DINAMIKA (STROJNIŠKA)

- ANALITIČNE METODE
- NUMERIČNE METODE
- EKSPERIMENTALNE METODE
  - L FIZ. PRINCIPI PRETVORBE VELIČIN
  - L SENZORJI, KALIBRACIJE
  - L MER. INSTRUMENTI, OJAČ., FILTRI, ..
  - L HARDWARE ZA ZAPIS PODATKOV.

* ANALIZA SIGNALOV (DATA ANALYSIS, DIGITAL SIGNAL PROCESSING)

- ZAJEMANJE SIGNALOV (DATA ACQUISITION)
- OBDELAVA SIGNALOV (SIGNAL PROCESSING)
- RAZLAGA REZULTATOV (INTERPRETATION)

RAZVRSTITEV SIGNALOV (DATA CLASSIFICATION)

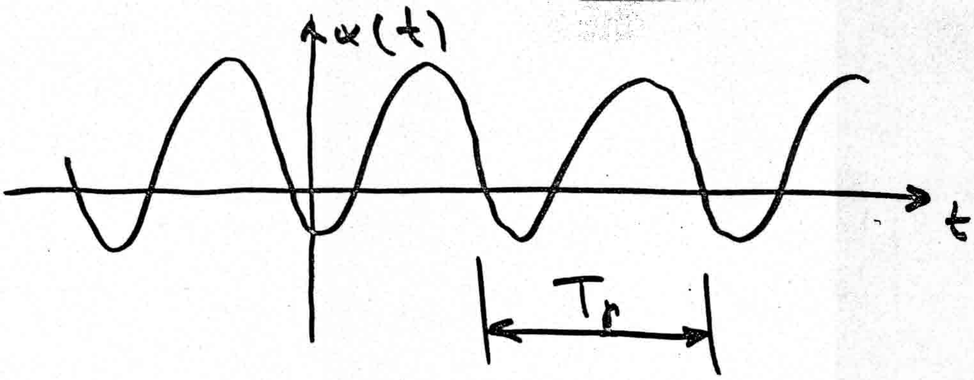
- SIGNALI
  - DETERMINISTIČNI
    - PERIODIČNI
    - NEPERIODIČNI
      - TRANSIENTNI
      - KVASI PERIODIČNI
  - NAKLJUČNI
    - STACIONARNI
    - NESTACIONARNI

IZ VSEBINE

- FOURIER-JEVE VRSTE (PERIODIČNI S.)
- FOURIER-JEV INTEGRAL (TRANSIENTI)
- $\delta(t)$ - DIRAC-OVA DELTA FUNKCIJA
- KONVOLUCIJA
- ČASOVNA OKNA (WINDOWING)
- A/D KONVERZIJA
- DINAMIČNI OBSEG (DYNAMIC RANGE)
- POPAČENJE, VZORČENJE (ALIASING)
- DISKRETNA FOURIER-JEVA TRANSFORMACIJA (DFT), ALGORITEM FFT
- NAKLJUČNI PROCESI (TIME SERIES ANALYSIS)
- VERJETNOST
- NAKLJUČNA SPREMENLJIVKA
- STACIONARNOST
- KORELACIJSKE FUNKCIJE
- SPEKTRALNE FUNKCIJE
- VHODNO-IZHODNE POVEZAVE (INPUT, OUTPUT)
- IDENTIFIKACIJA SISTEMOV

FOURIER - JEVE VRSTE

$x(t) = x(t + T_p)$ (REALNI PERIODIČNI SIGNALI)



$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cdot \cos\left(\frac{2\pi n t}{T_p}\right) + b_n \cdot \sin\left(\frac{2\pi n t}{T_p}\right) \right]$$

kjer

$\frac{a_0}{2}$ - SR. VREDNOST SIGNALA (D.C. LEVEL)

$$a_n = \frac{2}{T_p} \cdot \int_{-T_p/2}^{T_p/2} x(t) \cdot \cos\left(\frac{2\pi n t}{T_p}\right) \cdot dt$$

$$b_n = \frac{2}{T_p} \cdot \int_{-T_p/2}^{T_p/2} x(t) \cdot \sin\left(\frac{2\pi n t}{T_p}\right) dt$$

L.M.S - LEAST MEAN SQUARE OPTIMISATION

T_p - BAZIČNA, OSNOVNA PERIODA

$f_0 = \frac{1}{T_p}$ - BAZIČNA, OSNOVNA FREKVENCA
FUNDAMENTAL FREQUENCY

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} M_n \cdot \cos(2\pi n f_0 \cdot t - \Phi_n),$$

$$M_n = \sqrt{a_n^2 + b_n^2}, \quad \mathrm{tg}\,\Phi_n = \frac{b_n}{a_n}$$

KOMPLEKSNI ZAPIS FOURIER-JEVIH VRST

$$e^{\pm i\vartheta} = \cos\vartheta \pm i\cdot\sin\vartheta$$

če $c_0 = \frac{a_0}{2}$, $c_n = \frac{a_n - i\cdot b_n}{2}$, $c_n^* = \frac{a_n + i\cdot b_n}{2}$

