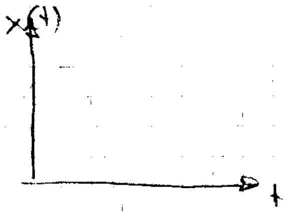
- Odziv sistema N=1 na periodično motnjo (kar per; $x = x \sin(\omega t + \varphi)$; $x = x(t + T_p)$)
  $x(t) = a_0/2 + \ldots$ Fourier
  $x(t) = c_0$ (primer)
- Nihanje lin. sis. zaradi udarne motnje
  ($+ \delta(t-t_1)$ ∞ $t \neq t_1$ $t = t_1$, $\int_0^\infty f(t)\delta(t-t_1)dt = f(t_1)$ Dirac)
- Impulzna prenosna f. (I:op = mv2 / Oscilator / nedušen x(t) g(t); dušen; x(t) g(t))
- Konvolucijski ∫ (Potrebne lastnosti f(t) Δt, ox. dI, superpozicija x(t) = lim = $\int_0^t$ x(t), Nato T, splošno + 2x x(t), ... x ...
- Uporaba udarne motnje
  - disk, - zvezni, - vapor test
- Odziv na vzbujanje podlage
  - mẍ + kx + dẋ = ...; mẍ = k(v−y) − d(v̇−ẏ) $g(t) = \frac{1}{m\omega_d} e^{-\delta\omega t} \sin\omega_d$
- Nihanje sis. z N > 1 (lastno) (N>1, P,N, lin, ui d, G) ... NLMVK k_i ... $k_{i+1}(x_{i+1} - x_i)$ − enačbe $m_i \ddot{x}_i$ ... Matrični zapis)
- MVK (prisotni vpl. koef − $x_{ij} = p_{ij} F_j$, togostni vp. − $F_{ij} = k_{ij} x_j$
- Določitev lastnih vrednosti, vektorjev .. ([M]{ẍ} + [k]{x} = 0; $x_{ij}(t) = X_{ij} T(t)$; ... ; {x} = {X} T(t)
- Modalna transformacija [Φ] = [X ...] vek; {x(t)} = [Φ][η]; [M][k], $M_i k_i$, η(t), ω_{0i}, zgled E
- Nihanje sis z N > 1 (vsiljeno)
  - fiz. kor. - gl. kor.

  FK; [M]{ẍ} + [k]{x} = {F̂} sin ωt; {x(t)} = {X} sin ωt **KAZALO**
  $X_i = \frac{D_i}{D}$ + Graf + Dušenje {ẍ} = −ω²{x}
  GK [M]{ẍ} + [k]{x} = {F}; {x} = [Φ][η]; M_i, k_i, α_i; $\eta_i(t)$, E, B_i
  zvezna porazdelitev
  ∞, f(x,t) 1 2 3 D
- Nihanje zveznih sistemov
- Prečno nihanje strune (...; ΣFiy = mÿ; y = y(x,t); Predpostavke; m = ρdx; Iščemo y(x,t), Nastavek y(x,t) = XT; ..., E)
- Torzijsko nihanje gredi (..., $\Sigma M_t = J\ddot{\varphi}$; $M_t = k\Delta\varphi = G I_t \frac{d\varphi}{dx}$; φ = φ(x,t); φ = Φ(x)T(t); Φ(x) = C ... → rešitev)
- Ravninsko upogibno nihanje (...; predpostavke, EB; ΣFiy = mÿ; $T = \frac{dM}{dx}$; w(x,t); w = X(x)T(t), T; $X(x) = Ce^{\lambda x}$, B
- Aproksimativne metode / MPM (Modeli, 1234 → Zaloge, ..., 3 slike, PPV(F=kt); PPM (x = ...)
- MPM Torzijsko nihanje (... {V}, ME [M], PE [M]; sist. ... f = ... ; {...} = [P]{V}; [PM]; RP ...
- MPM Upogibno nihanje ({V}; opis; PPBEP ...; $M_{Tg} = 0$, x = 0, L, y(t) [P]; PPMT, ... $T_0 - T_1 = m\ddot{y}_L = -\omega^2 m y$ [M]
- Nelinearno nihanje ($\ddot{x} + \omega_0^2 x + \mu x^3 = 0$; $x(t) = x_0(t) + \mu x_1(t)$; $\omega^2 = \omega_0^2 + \mu \alpha_1$; μ^{01}; $x_0(t) = A\cos\omega t$, $\ddot{x}_1 + \omega x_1$; α_1, D_1, C, $x_1(t)$ ...
- Metoda iteracij / vsiljeno ($m\ddot{x} + d\dot{x} + kx \pm \mu x^3 = F\cos\omega t$; $x_0(t) = A\cos\omega t$; ẍ, ω², A, graf)

Odziv sistema z N=1 na periodično motnjo

Izvor motnje - batni stroji
- mehanski stroji

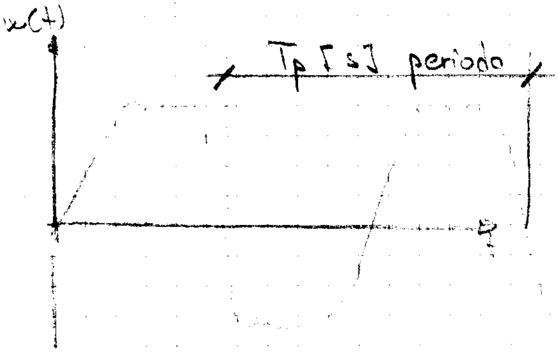


harmonično $x(t) = X \sin(\omega t + \varphi)$

X … amplituda

ω … krožna frekvenca

φ … faza



$x(t) = x(t + T_p)$

Fourier je dokazal, da lahko x(t) popišemo z neko vrsto

$$x(t) = \underbrace{\frac{a_0}{2}}_{\text{srednja vrednost funkcije}} + a_1 \cos\omega t + a_2 \cos 2\omega t + \dots b_1 \sin\omega t + b_2 \sin 2\omega t$$

osnovni ton

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

a_n, b_n … koeficienti Fourijeve vrste

$$a_0 = \frac{2}{T_p} \int_0^{T_p} x(t)\, dt$$

$$a_n = \frac{2}{T_p} \int_0^{T_p} x(t) \cos n\omega t\, dt$$

$$b_n = \frac{2}{T_p} \int_0^{T_p} x(t) \sin n\omega t\, dt$$

$\omega = \frac{2\pi}{T_p}$ osnovna fundamentalna frekvenca

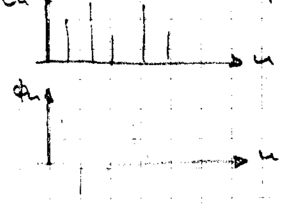
$$x(t) = c_0 \sum_{n=1}^{\infty} c_n \cos(n\omega t - \Phi_n)$$

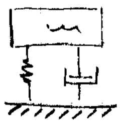
$$c_0 = \frac{a_0}{2}$$

$c_n = \sqrt{a_n^2 + b_n^2}$ - amplituda

$\mathrm{tg}\, \Phi_n = \frac{b_n}{a_n}$ - faza

Osnovni spektri





↑$F(t)$ periodicne funkcije

Iscemo $x(t)$ v ustaljenem stanju (samo partikularno resitev)

- linearen sistem
- eno prostostno stopnjo

$$m\ddot{x}+d\dot{x}+kx=F(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos n\omega t+b_n\sin n\omega t$$

$$x=x_p(t)$$

1) $m\ddot{x}+d\dot{x}+kx=\frac{a_0}{2}$

2) $m\ddot{x}+d\dot{x}+kx=a_n\cos n\omega t$; $n=1\ldots\infty$

3) $m\ddot{x}+d\dot{x}+kx=b_n\sin n\omega t$

$$x_i(t)=\frac{a_0}{2k}$$

$$x_{ii}(t)=\frac{a_n}{k}\,\frac{1}{\left(1-\left(\frac{n\omega}{\omega_0}\right)^2\right)^2+\left(2\delta\left(\frac{n\omega}{\omega_0}\right)\right)^2}\,\cos(n\omega t-\varphi_n)$$

$$x_{iii}(t)=\frac{b_n}{k}\quad -''-\quad \sin(n\omega t-\varphi_n)$$

$$x(t)=x_i(t)+x_{ii}(t)+x_{iii}(t)$$ sestavljen odziv

=

$$m\ddot{x}+d\dot{x}+kx=F_0\sin\omega t$$

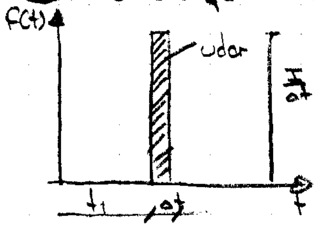
$$x(t)=X\sin(\omega t-\varphi)$$

$$X=\frac{F_0}{k}\beta\ ;\quad \mathrm{tg}\,\varphi=\frac{2\delta\frac{\omega}{\omega_0}}{1-\left(\frac{\omega}{\omega_0}\right)^2}\ ;\quad \beta=\frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2+\left(2\delta\frac{\omega}{\omega_0}\right)^2}}$$

