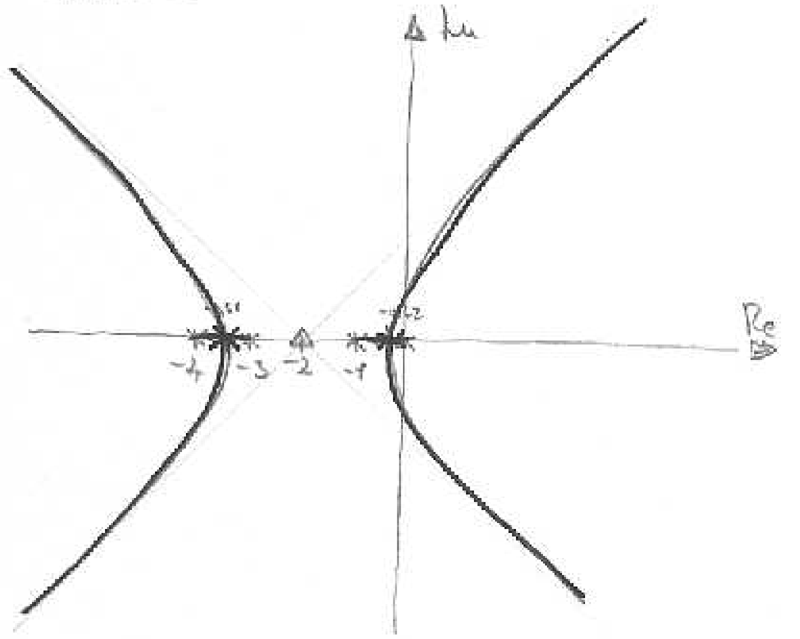
$p_1 = 0 \quad p_2 = -1 \quad p_3 = -3 \quad p_4 = -4$

$\delta_T = \dfrac{-1-3-4}{4} = -2$

$\alpha = \pi/4,\ 3\pi/4,\ 5\pi/4,\ 7\pi/4 \qquad \circ(-2, \alpha_k)$

$\alpha = 45^\circ, 135^\circ, 225^\circ, 315^\circ$

Narišemo:



$F_k = s(s+1)(s+3)(s+4) + k$

$F_k = (s^2+s)(s^2+7s+12) + k =$

$= s^4 + 7s^3 + 12s^2 + s^3 + 7s^2 + 12s + k =$

$F_k(s) = s^4 + 8s^3 + 19s^2 + 12s + k = \emptyset$

$F_k(\delta) = \delta^4 + 8\delta^3 + 19\delta^2 + 12\delta + k = \emptyset$

$\dfrac{dF_k(\delta)}{d\delta} = -4\delta^3 - 24\delta^2 - 38\delta - 12 = \emptyset$

to $(\delta+2)(4\delta^2 + 16\delta + 6)$

$\delta_1 = -2 \qquad \delta_{2,3} = \dfrac{-16 \pm \sqrt{160}}{8} \qquad \delta_2 = -0{,}42 \ ; \ \delta_3 = -3{,}58$

Hornerjev algoritem:

	-4	-24	-38	-12
	/	-16	-6	0
-2	8	32	12	$\boxed{0}$ ✓

Predpostavim ničle

$-4\delta^3 - 24\delta^2 - 38\delta - 12 : (\delta+2) = 4\delta^2 + 16\delta + 6$

$4\delta^2 + 8\delta^2$

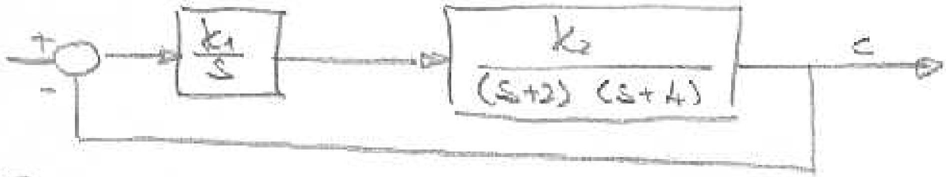
$\emptyset \quad -16\delta^2 - 38\delta$