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} c_n \cdot e^{\frac{i\,2\pi n t}{T_p}}}$$

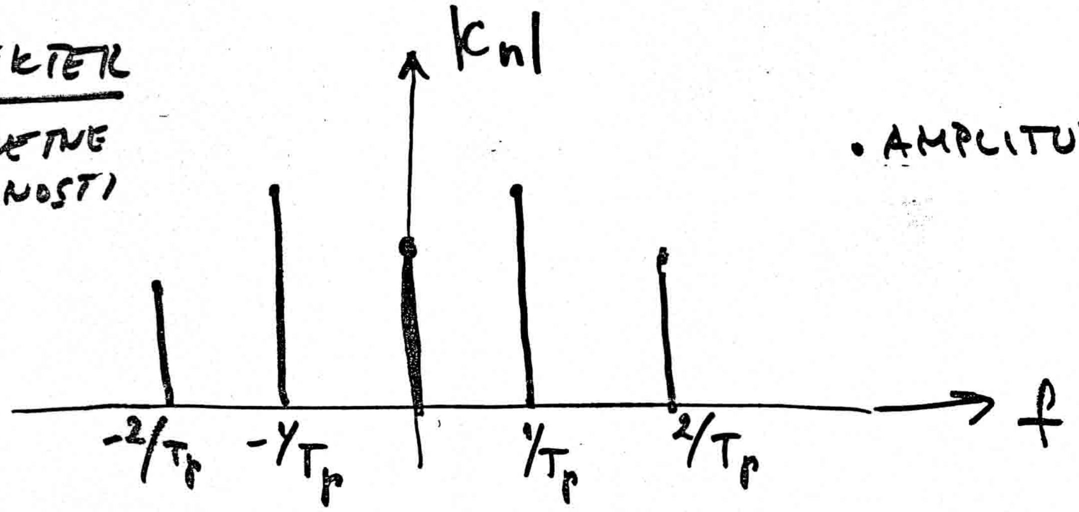
INVERSACIJA!

$$\boxed{c_n = \frac{1}{T_p}\cdot\int_{-T_p/2}^{T_p/2} x(t)\cdot e^{-\frac{i\,2\pi n t}{T_p}}\cdot dt}\quad,\qquad c_n^* = c_{-n}$$