Nihanje linearnega sistema zaradi udarne motnje

UPORABA KONVOLUCIJSKEGA INTEGRALA

Def. enotskega impulza ; Diracova delta funkcija



$\Delta t \to 0$ $\frac{I}{\Delta t}$ preko vseh mej

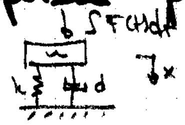
V realnosti nenaden udar zelo majhen

$$\delta(t-t_1) = \begin{cases} 0 & \text{za } t \neq t_1 \\ \infty & \text{za } t = t_1 \end{cases}$$

Integral produkta Diracove delta f z poljubno f(t)

$$\int_0^\infty f(t)\,\delta(t-t_1)\,dt = f(t) \qquad 0 < t_1 < \infty$$

Impulzna prenosna funkcija $g(t)$



$I = \Delta\vec{p} = m\vec{v}_2 - \cancel{m\vec{v}_1}$ mirno ; $I = \int F(t)dt$

$\vec{v}_2 = \frac{I}{m}$

Vzbujanje oscilatorja z impulzom

- NEDUŠEN OSCILATOR

$m\ddot{x} + kx = 0$; $x(t) = A\cos\omega_0 t + B\sin\omega_0 t$

$\omega_0 = \sqrt{k/m}$

$\left.\begin{array}{l} x(0) = 0 \\ \dot{x}(0) = I/m \end{array}\right\} \to A, B$

Odziv: $x(t) = \frac{I}{m\omega_0} \sin\omega_0 t$

IPF: $g(t) = \frac{x(t)}{I} = \frac{1}{m\omega_0} \sin\omega_0 t$

- DUŠEN OSCILATOR

$m\ddot{x} + d\dot{x} + kx = 0$; $x(t) = e^{-\delta\omega_0 t}[A\cos\omega t + B\sin\omega t]$

$\to A, B \to$ ZP.

Odziv: $x(t) = \frac{I}{m\omega_d} e^{-\delta\omega_0 t} \sin\omega_d t$

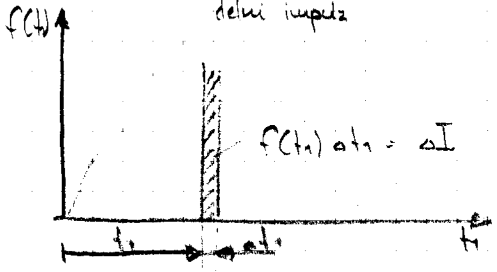
IPF: $g(t) = \frac{x(t)}{I} = \frac{1}{m\omega_d} e^{-\delta\omega_0 t} \sin\omega_d t$

δ .. razmernik dušenja

Konvolucijski integral

Potrebne lastnosti sistema za uporabo konvolucijskega integrala so:
- linearnost
- časovna invariantnost
- kavzalnost

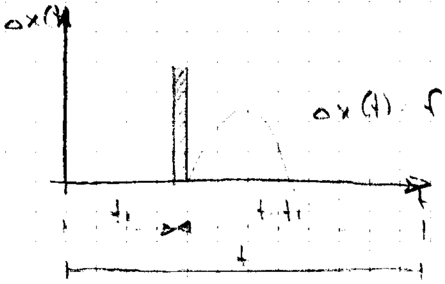
$$\Delta x(t) = \underbrace{f(t_1)\,\Delta t_1}_{\text{delni impulz}}\, g\overset{\text{prav argument}}{(t-t_1)}$$



$$\Delta x(t) = \Delta I\, g(t) \;;\; g(t) = \frac{x(t)}{I} = \frac{\text{odziv}}{\text{impulz}}$$

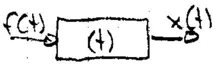
Z uporabo superpozicije dobimo

$$\underbrace{x(t)}_{\text{odziv}} = \lim_{\Delta t \to \infty} \sum_{i=1}^{n} f(t_i)\, g(t-t_i)\, \Delta t_i = \underbrace{\int_0^t f(t_1)\, g(t-t_1)\, dt_1}_{\text{konvolucijski integral}}$$



$$\Delta x(t) = f(t_1)\,\Delta t_1\, g(t-t_1)$$

Zapis konvolucije s pomočjo spominske spremenljivke τ



Splošno lahko zapišemo

$$x(t) = \int_{-\infty}^{t} f(t_1)\, g(t-t_1)\, dt_1$$

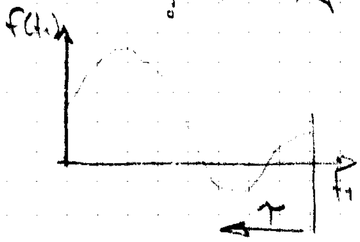
Uvedemo novo časovno spremenljivko $\tau = t - t_1$, $d\tau = -dt_1$

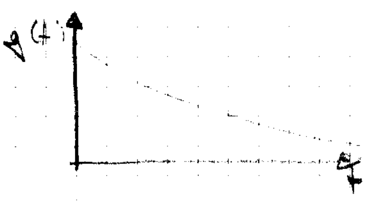
$$x(t) = \int_0^{\infty} f(t-\tau)\, g(\tau)\, d\tau \quad , \; t_1 = 0, \tau = t$$

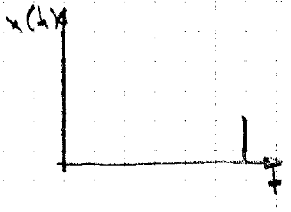
$$x(t) = \int_0^{t} f(t-\tau)\, g(\tau)\, d\tau \quad ; \; t_1 = t, \tau = 0$$

KAZALNI SISTEM
- odziva se lahko po motnji ali ali takoj ko damo motnjo

$g(\tau)$ ... dinamični spomin sistema



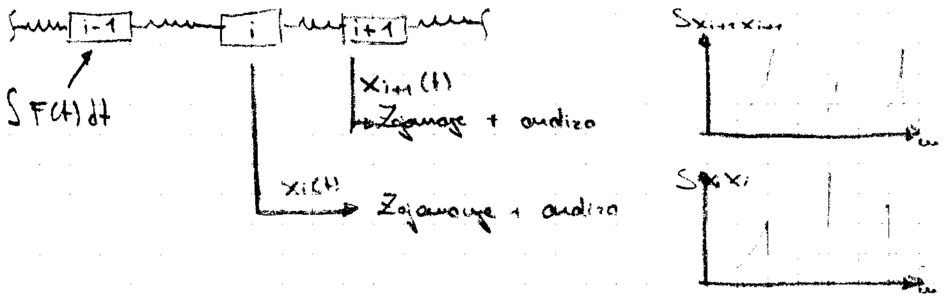




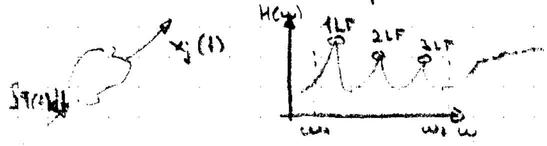
$$x(t) = \int_0^t f(t-\tau)\, g(\tau)\, d\tau = \lim_{\Delta\tau_i \to \infty} \sum_{i=1}^{n} f(t - i\Delta\tau)\, g(i\Delta\tau)\, \Delta\tau = [\cdots]\,\Delta\tau$$