$16\delta^2 + 32\delta$

$\emptyset \quad -6\delta - 12$

$6\delta - 12$

$\emptyset \quad \emptyset$



Za razklenjen sistem velja

$$G_o(s) = \frac{k_1 k_2}{s(s+2)(s+4)} \quad , \; k_1 \cdot k_2 = k$$

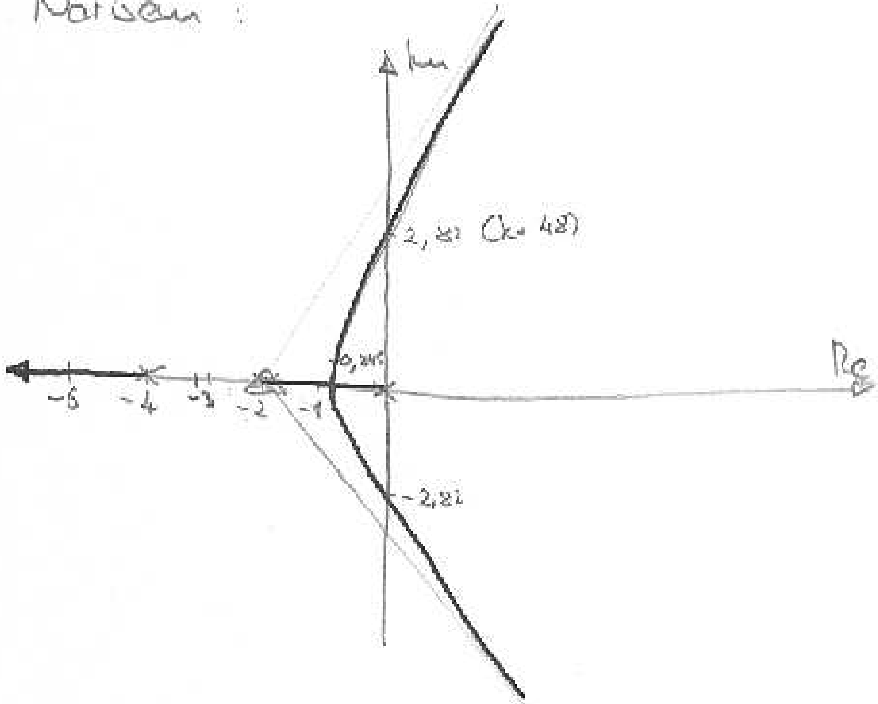
$m = 0$

$n = 3$ $\qquad p_1 = 0 \; ; \; p_2 = -2 \; ; \; p_3 = -4$

$n - m = 3$

$$\sigma_r = \frac{-6}{3} = -2 \quad ; \quad \alpha = 60^\circ, 180^\circ, 300^\circ$$

Narišem:



$$Fk(s) = s(s+2)(s+4) + k$$

$$Fk(\sigma) = \sigma(\sigma+2)(\sigma+4) + k = (\sigma^2 + 2\sigma)(\sigma+4) + k = \sigma^3 + 6\sigma^2 + 8\sigma + k$$

$$\frac{dk(\sigma)}{d\sigma} = -3\sigma^2 - 12\sigma - 8 = 0$$

$$\sigma_{1,2} = \frac{12 \pm \sqrt{144 - 4 \cdot 3 \cdot 8}}{-6} = \frac{12 \pm \sqrt{48}}{-6}$$