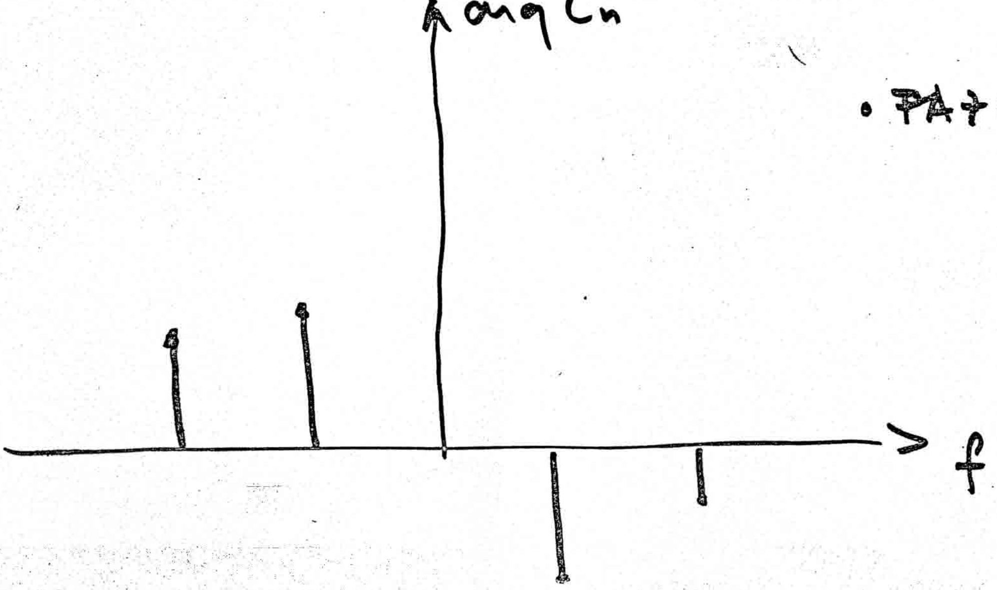
SPEKTER

DISKRETNE VREDNOSTI

• AMPLITUDNI



• FAZNI



MOČNOSTNI SPEKTER

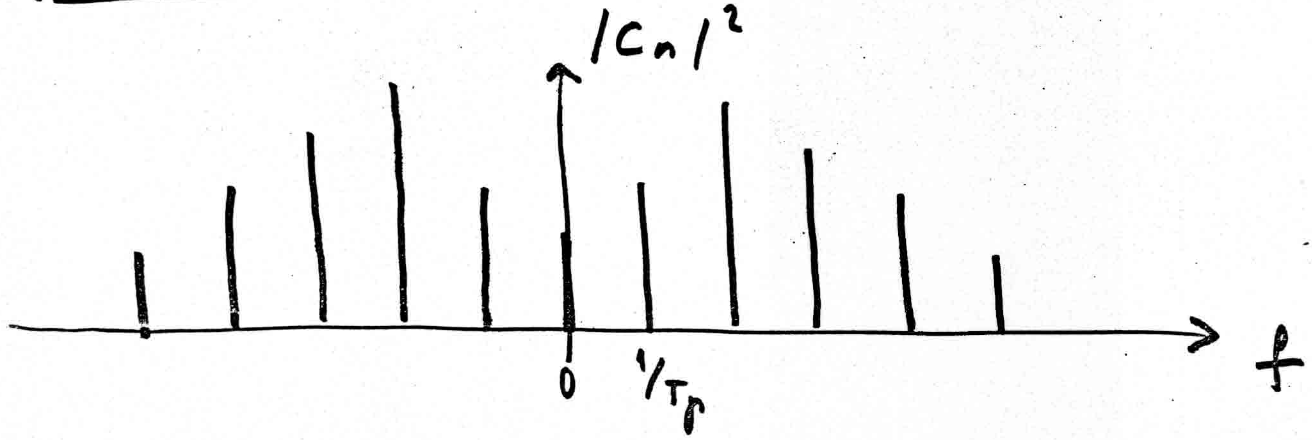
$$x(t)=U \longrightarrow \boxed{R=1} \longrightarrow$$

$$\overline{P} = \frac{1}{T_p}\cdot\int_{-T_p/2}^{T_p/2} x^2(t)\cdot dt \quad , \quad x^2(t) = x(t)\cdot x^*(t)$$

PRINCIP ORTOGONALNOSTI

$$\Rightarrow \boxed{\frac{1}{T_p}\cdot\int_{-T_p/2}^{T_p/2} x^2(t)\cdot dt = \sum_{n=-\infty}^{\infty} |C_n|^2}$$

PARCEVAL-ov TEOREM



MOČNOSTNI SPEKTER - DEKOMPOZICIJA PROCESA PO FREKVENCAH

ČE ŽELIMO $f_i \geq 0 \Rightarrow$

$C_0 \rightarrow C_0$

$C_n\Big|_{-\infty}^{\infty} = 2\cdot C_n\Big|_{0}^{\infty} \quad , f_i > 0$

FOURIER-JEV INTEGRAL

$$x(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{i\frac{2\pi n t}{T_p}}, \quad C_n = \frac{1}{T_p}\cdot\int_{-T_p/2}^{T_p/2} x(t)\cdot e^{-\frac{i2\pi n t}{T_p}}\cdot dt$$

potem

$$C_n = \lim_{\substack{T_p\to\infty \\ \Delta f\to 0}} \Delta f \cdot \int_{-T_p/2}^{T_p/2} x(t)\cdot e^{-i2\pi\cdot f_n\cdot t}\cdot dt \quad \Big/ : \Delta f$$

$$f_n = n\cdot\frac{1}{T_p}$$

$$\lim_{\Delta f\to 0}\left(\frac{C_n}{\Delta f}\right) = \underline{X(f)} = \int_{-\infty}^{\infty} x(t)\cdot e^{-i2\pi f t}\cdot dt$$

! AMPLITUDNA GOSTOTA

TAKO IMAMO TRANSFORMACIJSKI PAR

$$x(t) = \lim_{\Delta f\to 0}\sum_{n} \underbrace{X(f)\cdot\Delta f}_{C_n}\cdot e^{i2\pi f_n\cdot t}$$

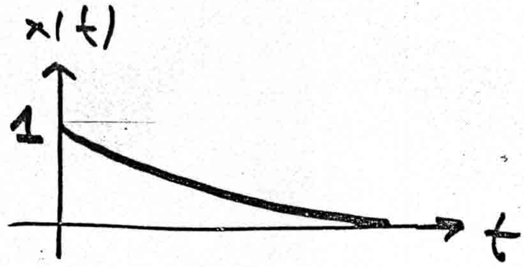
$$\boxed{\begin{aligned} x(t) &= \int_{-\infty}^{\infty} X(f)\cdot e^{i2\pi f t}\cdot df \\ X(f) &= \int_{-\infty}^{\infty} x(t)\cdot e^{-i2\pi f t}\cdot dt \end{aligned}}$$

LAHKO TUDI $\omega = 2\pi f$, ...