Uporaba udarne metode

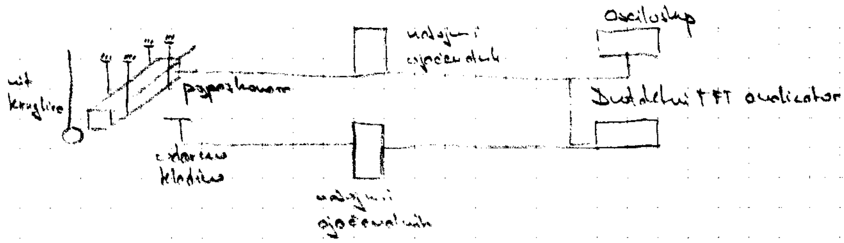
- DISKRETNI SISTEM



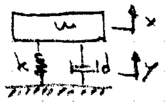
- ZVEZNI SISTEM - eksperimentalna strukturna dinamika



- NEPORUŠNO TESTIRANJE POŠKODOVANIH STRUKTUR



Odziv na vzbujanje podlage



Gibalna enačba: $m\ddot{x} = -k(x-y) - d(\dot{x}-\dot{y})$

$z = x - y$

$m\ddot{z} + kz + d\dot{z} = -m\ddot{y}$

$m\ddot{z} + kz + d\dot{z} = F$

Za podkritično viskozno dušen sistem je impulzna prenosna funkcija

$g(t) = \frac{1}{m\omega_d} e^{-\delta\omega_0 t} \sin(\omega_d t)$

Rešitev gibalne enačbe je:

$x(t) = \int_0^t f(t_1)\, g(t-t_1)\, dt_1$

$z(t) = -\frac{1}{\omega_d} \int_0^t \ddot{y}(t_1)\, e^{-\delta\omega_0 (t-t_1)} \sin(\omega_d (t-t_1))\, dt_1$

Nihanje sistemov z več prostostnimi stopnjami (Lastno)

$N > 1$ št koordinat, ki so neodvisne od drugih

P ... št. prostostnih stopenj
N ... št. koordinat

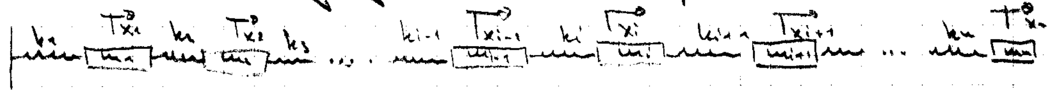
Načeloma: Imamo toliko moi $\{x\}^{(i)}$; $i = 1 \dots N$, kolikor je koordinat

Želja: Obvladovanje velikih sistemov „veliko prostostnih stopenj - zvezni sistem"

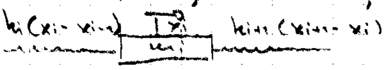
Predpostavimo: - linearnost
- nedušeni sistemi

1) PREDPOSTAVIMO GIBALNE ENAČBE

- Newton - Lagrange - Metoda vplivnih koeficientov



$x_i = x_i(t)$; $i = 1 \dots N$; $x_{i+1} > x_i$



$$m_i \ddot{x}_i = -k_i(x_i - x_{i-1}) + k_{i+1}(x_{i+1} - x_i) + F_i(t) \quad ; \; i = 1 \dots N$$

$$m_i \ddot{x}_i = -k_i x_{i-1} + (k_i + k_{i+1}) x_i - k_{i+1} x_{i+1} = F(t) \quad ; \; i = 1 \dots N$$

Uredimo sistem - vse člene z koordinato (x_i...) in njihove pospeške damo na levo

Matrični zapis

$$[M] \{\ddot{x}\} + [K]\{x\} = \{0\}$$

$$[M] = \begin{bmatrix} m_{11} & m_{12} \dots & m_{1N} \\ m_{21} & m_{22} \dots & m_{2N} \\ \vdots & \vdots & \vdots \\ m_{N1} & m_{N2} \dots & m_{NN} \end{bmatrix} \qquad \{\ddot{x}\} = \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \vdots \\ \ddot{x}_i \\ \vdots \\ \ddot{x}_N \end{Bmatrix}$$

masna matrika

Vektor pospeškov

Če je masna matrika diagonalna → je masni sistem nevezan. (V vsaki dif. en. nastopa samo ena mas

$$[K] = \begin{bmatrix} k_{11} & k_{12} \dots & k_{1N} \\ k_{21} & k_{22} \dots & k_{2N} \\ \vdots & \vdots & \vdots \\ k_{N1} & k_{N2} \dots & k_{NN} \end{bmatrix}$$

Princip Maxwellovega teorema recipročnosti

$k_{ij} = k_{ji}$; $i \neq j$; $i,j = 1 \dots N$

Matrika je simetrična

$$\{x\} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{Bmatrix} \qquad \{F\} = \begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_N \end{Bmatrix} \qquad \{0\}$$

Vektor pomikov

Vektor zunanjih sil. Za lastno nihanje $\{F\} = \{0\}$

Matrika ničel

Metoda vplivnih koeficientov

- PROŽNOSTNI VPLIVNI KOEFICIENTI p_{ij}
  - na j-tem mestu naj deluje na sistem sila F_j
  - x_{ij} .. pomik na mestu i, zaradi delovanja sile na mestu j

$x_{ij} = p_{ij} F_j$; če deluje na sistem več sil $x_i = \sum_{j=1}^{N} x_{ij} = \sum_{j=1}^{N} p_{ij} F_j$ $i = 1, .. N$

Sledi: $\{x\} = [P]\{F\}$; $[P]$ prožnostna matrika sistema

- TOGOSTNI VPLIVNI KOEFICIENTI k_{ij}

x_j .. pomik na mestu j

F_{ij} .. sila na mestu i, zaradi pomika na mestu j

$F_{ij} = k_{ij} x_j$; za več pomikov $F_{ij} = \sum_{j=1}^{N} k_{ij} x_j$; $j = 1..N$

Sledi $[P] = [k]^{-1}$; $[k]$ ... togostna matrika ; $\{F\} = [k]\{x\} = [k]\overbrace{[P]\{F\}}^{\{x\}}$ $[k][P] = [I]$

$[I]$ .. enotska matrika

Določitev lastnih vrednosti

$[M]\{\ddot{x}\} + [k]\{x\} = \{0\}$

Iščemo $x_i = x_i(t)$; $i = 1..N$, ki od začetnih pogojih $x_i(0), \ddot{x}_i(0)$

$i = 1, .., N$ rešijo zgornjo enačbo

Osnovna ideja:

$x_i(t) = X_i T(t)$ $\frac{x_i(t)}{x_j(t)} \neq f(t)$ nihanja so sinhrona

$x_j(t) = X_j T(t)$

X_i ... i-ta amplituda pomika

T .. funkcija časa

Ideja: ločitev časa od pomikov $\{x\} = \{X\} T(t)$

$\{X\} = \begin{Bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{Bmatrix}$

$[M]\{X\}\ddot{T}(t) + [k]\{X\}T = 0$

Skalarni zapis za i-to vrstico

$(\sum_{j=1}^{N} m_{ij} X_j)\ddot{T}(t) = -(\sum_{j=1}^{N} k_{ij} X_j) T(t) = 0$

Separacija spremenljivk

$$\underbrace{\frac{\ddot{T}(t)}{T(t)}}_{L(t)} = \underbrace{-\frac{\sum_{j=1}^{N} k_{ij} X_j}{\sum_{j=1}^{N} m_{ij} X_j}}_{D(x)} = \text{konst} = -\omega^2$$

L .. čas

D ... kraj

$\ddot{T} + \omega^2 T = 0$ homogena enačba II reda

$T(t) = A\cos\omega t + B\sin\omega t$; AB konstanti, $\{x\} = \{X\}\sin\omega t$ nastavek, ki upore harmoničnost

$\sum_{j=1}^{N} (k_{ij} - \omega^2 m_{ij}) X_j = 0$; $i = 1..N$

$([k] - \omega^2 [M])\{X\} = \{0\}$ $/[M]^{-1}$

$([A] - \lambda [I])\{X\} = \{0\}$; $[A] = [k][M]^{-1}$ dinamična matrika sistema

$[I] = [M][M]^{-1}$ .. enotska matrika

$\lambda = \omega^2$ lastna vrednost

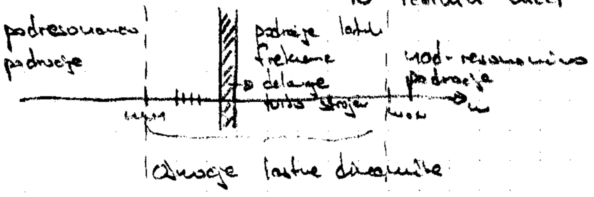
Iz dif. en. smo prišli v algebrajske en.

$$\det([A]-\lambda[I]) = 0 \rightarrow \begin{bmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22}-\lambda & \cdots & a_{2N} \\ \vdots & & & \vdots \\ a_{N1} & \cdots & a_{N2} & \cdots & a_{NN} \end{bmatrix} = 0 \; ; \; \det = 0 , P(\lambda^N) = 0$$

Netrivialne rešitve

N realnih ničel

$\{\lambda_1 < \lambda_2 < \lambda_3 < \ldots < \lambda_N\}$
1.L.V. ... N.L.S.S.