$\sigma_1 = -3{,}15$

$\sigma_2 = -0{,}845$

$$Fk(s) = s^3 + 6s^2 + 8s + k$$

$$Fk(j\omega) = -j\omega^3 - 6\omega^2 + 8j\omega + k$$

Re: $-6\omega^2 + k = 0$

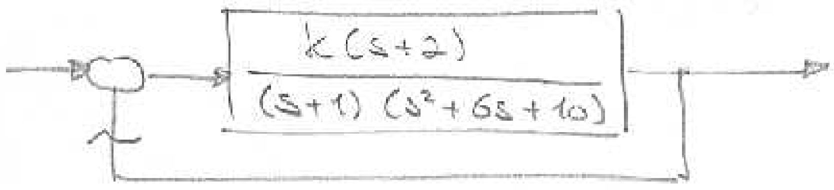
Im: $-\omega^3 + 8\omega = 0$

$\rightarrow -\omega^2 + 8 = 0 \; ; \; \omega^2 = 8 \; ; \; \omega = \pm 2{,}82$

$k = 6\omega^2 = 48$

$$\sum p_i = \sum R_i$$

$$0 - 2 - 4 = +2{,}82j - 2{,}82j + R_3 \; ; \; R_3 = -6$$



Za razklenjen sistem velja

$$G_p(s) = \frac{k(s+2)}{(s+1)(s^2+6s+10)}$$

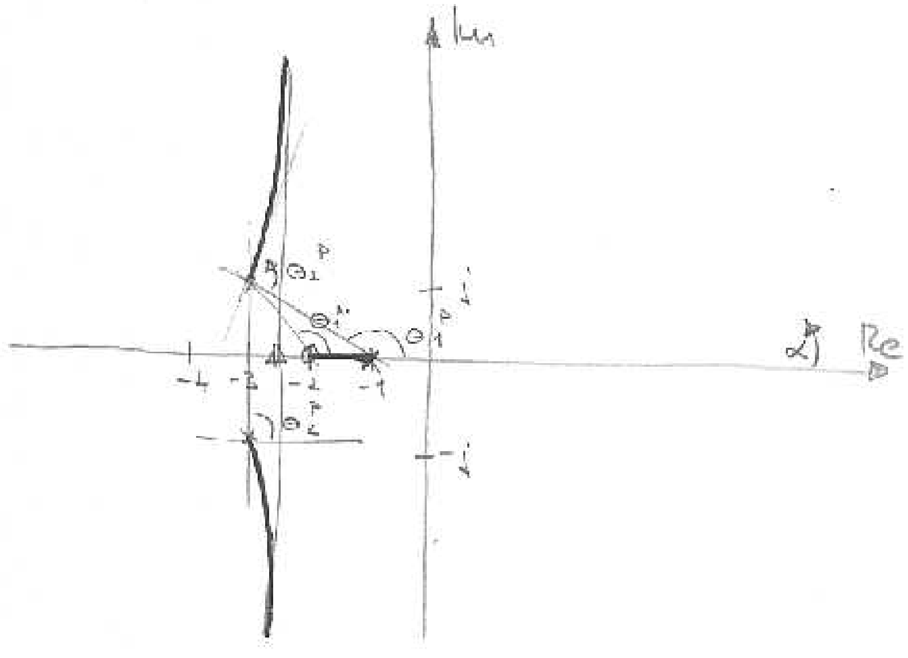
$m = 1$ $\qquad$ $n_1 = -2$

$n = 3$ $\qquad$ $p_1 = -1$; $\qquad p_{2,3} = \frac{-6 \pm \sqrt{-4}}{2} = \frac{-6 \pm 2j}{2}$

$n - m = 2$ $\qquad$ $p_2 = -3 + j \qquad p_3 = -3 - j$

$$\sigma_T = \frac{-1 - 3 \cancel{+j} - 3 \cancel{-j} + 2}{2} = -5/2 = -2,5$$

$\alpha = 90°, 270°$



$\Theta_1^n - \Theta_2^p - \Theta_1^p - \Theta_3^p = \pi$; $k=0$

$135° - \Theta_2^p - 153,4° - 90° = 180°$

$\Theta_2^p = -288° = 71,6°$

• Podan je naslednji sistem:

$$G_P = \frac{1{,}6k(s+10)}{s(s+1)(s+4)^2}$$

Navodilo: Jebi se!