! $(\Delta t)\cdot(\Delta f) > 1/2$ UNCERTAINTY PRINCIPLE

PRIMER

$$x(t) = \begin{cases} e^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



$$\bar{X}(f) = \int_0^\infty e^{-at} \cdot e^{-i2\pi f t} \cdot dt = \int_0^\infty e^{-t(a+i2\pi f)} \cdot dt =$$

$$= \frac{1}{-(a+i2\pi f)} \cdot e^{-(a+i\pi 2f) \cdot t} \Big|_0^\infty = \frac{1}{a+i2\pi f} =$$

$$= Re\{\bar{X}(f)\} + i \cdot Im\{\bar{X}(f)\} \quad (i) \quad \text{ali pa}$$

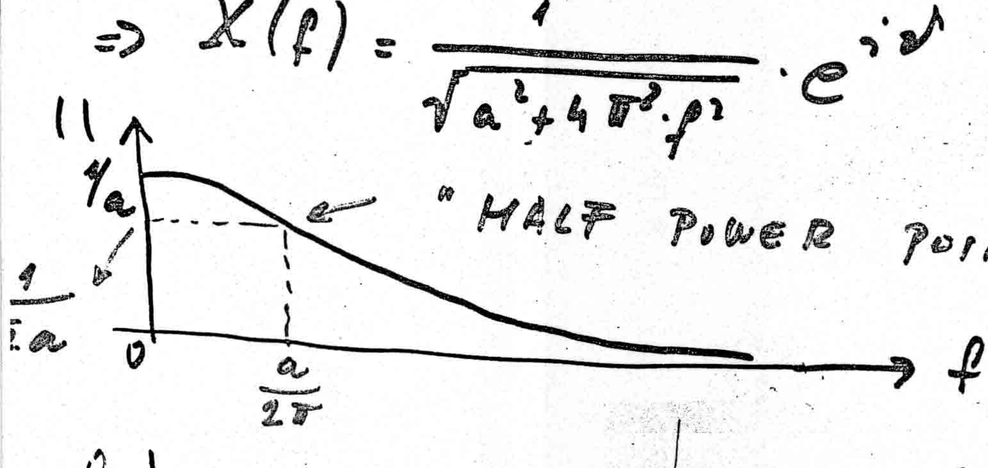
$$|\text{Modul}| \cdot e^{i\,\arg} \quad (ii)$$

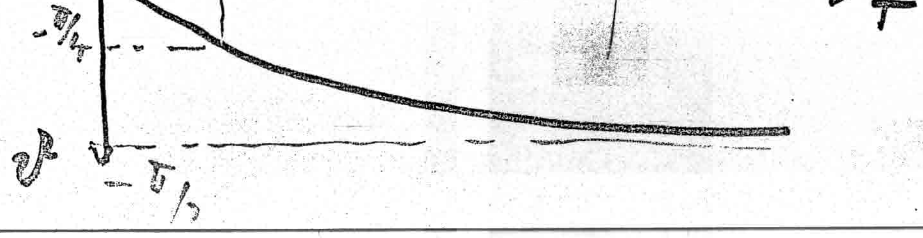
i) $$\bar{X}(f) = \frac{a - i2\pi f}{a^2 + 4\pi^2 f^2} = \frac{a}{a^2+4\pi^2 f^2} - i \cdot \frac{2\pi f}{a^2 + 4\pi^2 \cdot f^2}$$

ii) $$|\text{Modulus}| = \sqrt{Re^2\{z\} + Im^2\{z\}} = \frac{1}{\sqrt{a^2+4\pi^2 \cdot f^2}}$$

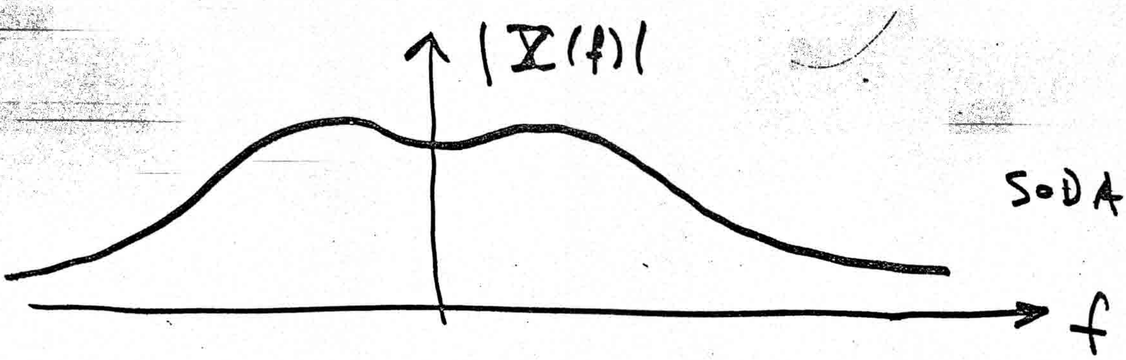
$$tg\,\vartheta = \frac{Im\{z\}}{Re\{z\}} = -\frac{2\pi f}{a}$$

$$\Rightarrow \bar{X}(f) = \frac{1}{\sqrt{a^2+4\pi^2 \cdot f^2}} \cdot e^{i\vartheta}, \quad tg\,\vartheta = -\frac{2\pi f}{a}$$