$\omega_{0i} = \sqrt{\lambda_i} \quad ; \; i = 1..N$

Trivialna rešitev če $\omega = 0$

$x_1 = 0, \; x_2 = 0 \ldots$

Določitev lastnih vektorjev

$([A]-\lambda[I])\{X^{(i)}\} = 0$ $\quad$ $\{X\}$ ... vektor amplitud ; $\{X^{(i)}\}$ ... i-ti lastni vektor

$$z_{11} X_1^{(i)} + z_{12} X_2^{(i)} + \ldots + z_{1N} X_N^{(i)} = 0$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$z_{N1} X_1^{(i)} + z_{N2} X_2^{(i)} + \ldots + z_{NN} X_N^{(i)} = 0$$

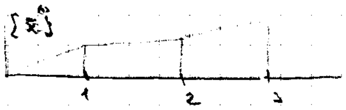
Dobimo:

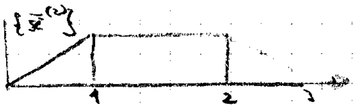
$$\{X^{(i)}\} = X_1^{(i)} \{\bar{X}^{(i)}\} = X_1^{(i)} \cdot \begin{Bmatrix} 1 \\ X_2^{(i)}/X_1^{(i)} \\ X_N^{(i)}/X_{N-1}^{(i)} \end{Bmatrix}$$

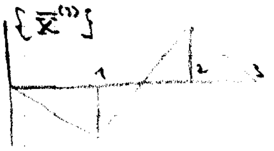
Lastni vektor predstavlja . Sistem niha s tako V
razmerje med amplitudami / ki ji pripada λ_i

Prikaz lastnih vektorjev

Primer:







Ortogonalnost lastnih vektorjev

$[M]\{\ddot{x}\} + [k]\{x\} = \{0\}$

$[k]\{X^{(i)}\} = \lambda[M]\{X^{(i)}\} \quad ; \lambda = \omega^2$

$\{X^{(j)}\}^T [K] \{X^{(i)}\} = \lambda_i \{X^{(j)}\}^T [M] \{X^{(i)}\}$ Ⓐ

$\{X^{(i)}\}^T [k] \{X^{(j)}\} = \lambda_j \{X^{(i)}\}^T [M] \{\bar{X}^{(j)}\}$ Ⓑ

[M] in [k] simetrične

kolikor zamenjamo vrstni red med množenjem in transponiranja

Dobimo: $\{X^{(j)}\}^T [k] \{X^{(i)}\} = \{X^{(i)}\}^T [k] \{X^{(j)}\}$

odštevamo: $0 = (\lambda_i - \lambda_j) \{X^{(i)}\}^T [M] \{\bar{X}^{(j)}\} \quad ; i \neq j$

Sledi: $\{\bar{X}^{(i)}\} \; [M] \; \{\bar{X}^{(j)}\} = 0 \quad ; i \neq j \; ; \; i,j = 1..N$ ⟶ Ortogonalnost lastnih vektorjev preko masne matrike

Če delimo z λ_i $\{\bar{X}^{(i)}\}^T [k] \{X^{(j)}\} = 0$ ⟶ Ortogonalnost lastnih vektorjev preko togostne matrike

Modalna transformacija

Prehod v glavne (modalne) koordinate

Modalno matriko dobimo, če v vrsto zložimo lastne vektorje

$$[\Phi] = \left[\{X^{(1)}\}, \{X^{(2)}\}, \dots, \{X^{(n)}\}\right] \longrightarrow + \text{ modalna transformacija}$$

$$\{x(t)\} = [\Phi]\{\eta(t)\}$$

$\{x(t)\}$ ... vektor pomikov ... fizikalne koordinate

$[\Phi]$ ... modalna matrika, koordinate

$\{\eta(t)\}$ ... glavne koordinate

Začnemo: $[M]\{\ddot{x}\} + [K]\{x\} = \{0\}$

$$\{\ddot{x}\} = [\Phi]\{\ddot{\eta}\}$$

Sledi: $[M][\Phi]\{\ddot{\eta}\} + [K][\Phi]\{\eta\} = \{0\} \quad / [\Phi]^T$

$$[\Phi]^T[M][\Phi]\{\ddot{\eta}\} + [\Phi]^T[K][\Phi]\{\eta\} = \{0\}$$

$[\Phi]^T[M][\Phi] = \begin{bmatrix} \searrow & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & \searrow \end{bmatrix}$ generalizirana (posplošena) masna matrika

$[\Phi]^T[K][\Phi] = \begin{bmatrix} \searrow & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & \searrow \end{bmatrix}$ generalizirana (posplošena) togostna matrika

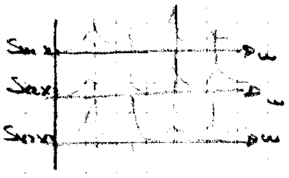
Definiramo: $M_i = \{X^{(i)}\}^T[M]\{X^{(i)}\}$ generalizirana masa

$k_i = \{X^{(i)}\}^T[K]\{X^{(i)}\}$ generalizirana togost

Sledi: $[{}^{\searrow}M_{\searrow}]\{\ddot{\eta}\} + [{}^{\searrow}k_{\searrow}]\{\eta\} = \{0\}$; $M_i\ddot{\eta}_i + k_i\eta_i = 0 \quad i = 1..n$

Splošna rešitev: $\eta_i(t) = A\cos\left(\frac{k_i}{M_i}t\right) + B\sin\left(\frac{k_i}{M_i}t\right) = C_i\cos(\omega_{0i}t - \varphi_i) \qquad \omega_{0i} = \sqrt{\frac{k_i}{M_i}}$

Zgled: $\begin{Bmatrix} x_1 \\ x_2 \\ x_i \\ x_n \end{Bmatrix} = \left\{X_i^{(1)}, \dots, X_i^{(2)}, \dots, X_i^{(n)}\right\} \begin{Bmatrix} \eta_1(t) \\ \eta_2(t) \\ \eta_i(t) \\ \eta_n(t) \end{Bmatrix}$



Vrhovi so odziv sistema pri določeni frekvenci (tu je lastna)

Usiljeno nihanje sistema z N>1 prostostnih stopenj (harmonično vzbujanje)

a) FIZIKALNE KOORDINATE
b) GLAVNE KOORDINATE

FIZIKALNE KOORDINATE

$[M]\{\ddot{x}\} + [k]\{x\} = \{F\} = \{F_0\} \sin \omega t$

{F} .. vektor vzbujevalnih sil (amplitud)

$\{F\} = \{F(t)\} = \begin{Bmatrix} F_{01} \\ F_{02} \\ \vdots \\ F_{0N} \end{Bmatrix} \sin \omega t = \{F_0\} \sin \omega t$

ω .. vzbujevalna frekvenca

Vsaj ena ≠0 sicer lastno nihanje

{X} vektor amplitud

Matematično je to sistem vezanih nehomogenih dif. en
Iščemo partikularno rešitev
Nastavek za enačbo:

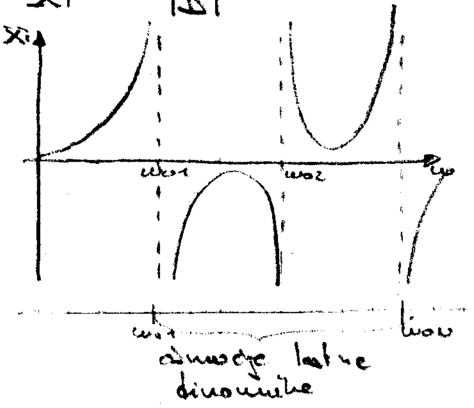
$\{x(t)\} = \{X\} \sin \omega t$

$\{\ddot{x}\} = -\omega^2 \{x\}$

$(-\omega^2 [M] + [k]) \{x\} \cancel{\sin \omega t} = \{F_0\} \cancel{\sin \omega t}$