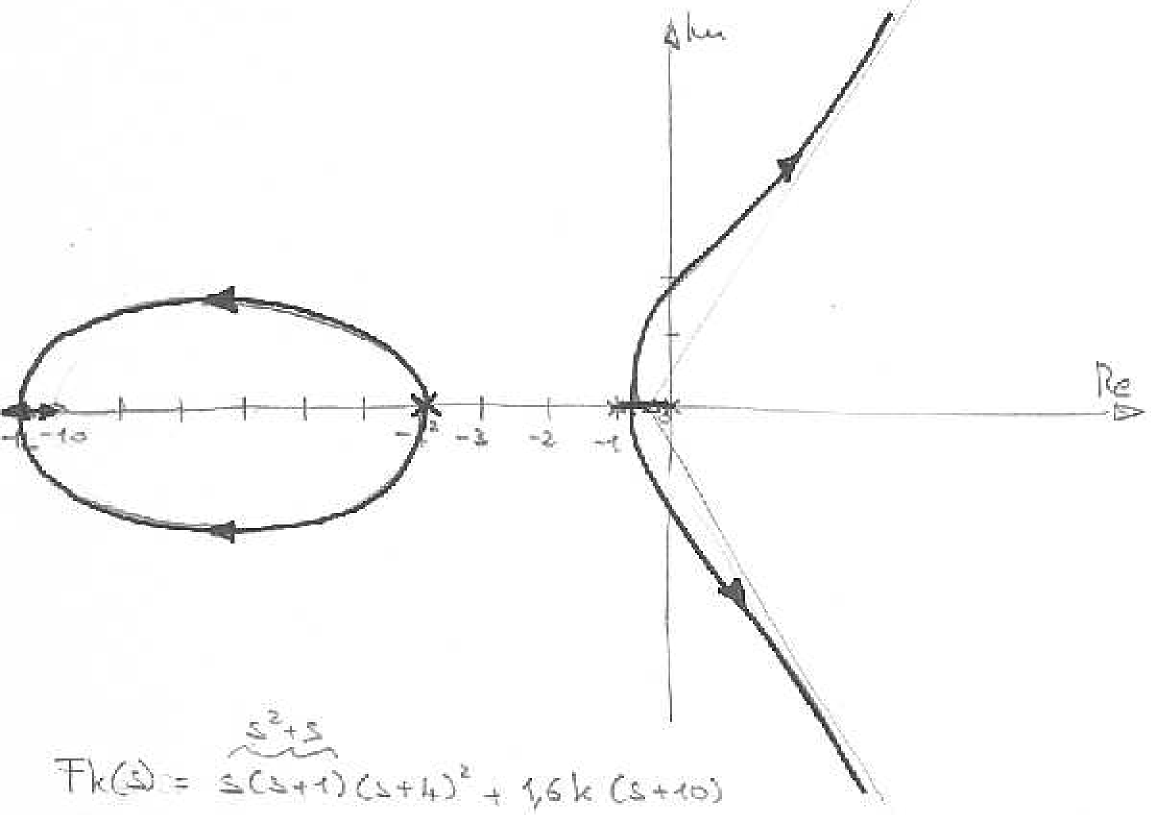
$m = 1 \qquad n_1 = -10$

$n = 4 \qquad p_1 = 0\ ;\ p_2 = -1\ ;\ p_3 = -4\ ;\ p_4 = -4$

$n - m = 3$

$$\delta_r = \frac{-1-4-4+10}{3} = 1/3 = 0{,}33$$

$\alpha = 60°, 180°, 300°$



Zgolj aproksimacija!