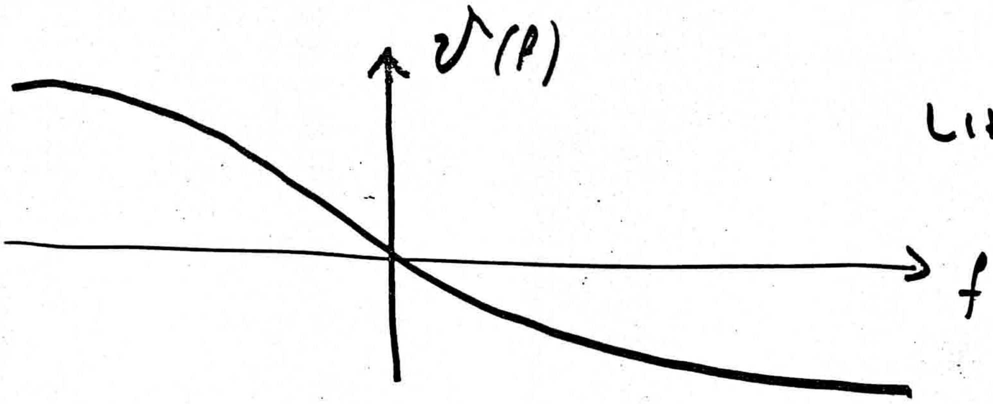




PRIKAZ $X(f)$ V SPLOŠNEM

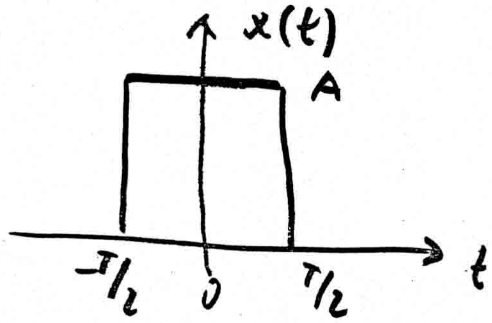


SODA FUNK.



LIHA FUNK.

ŠE EN PRIMER!