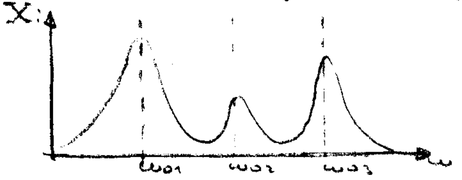
$(-\omega^2 [M] + [k]) \{x\} = \{F_0\}$

$X_i = \frac{|D_i|}{|D|}$ ko je Det proti 0 gre to odziv → ∞ amplituda



Če imamo prisotno še dušenje (vedno v praksi)

DUŠENJE – viskozno dušenje model
– suho drsno trenje
– strukturno dušenje



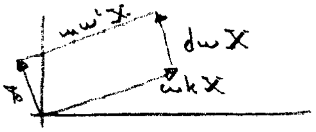
$[M]\{\ddot{x}\} + (\text{Disipacija}) + [k]\{x\} = \{F\}$

$\left.\begin{matrix} + [C]\{\dot{x}\} \\ + [D]\{x\} \end{matrix}\right\}$ za viskozno dušenje

[C,D] .. dušilna matrika

PROPORCIONALNO VISKOZNO DUŠENJE
RAYLEIGHOVO DUŠENJE

$[C] = \alpha [M] + \beta [k]$



Resonančno stanje

Dušenje ima največji vpliv ravno v resonančnem področju.
izven področja vpliv pade

GLAVNE KOORDINATE

$\{x\} = [\Phi][\eta]$

$[M]\{\ddot{x}\} + [K]\{x\} = \{F\}$

$\rightarrow [M][\Phi][\ddot{\eta}] + [k][\Phi]\{\eta\} = \{F\} \quad / [\Phi]^T$

$[{}^{*}M_{*}][\ddot{\eta}] + [{}^{*}k_{*}][\eta] = \{F\}[\Phi]^T$

$M_i \ddot{\eta} + k_i \eta = [X^{(i)}]^T \{F\} \quad / : M_i$

$\ddot{\eta} + \frac{k_i}{M_i}\eta = Q_i$ → ito posplošena zunanja sila

$\ddot{\eta} + \omega_{0i}^2 \eta = Q_i$

$[X^{(i)}]$ … lastni vektor

Q_i … posplošena sila

Partikularno rešitev:

$\eta_i(t) = E_i \sin \omega_{0i} t$

$E_i = \frac{\{X_i\}^T \{F\}}{X_i} \beta_i \quad ; \quad \beta_i = \frac{1}{\sqrt{1-\left(\frac{\omega}{\omega_{0i}}\right)^2}}$

Naš sistem je N
razsežen in ima N
prostostnih stopaj

Nihanje zveznih sistemov

Osnovne lastnosti:

- masa, togost, dušenje so zvezno porazdeljeni
- ∞ prostostnih stopenj
- ∞ lastnih oblik
- koordinate so funkcije kraja in časa

1D SISTEM

- struna - prečno nihanje
- palica - osno nihanje
- gredi - torzijsko nihanje
- nosilec - upogibno nihanje

$\left.\begin{matrix}\text{struna, palica, gredi}\end{matrix}\right\} \frac{d^2 q}{dx^2} \propto \frac{d^2 q}{dt^2}$

$\left.\text{nosilec}\right\} \frac{d^4 q}{dx^4} \propto \frac{d^2 q}{dt^2}$

2D SISTEM

- plošče, stene, lupine, membrane

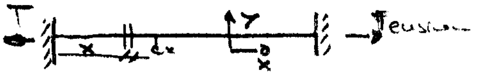
3D SISTEM

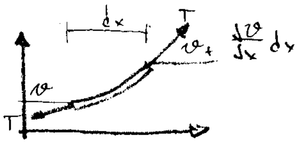
- oceani, možgani

Lastno nedušeno linearno nihanje

- prečno nihanje strune
- torzijsko nihanje gredi
- ravninsko upogibno nihanje

Prečno nihanje strune





PREDPOSTAVKE:

- osna sila T = konst
- majhni koti, mala nihanja
- $\mu = m/l$
- ni disipacije
- idealno gibko, ni prečne sile

Newtonov zakon

$\sum F_{iy} = m\ddot{y}$; $y = y(x,t)$

$-T\sin\vartheta + T\sin\left(\vartheta + \frac{d\vartheta}{dx}dx\right) = \mu\,dx\,\frac{d^2y}{dt^2}$; majhni koti $\sin\vartheta = tg\,\vartheta = \vartheta$

$\cancel{-T\vartheta} + T\left(\cancel{\vartheta} + \frac{d\vartheta}{dx}dx\right) = \mu\,dx\,\frac{d^2y}{dt^2}$; $\vartheta = \frac{dy}{dx}$

$T\frac{d\vartheta}{dx}\cancel{dx} = \mu\cancel{dx}\frac{d^2y}{dt^2}$; $c^2 = \frac{T}{\mu}$ ... hitrost širjenja valov

$c^2\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2}$

$\mapsto \boxed{\frac{d^2y}{dx^2} = \frac{1}{c^2}\frac{d^2y}{dt^2}}$ Valovna enačba / Gibalna enačba prečnega nihanja strune

Iščemo $y = y(x,t)$ pri danih R.P., Z.P. reši

Nastavek: $y(x,t) = X(x)\,T(t)$

' odvod po kraju

·· odvod po času

$X''T = \frac{d^2y(x,t)}{dx^2}$

$X\ddot{T} = \frac{d^2y(x,t)}{dt^2}$ $\rightarrow X''T = \frac{1}{c^2}X\ddot{T} \rightarrow \frac{X''}{X} = \frac{1}{c^2}\frac{\ddot{T}}{T} = konst = -\left(\frac{\omega}{c}\right)^2$

Enačba v času: $\ddot{T} + \omega^2 T = 0$; rešitev $T(t) = A\cos\omega t + B\sin\omega t$

A, B .. funkciji Z.P. za harmonično gibanje

Enačba v kraju: $X'' + \left(\frac{\omega}{c}\right)^2 X = 0$; rešitev $X(x) = C\cos\left(\frac{\omega}{c}x\right) + D\sin\left(\frac{\omega}{c}x\right)$

Z.P. $y(x,0) = ..$
$\dot{y}(x,0) = ...$

Podstavimo: $X(0) = 0 \rightarrow C = 0$

$X(l) = D\sin\left(\frac{\omega}{c}l\right) = 0$

$\mapsto \sin\frac{\omega l}{c} = 0 = n\pi$

C, D .. funkciji R.P

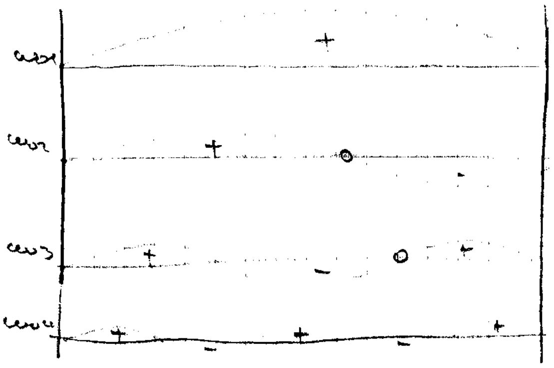
R.P. $y(0,t) = ..$
$y(l,t) = ..$

Lastne frekvence sistema

$\omega_{0i} = n\frac{\pi}{l}\sqrt{\frac{T}{\underbrace{\mu}_{c^2}}}$; n = 1...