$$Fk(s) = \underbrace{s(s+1)}_{s^2+s}(s+4)^2 + 1{,}6k(s+10)$$

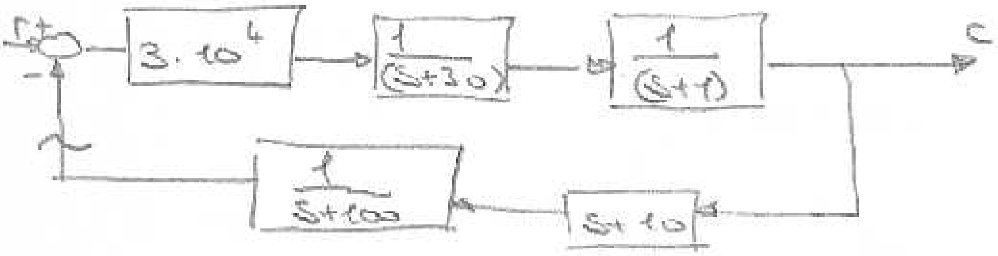
$$Fk(s) = s^4 + 8s^3 + 16s^2 + s^3 + 8s^2 + 16s + 1{,}6ks + 16k$$

$$k(\delta) = \left[-\delta^4 - 9\delta^3 - 24\delta^2 - s(16+1{,}6k)\right] : 1/16 = \emptyset$$

Ničle:

WTH!!

• Imamo naslednji Bodejev sistem:



Delam za razklenjen sistem:

$$G_P(s) = 3\cdot 10^4 \cdot \left(\frac{1}{s+30}\right)\left(\frac{1}{s+1}\right)(s+10)\left(\frac{1}{s+100}\right)$$ / Izpostavim: $\frac{10}{30\cdot 100}$

$$G_P(s) = 100 \quad \frac{1}{\frac{1}{30}s+1} \quad \frac{1}{s+1} \quad \frac{1}{10}s+1 \quad \frac{1}{\frac{1}{100}s+1}$$

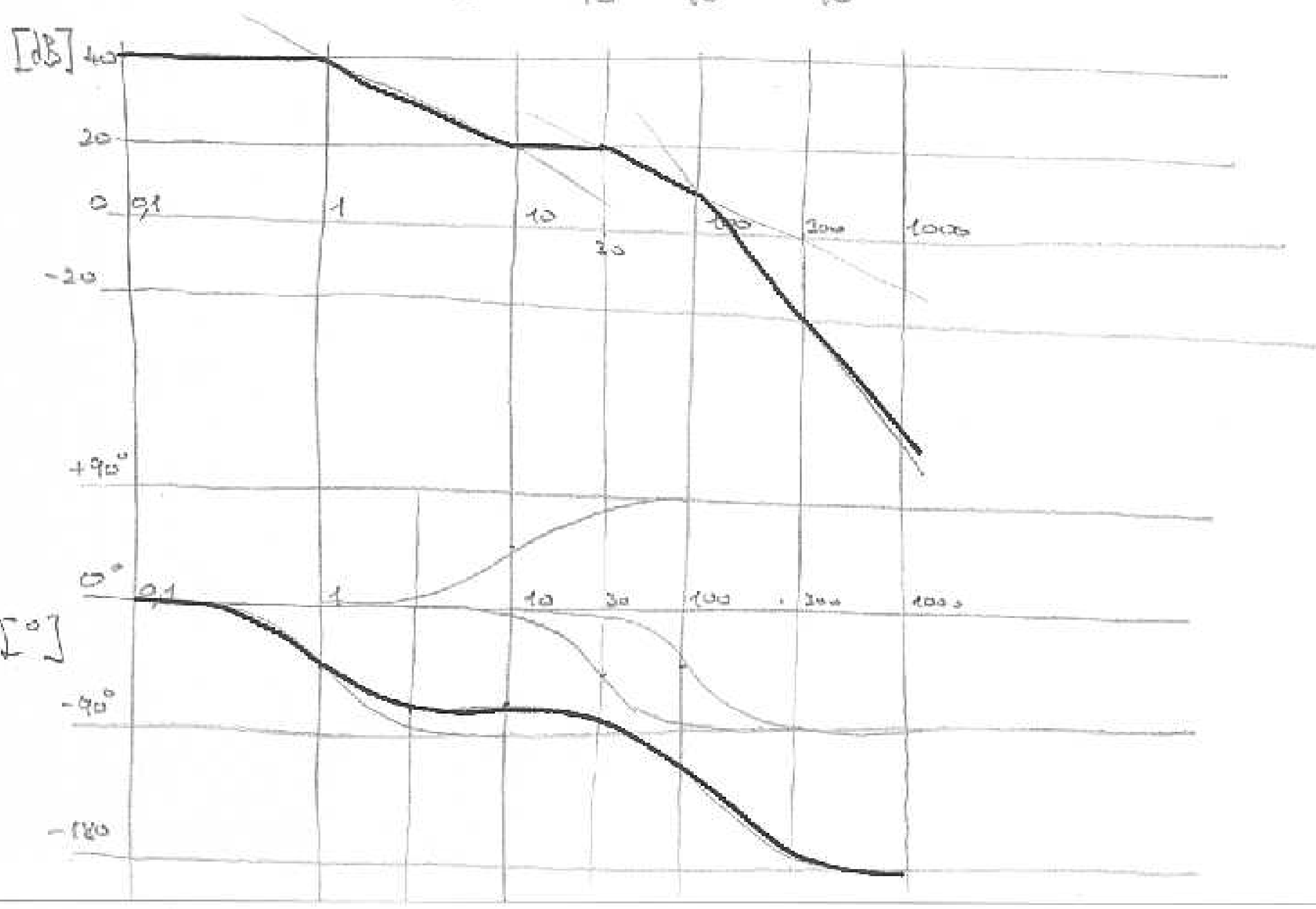
(1) (2) (3) (4) (5)