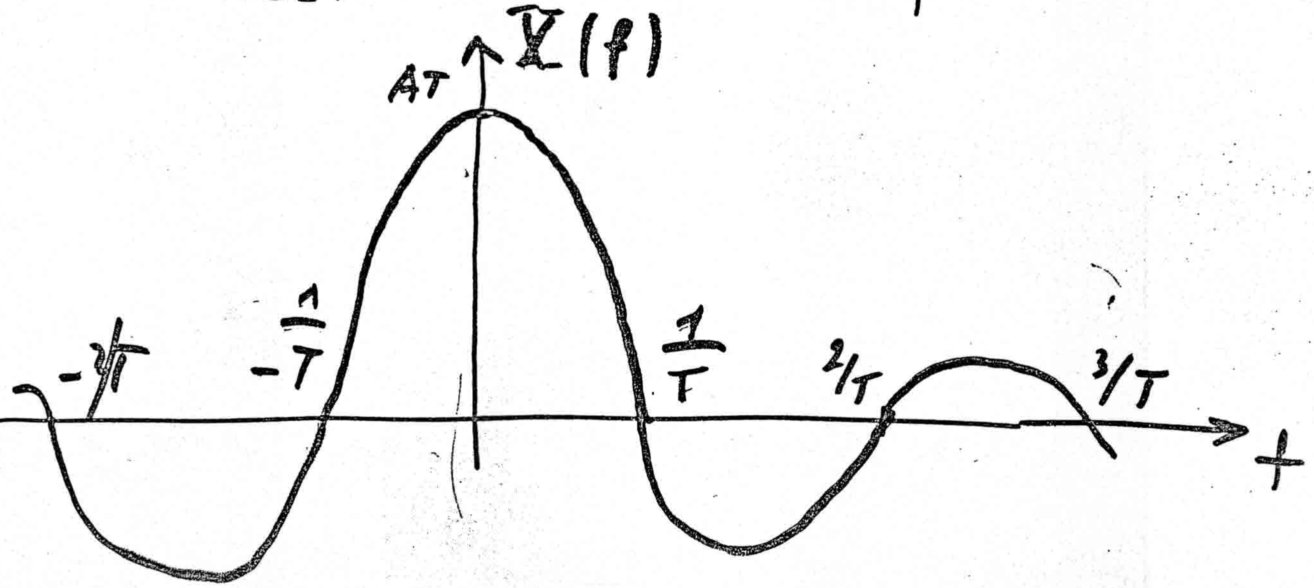


$$X(f) = ?$$

$$X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j 2\pi f t} \cdot dt$$

POKAŽITE, DA JE REALEN

$$X(f) = \frac{A \cdot T \cdot \sin(\pi f T)}{\pi \cdot f \cdot T}$$



ENERGIJSKI SPEKTER

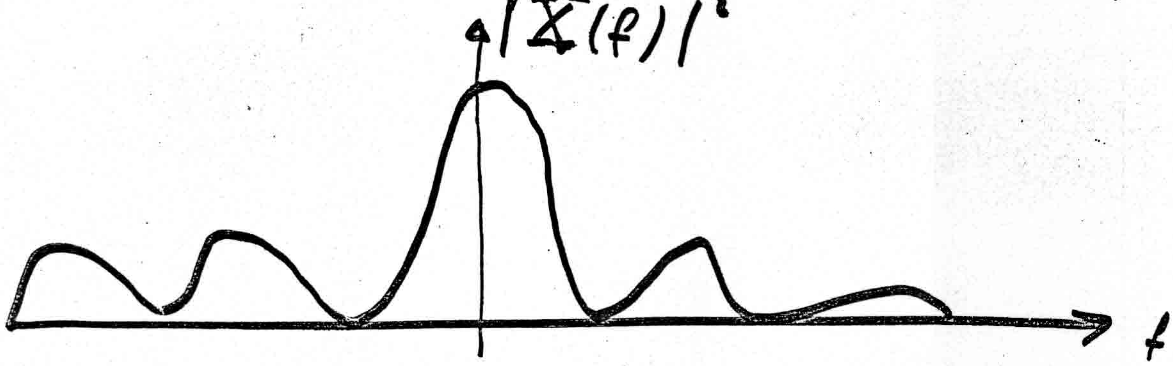
$$\int_{-\infty}^{\infty} x^2(t)\, dt$$ – CELOTNA ENERGIJA SIGNALA

če $x^2(t) = x(t) \cdot x^*(t)$, $\Rightarrow$

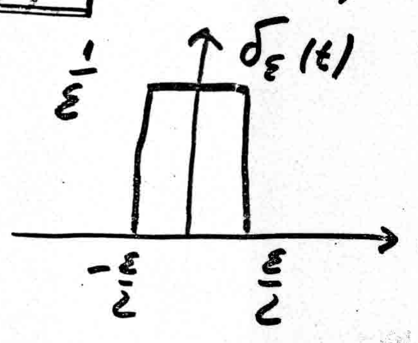
$$\int_{-\infty}^{\infty} x^2(t)\, dt = \int_{-\infty}^{\infty} |X(f)|^2 \cdot df$$

PR.

$|X(f)|^2$ – ENERGIJA NA ENOTO PASOVNE ŠIRINE (ENERGY PER UNIT BANDWIDTH) = = ENERGIJSKA SPEKTRALNA GOSTOTA



DELTA FUNKCIJA $\delta(t)$ (DIRAC)



$\delta(t) = 0 \quad , \quad t \neq 0$

$\delta(t) = \lim_{\varepsilon \to 0} \delta_\varepsilon(t)$

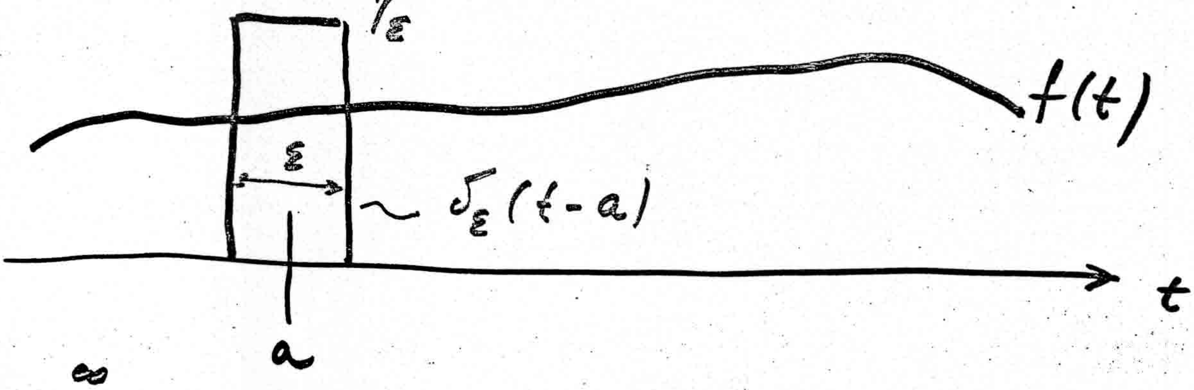
i) $\int_{-\infty}^{\infty} \delta(t) \cdot dt = 1$