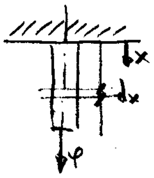
1 Lastna i = 1 $\omega_{01} = \frac{\pi}{l}\sqrt{\frac{T}{\mu}}$

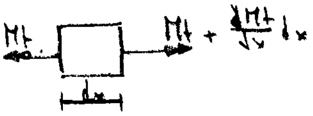
Lastne oblike $X(x) = D\sin\left(\frac{\omega_{0i}}{c}x\right)$



o ... vozli so točke mirovanja

Torzijsko nihanje gredi





J... osni vztrajnostni moment

Podano J, l, G

$\sum Mit = J\ddot{\varphi}$

$\cancel{-Mt} + \cancel{Mt} + \frac{dMt}{dx}dx = J/l \; dx \frac{d^2\varphi}{dt^2}$

trdnost: $Mt = kt\varphi = \frac{GIt}{l}\varphi = GIt \frac{d\varphi}{dx}$

$\frac{d}{dx}\left(GIt \frac{d\varphi}{dx}\right) = J/l \frac{d^2\varphi}{dt^2}$; $GIt = konst$

$\frac{GIt\,l}{J} \frac{d\varphi^2}{dx^2} = \frac{d^2\varphi}{dt^2} \rightarrow c^2 \frac{d^2\varphi}{dx^2} = \frac{d^2\varphi}{dt^2}$; $c^2 = \sqrt{\frac{GIt\,l}{J}}$

Valovna enačba

Iščemo $\varphi = \varphi(x,t)$

$\varphi = \Phi(x)\,T(t)$; $\Phi(x) = C\cos\left(\frac{\omega}{c}x\right) + D\sin\left(\frac{\omega}{c}x\right)$

R.P. $\Phi(0,t) = 0$; $C = 0$

$\Phi'(x=l) = 0$

⊗ $\int_D Mt = GIt \frac{d\varphi}{dx} = 0$

⊗ $\Phi'(x) = D\frac{\omega}{c}\cos\left(\frac{\omega}{c}x\right)$

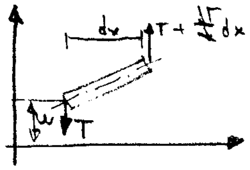
$\Phi(x=l) = D\frac{\omega}{c}\cos\left(\frac{\omega}{c}l\right) = 0$; $\cos\frac{\omega l}{c} = 0$; $\frac{\omega}{c}l = \frac{2i-1}{2}\pi$; $i = 1..\infty$

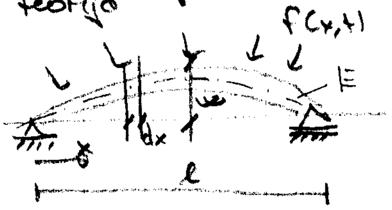
$\omega_{0i} = \frac{(2i-1)}{2}\frac{\pi}{l}\sqrt{\frac{GIt\,l}{J}}$ lastne frekvence

$c^2 = \sqrt{\frac{G}{\rho}}$; za okrogli polni presek

Ravninsko upogibno nihanje

Euler - Bernoullyjeva teorija





Predpostavke:

- mali koti
- planost prerezov
- zanemarimo strižne deformacije
- zanemarimo rotacijsko vztrajnost

y smer: $\sum Fiy = m\ddot{y}$

$\cancel{-T} + \cancel{T} + \frac{dT}{dx}dx = \eta\, dx = \overbrace{\rho A(x)\, dx}^{dV} \frac{dw^2}{dt^2}$; $T = \frac{dM}{dx}$; $M = -EI\frac{dw^2}{dx^2}$

$$\boxed{\frac{d}{dx^2}\left(EI(x)\frac{d^2w}{dx^2}\right) + \rho A(x)\frac{dw^2}{dt^2} = 0}$$ E-B enačba

T... prečna sila

$\eta = \frac{m}{l}$ ... masa na dolžino

I(x)... vztrajnost. prereza [m⁴]

če je prerez in vztrajnostni moment konstanten + homogen material

$EI\frac{d^4w}{dx^4} + \rho A\frac{d^2w}{dt^2} = 0 \rightarrow \boxed{\frac{EI}{\eta}\frac{d^4w}{dx^4} + \frac{d^2w}{dt^2} = 0}$

Metode	upog. mom.	strižn. sila	rotac. vztr.	strič. defor.
EB	+	+	−	−
strižno	+	+	−	+
rayleigh	+	+	+	−
timoshenko	+	+	+	+

Iščemo $w = w(x,t)$

Ločimo kraj in čas $w(x,t) = X(x)\,T(t)$

$\frac{d^4w}{dx^4} = X''''T$

$\frac{d^2w}{dt^2} = X\ddot{T}$

$\rightarrow \frac{EI}{\eta}X''''T + X\ddot{T} = 0 \;/\; XT$; $c^2 = \frac{EI}{\eta}$

$c^2\frac{X''''}{X} = -\frac{\ddot{T}}{T} = konst = \omega^2$

$\underset{L(x)}{}$ L(x) D(t)

- $\ddot{T} + \omega^2 T = 0$ čas

$T(t) = A\cos\omega t + B\sin\omega t$

A,B ... konstanti + funkcija zač. pog.

- $X^{IV} - \beta^4 X = 0$; $\beta^4 = (\frac{\omega}{c})^2$ kraj

Nastavek: $X(x) = Ce^{\lambda x}$
$X^{IV}(x) = C\lambda^4 e^{\lambda x}$ $\rightarrow Ce^{\lambda x}(\lambda^4 - \beta^4) = 0$

to $\lambda^4 - \beta^4 = 0$
$\lambda_{1,2} = \pm \beta$
$\lambda_{3,4} = \pm i\beta$ imaginarna korena

Splošna rešitev:

$X(x) = C_1 e^{\beta x} + C_2 e^{-\beta x} + C_3 e^{i\beta x} + C_4 e^{-i\beta x}$

$X(x) = D_1 \operatorname{ch}\beta x + D_2 \operatorname{sh}\beta x + D_3 \cos\beta x + D_4 \sin\beta x$

$D_{1...4}$ ... konstante in f(RP)

Aproksimativne metode

Modeli - analitični - eksaktni
- aproksimativni - numerični
- eksperimentalni - empirični

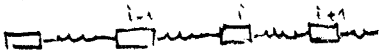
proti realnim sistemom

1) Rayleigh - Ritz
2) Dunkerley (Kull)
3) Metoda matričnih iteracij

4) Metoda prenosnih matrik MPM

- zasluge Holzer, Tolle
- za linijske sisteme
- osno nihanje palic
- torzijsko nihanje
- upogib nosilcev

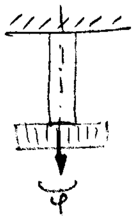
- uporabna za statično, lastno nihanje in vsiljeno nihanje

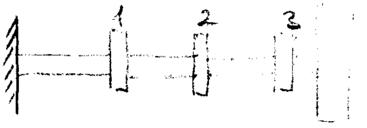


diskretni sistem za osno obremenitev

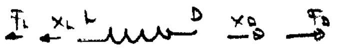
$[k] = [\ \ddots\ k_i \ \ddots\ ]$

Osnovna ideja mehanike





- Prenos preko vzmeti



$F_D - F_L = 0$; $F_D = F_L$

$x_D - x_L = \frac{F_L}{k_i}$

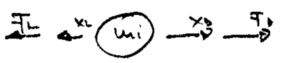
$x_D = x_L + \frac{F_L}{k_i}$

$$\begin{Bmatrix} x_D \\ F_D \end{Bmatrix} = \begin{bmatrix} 1 & 1/k_i \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} x_L \\ F_L \end{Bmatrix}$$

$\{U\}_D = [P]\{U\}_L$

osnovna prenosna matrika

- Prenos preko mase



$x_D - x_L = 0$

$F_D - F_L = m_i \ddot{x}_L$; $F_D = -\omega^2 m_i x_L + F_L$

$$\begin{Bmatrix} x_D \\ F_D \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ -m_i\omega^2 & 1 \end{bmatrix} \begin{Bmatrix} x_L \\ F_L \end{Bmatrix}$$

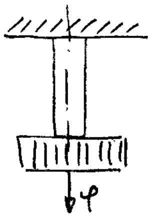
$[M]$

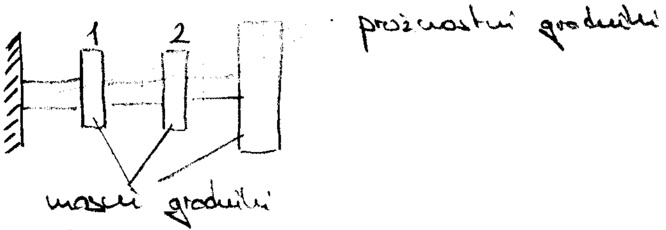
masna prenosna matrika

$\{U\}$ ... vektor stanja

$\{U\} = \begin{Bmatrix} x(t) \\ F(t) \end{Bmatrix}$

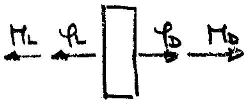
MPM - Torzijska nihanja





Vektor stanja $\{v\} = \begin{Bmatrix}\varphi \\ M_t\end{Bmatrix}$; Ovrednotimo produkt $\{v\}$ preko osnovnih gradnikov