Tabela: (Glej tabelo!!)

Členi:	1	2	3	4	5
ω_L	/	30	1	10	100
Naklon:	/	-20	-20	20	-20
0		0	0	0	0
φ_L	/	-45	-45	45	-45
∞		-90	-90	90	-90

; $\omega_L = 1/T$; T - pred s (vrednost)

Ojačanje = 20 log₁₀ Člena 1 = 40 dB



- Bodejev diagram za

$$G_P(s) = \frac{64\,(1+0{,}5\,s)}{s\,(1+5s)\,(1+0{,}05\,s+0{,}0125\,s^2)}$$

$$G_P(s) = 64\,(1+0{,}5s)\;\frac{1}{s}\;\frac{1}{(1+5s)}\;\frac{1}{(1+0{,}05\,s+0{,}0125\,s^2)}$$

× $1+0{,}05\,s+0{,}0125\,s^2$

$1+2\xi\,{}^{s}/_{\omega_L}+{}^{s^2}/_{\omega_L^2}$

Menjamo obliko da bomo lahko sploh uporabili tabelo

$0{,}05 = 2\xi\,{}^{1}/_{\omega_L}$ $\qquad \xi = 0{,}22$

$0{,}0125 = {}^{1}/_{\omega_L^2} \rightarrow \omega_L = \sqrt{80} = 8{,}94$

$$\Rightarrow G_P(s) = 64\,(1+0{,}5\,s)\,\frac{1}{s}\,\frac{1}{(1+5s)}\;\frac{1}{1+2\cdot 0{,}22\,{}^{s}/_{8{,}94}+{}^{s^2}/_{8{,}94^2}}$$

① ② ③ ④ ⑤

Tabela:

člen	1	2	3	4	5
ω_L	/	2	/	0,2	8,94
Naklon	/	+20	-20	-20°	-40
0		0	-90	0	0
φ_L	/	45	-90	-45	-90
0		90	-90	-90	-180

Ojačanje = 20 $\log_{10}$ 64 = 36 dB