ii) $\delta(t) = \delta(-t)$

iii) $$\int_{-\infty}^{\infty} f(t)\cdot \delta^{k}(t-a)\cdot dt = f^{k}(a)$$

VELJA TUDI ZA k-TE ODVODE

↑ SIFTING INTEGRAL



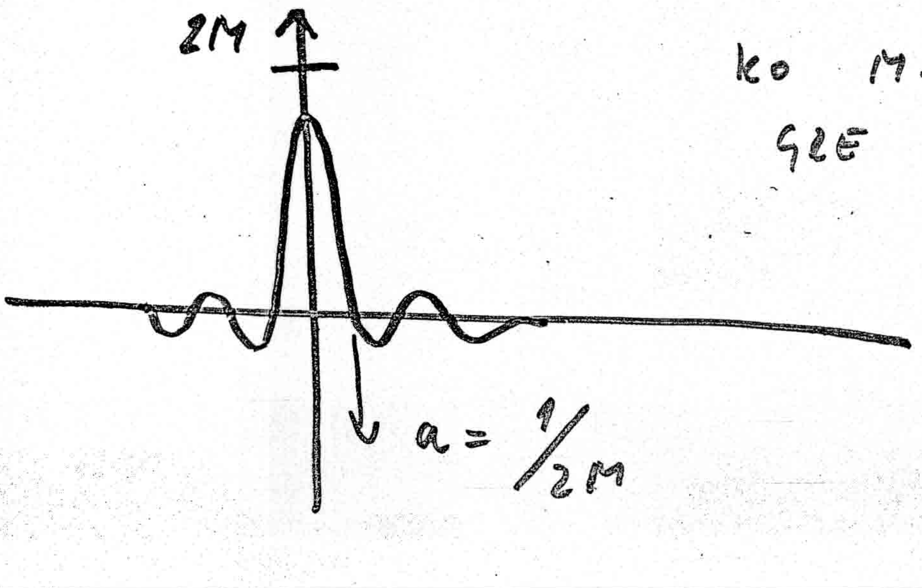
iv) $$\int_{-\infty}^{\infty} e^{i2\pi at}\cdot dt = \delta(a)$$

NE PO OBIČAJNI POTI, AMPAK

$$\lim_{M\to\infty}\int_{-M}^{M} e^{i2\pi at}\cdot dt = \lim_{M\to\infty}\int_{-M}^{M}\left[\cos(2\pi at) + i\cdot \underbrace{\sin(2\pi at)}_{0}\right] dt$$

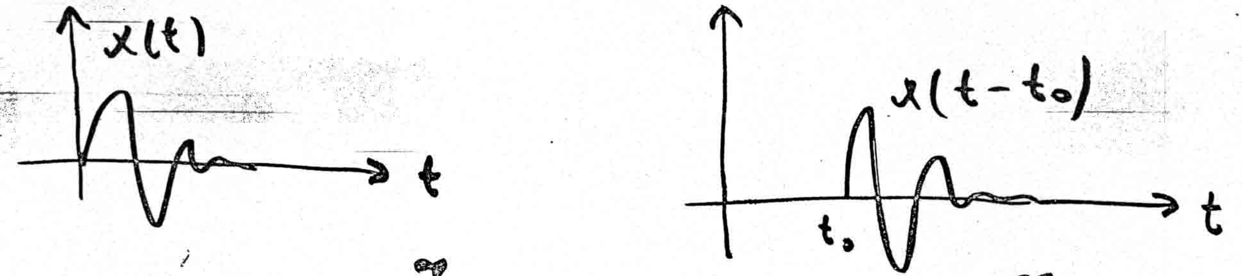
$$= \lim_{M\to\infty} 2\cdot\int_{0}^{M}\cos(2\pi at)\, dt = \lim_{M\to\infty} 2\cdot \frac{\sin(2\pi at)}{2\pi a}\Big|_{0}^{M} =$$

$$= \lim_{M\to\infty} 2M\,\frac{\sin(2\pi aM)}{2\pi aM}$$



KO $M\to\infty$, GRAF GRE V δ FUNKCIJO

MALO O ČASOVNEM ZAOSTANKU



$$\mathcal{F}\{x(t-t_0)\} = \int_{-\infty}^{\infty} x(t-t_0)\cdot e^{-i2\pi f t}\cdot dt = \int_{0}^{\infty} x(u)\cdot e^{-i2\pi f (t_0+u)}\cdot du$$