MASNI ELEMENT



$\varphi_D - \varphi_L = 0 \rightarrow \varphi_D = \varphi_L$

$M_D - M_L = J\ddot{\varphi} = -\omega^2 J\varphi$ Predpostavimo harmonično nihanje

$\varphi_L = \varphi_L \sin\omega t$

ŽELIMO $\{v\}_D = [\text{prenosna matrika}]\{v\}_L$

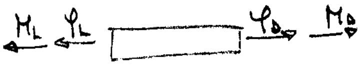
$\varphi_D \; \varphi_L$

$M_D = -\omega^2 J\varphi_L + 1 M_L$

$$\begin{Bmatrix}\varphi \\ M\end{Bmatrix}_D = \begin{bmatrix}1 & 0 \\ -J\omega^2 & 1\end{bmatrix}\begin{Bmatrix}\varphi \\ M\end{Bmatrix}_L$$

$[M]$ .. prenosna matrika preko mase

PROŽNOSTNI ELEMENT



k_t .. torzijska togost

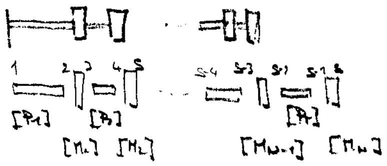
$M_D - M_L = 0 \quad M_D = M_L$

$\varphi_D - \varphi_L = \frac{M_L}{k_t} \rightarrow \varphi_D = \varphi_L + \frac{M_L}{k_t}$ Iz torzije $M_t = k_t\varphi = \frac{G I_t}{l}\varphi$

$$\begin{Bmatrix}\varphi \\ M\end{Bmatrix}_D = \begin{bmatrix}1 & 1/k_t \\ 0 & 1\end{bmatrix}\begin{Bmatrix}\varphi \\ M\end{Bmatrix}_L$$

$[P]$ .. prenosna matrika preko polja

začnemo na robu sistema



$\{v\}_D = [P]\{v\}_L$ posamezno enačbo

$\{v\}_2 = [P_1]\{v\}_1$

$\{v\}_3 = [M_1]\{v\}_2 = [P_1][M_1]\{v_1\}$

$\vdots \qquad \vdots \qquad \vdots$

$\{v\}_S = [M_n]\{v\}_{S-1} = \underbrace{[M_n][P_n][M_{n-1}]\cdots[M_2][P_2][M_1][P_1]}_{PM \text{ celega sistema}}\{v\}_1$

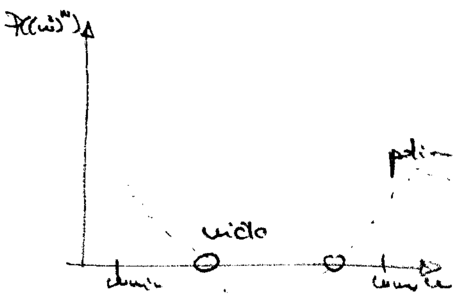
$\{v\}_D = [PM]\{v\}_L$

Robni pogoji:

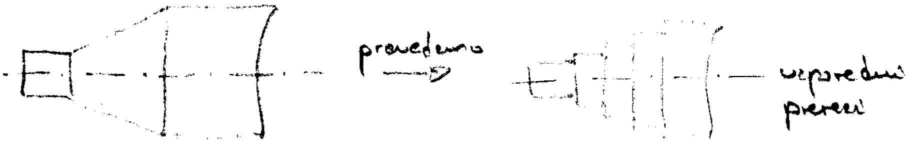
$$\begin{Bmatrix}\varphi_D \\ M_D\end{Bmatrix} = \begin{Bmatrix}\varphi_D \\ 0\end{Bmatrix} = \begin{bmatrix}p_{m11} & p_{m12} \\ p_{m21} & p_{m22}\end{bmatrix}\begin{Bmatrix}0 \\ M_L\end{Bmatrix}$$

1) $\varphi_D = p_{m12} M_L$

2) $0 = p_{m22} M_L$; $p_{m22} = 0 \rightarrow P((\omega^2)^n) = 0$ polinom n presečišče
stopnje
frekvence



Metoda prenosne matrike PMP za upogibno vibracijo

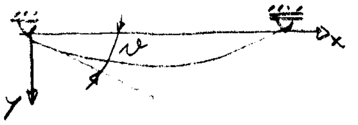


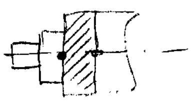
Vektor stanja:

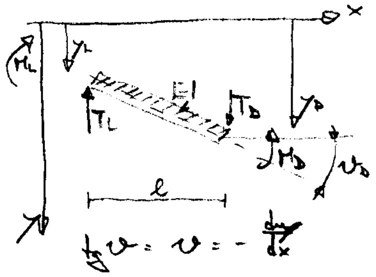
$$\{v\} = \begin{Bmatrix} y \\ \vartheta \\ M \\ T \end{Bmatrix}$$

- upogibna deformacija
- zasuk
- upogibni moment
- prečna sila

PRENOS PREKO BREZMASNEGA ELASTIČNEGA POLJA







$\operatorname{tg}\vartheta = \vartheta = -\frac{dy}{dx}$

$$M = -EI\,y''$$

$$T = -EI\,y''' = \frac{dM}{dx}$$

$$g = -\frac{dT}{dx} = EI\,y^{IV}$$

g.. kontinuirna obremenitev

g=0 brezmasni element

Začetek

$g=0\ ;\quad EI\,y^{IV}=0 \longrightarrow y^{IV}=0 \qquad 4\times\int$

$$y(x) = A + Bx + Cx^2 + Dx^3$$

Levi rob premaknemo v $x=0$

$y(x=0) = y_L = A$
$\vartheta(x=0) = \vartheta_L = -y'(x=0) = B$
$M(x=0) = M_L = -EI\,2C$
$T(x=0) = T_L = -6EI\,D$

Desni rob premaknemo v $x=L$

$y(x=L) = y_D = y_L - \vartheta_L L - \frac{M_L}{2EI}L^2 - \frac{T_L}{6EI}L^3$
$\vartheta(x=L) = \vartheta_D = y'(x=L) = \vartheta_L + \frac{M_L}{EI}L + \frac{T_L}{2EI}L^2$
$M(x=L) = M_D = M_L + T_L L$
$T(x=L) = T_D = T_L$

Upogibnica zgleda

$$y(x) = y_L - \vartheta_L x - \frac{M_L}{2EI}x^2 - \frac{T_L}{6EI}x^3$$

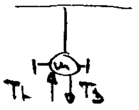
Pazi minuse

$$\{v\}_D = [P]\{v\}_L \quad ; \quad [P] = \begin{bmatrix} 1 & L & L^2/2EI & L^3/6EI \\ 0 & 1 & L/EI & L^2/2EI \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Prenosna matrika preko brezmasnega polja

$$\{v\} = \begin{Bmatrix} y \\ \vartheta \\ M \\ T \end{Bmatrix}$$

PRENOS PREKO MASNE TOČKE

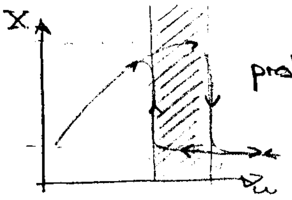


$y_D = y_L$
$\vartheta_D = \vartheta_L$
$M_D = M_L$
$T_D - T_L = m\ddot{y}_L = -m\omega^2 y_L$ (predpostavka harmoničnega gibanja)
$T_D = -m\omega^2 y_L + T_L$

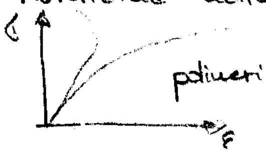
$$\{v\}_D = [M]\{v\}_L \qquad [M] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ m\omega^2 & 0 & 0 & 1 \end{bmatrix}$$

Prenosna matrika preko masne točke

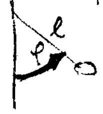
Nelinearno nihanje

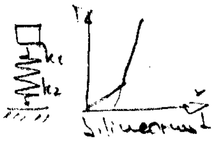


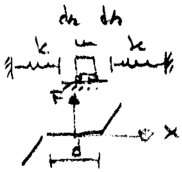
- Materialna nelinearnost



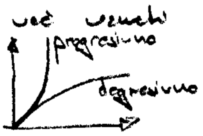
- kinematična nelinearnost
  - veliki pomiki (nihalo)

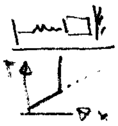


- Geometrijske nelinearnosti
- 

- 

- več vzmeti



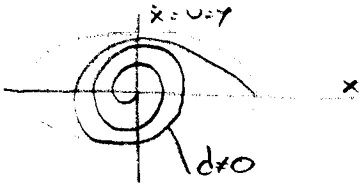
- 

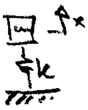
- Rotordinamika

METODE OBRAVNAVANJA

- Geometrijske metode (ravnine, fazni prostori)
- Eksaktne, analitične metode
- Aproksimativne metode (lastna nihanja, metoda iteracij, num. metode)

FAZNA RAVNINA





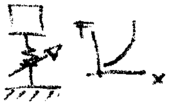
$\dot{x}=y$

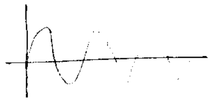
$\dot{y}=\ddot{x}=-\omega^2 x \rightarrow \frac{dy}{dx}=-\omega^2\frac{x}{y}$

to $y^2+\omega_0^2x^2=c$ elipsa

NN
- lastna z $N=1$
- Lindstedt perturbacijska metoda (mali odklon od časa)

$$\boxed{\ddot{x}+\omega_0^2x+\mu x^3=0}$$





μx^3 nelinearnost

$$\boxed{x(t)=x_0(t)+\mu x_1(t)}+\overset{\longrightarrow\,//}{\mu^2x_2(t)+\ldots}$$

; če $\mu=0$ lin. sis.

μ .. parameter nelin..

$\omega=\sqrt{k/m}$

$$\boxed{\omega^2=\omega_0^2+\mu\alpha_1}+\overset{\longrightarrow\,//}{\mu\alpha_2+\ldots}$$

$x_{1,2}$ .. f(ZP)

ω_0^2 .. lastna frekvenca linearnega problema

Sledi:

$(\ddot{x}_0+\mu\ddot{x}_1)+\omega_0^2(x_0+\mu x_1)+\mu(x_0+\mu x_1)^3=0$

$\ddot{x}_0+\mu\ddot{x}_1+(\omega^2-\mu\alpha_1)(x_0+\mu x_1)+\mu(x_0^3+3x_0\mu x_1+3x_0(\mu x_1)^2+(\mu x_1)^3)=0$

ω .. lastna frekvenca nelinearnega problema

μ^0 : $\ddot{x}_0+\omega^2x_0=0$

μ^1 : $\ddot{x}_1+\omega^2x_1-\alpha_1x_0+x_0^3$

μ .. eksperimentalno dolo

$x_0(0) = 0$

$x_1(0) = 0$

$x_0(t) = A\cos\omega t + B\sin\omega t$
$x_0(t) = A\cos\omega t$ (samo cos odziv) (generična rešitev)

$\ddot{x}_1 + \omega^2 x_1 = \alpha_1 x_1 - x_0^3 = \alpha_1 A\cos\omega t - (A\cos\omega t)^3$

$\cos^3 x = 3/4 \cos x + 1/4 \cos 3x$

$\ddot{x}_1 + \omega^2 x_1 = \alpha_1 A\cos\omega t - A^3 3/4 \cos\omega t - A^3 1/4 \cos 3\omega t$

$\ddot{x}_1 + \omega^2 x_1 = \cos\omega t \underbrace{(\alpha_1 A - A^3 3/4)}_{x_1(t) = k_1 t \cos\omega t} - \underbrace{A^3 1/4}_{\text{amplituda vzbujevalne sile}} \underbrace{\cos 3\omega t}_{\text{frekvenca vzbujanja}}$

Zagotoviti fizikalno sprejemljive pomike

$\ddot{x}_1 + \omega^2 x_1 = 0$

$\alpha_1 A - A^6 3/4 = 0$

$\alpha_1 = 3/4 A^2$

$\ddot{x}_1 + \omega^2 x_1 = - A^2 1/4 \cos 3\omega t$

$x_1(t) = x_{1H}(t) + x_{1P}(t)$

$x_{1H} = D_1 \cdot \cos\omega t + D_2 \sin\omega t$

$x_{1P} = C \cos 3\omega t$

$(-9C\omega^2 + C\omega^2)\cos 3\omega t = -A^3/4 \cos 3\omega t \quad ; \quad C = A^3/32\omega^2$

$x_1(t) = D_1\cos\omega t + D_2 \cos\omega t + A^3/32\omega^2 \cos 3\omega t$

$x_1(0) = 0 \rightarrow = D_1 + A^3/32\omega^2 + \cancel{D_2} \Rightarrow D_1 = - A^3/32\omega^2$

$\dot{x}_1(0) = 0 \rightarrow = D_2\,\omega \dots \rightarrow D_2 = 0$

$x(t) = \left(A - \frac{\mu A^3}{32\omega^2}\right)\cos\omega t + \frac{\mu A^3}{32\omega^2}\cos 3\omega t$ } Skupna rešitev

$x(t) = x_0(t) + \mu x_1(t)$

Metoda iteracij / vsiljeno nihanje

Duffing

$m\ddot{x} + \cancelto{0}{d\dot{x}} + kx \pm \mu_1 x^3 = F_1 \cos\omega t \quad / : m$

$\ddot{x} + \omega_0^2 x \pm \mu x^3 = F \cos\omega t$

Odziv v ustaljenem stanju

Začetni približek: $x_0(t) = A \cos\omega t$

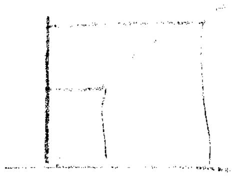
$\dot{x} = -\omega_0^2 x \mp \mu x^3 + F\cos\omega t$

$\dot{x} = -\omega_0^2 A\cos\omega t \mp \mu A^3 \cos^3\omega t + F\cos\omega t$

$\dot{x} = (-\omega_0^2 A \mp \mu 3/4 A^2 + F)\cos\omega t \mp 1/4 \mu A^3 \cos 3\omega t \quad \int 2x$

$x_1(t) = -1/\omega^2 (-\omega_0^2 A \mp 3/4 \mu A^2 + F)\cos\omega t \pm \cancel{1/9\omega^2}\ \cancel{1/4 \mu} A^3 \cos 3\omega t + \cancel{C_1 t} + \cancel{C_2}$

$x_1(t) = -1/\omega^2 (-\omega_0^2 A \mp 3/4 \mu A^3 + F)\cos\omega t$



log konvergenca

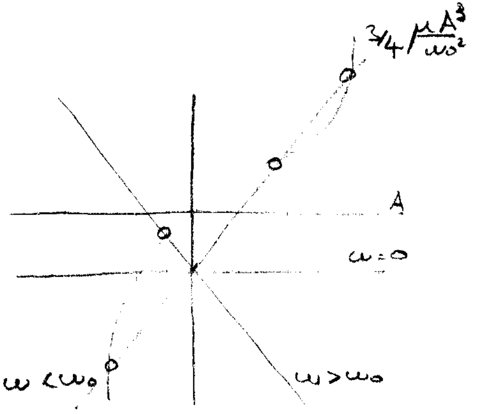
$x_0(t) = A\cos\omega t$

$A = 1/\omega^2 (\omega_0^2 A \pm 3/4 \mu A^3 - F) \quad / \omega^2/A$

$\omega^2 = \omega_0^2 \pm 3/4 \mu A^2 - F/A$

1) $F=0$ homogena enačba

2) $\mu=0$ lin. nihanje



$$A \frac{\omega^2}{\omega_0^2} = A \pm 3/4 \frac{\mu A^3}{\omega_0^2} - \frac{F}{\omega_0^2}$$

$$3/4 \frac{\mu A^3}{\omega_0^2} = A\left(1 - \frac{\omega^2}{\omega_0^2}\right) - \frac{F}{\omega_0^2}$$

$$\downarrow \quad = D$$