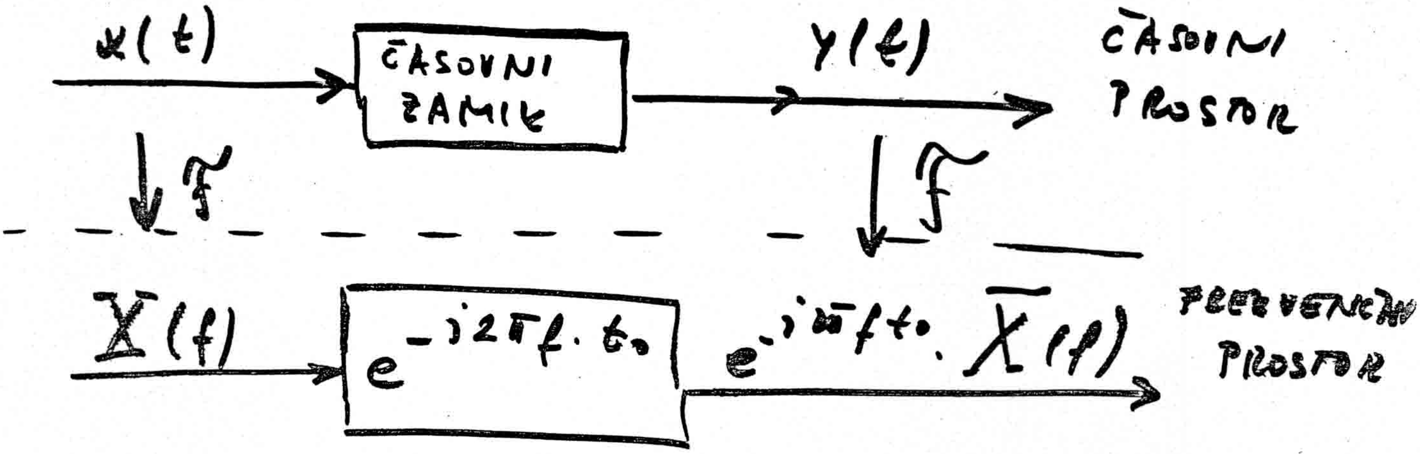
$t - t_0 = u$

$dt = du$

če

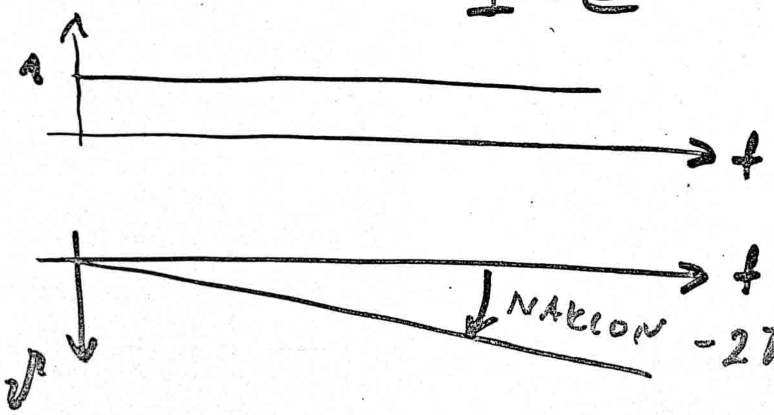
$X(f) = \mathcal{F}\{x(t)\}$

$$\Rightarrow \mathcal{F}\{x(t-t_0)\} = e^{-i2\pi f\cdot t_0}\cdot X(f)$$



ZAPIS V POLARNI OBLIKI

$$e^{-i2\pi f t_0} = 1\cdot e^{-i2\pi f t_0}$$



$$\frac{d\vartheta}{df} = -2\pi t_0$$

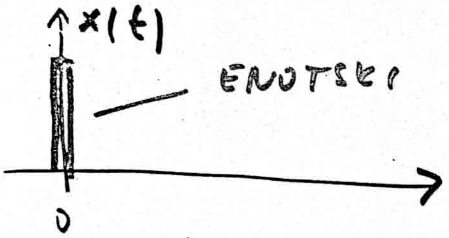
NAKLON $-2\pi t_0 \Rightarrow t_0 = -\left(\frac{1}{2\pi}\cdot\frac{d\vartheta}{df}\right)$

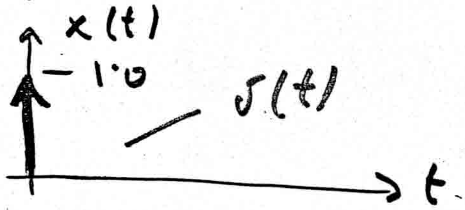
t_0 = GROUP DELAY

KONVOLUCIJA (CONVOLUTION)

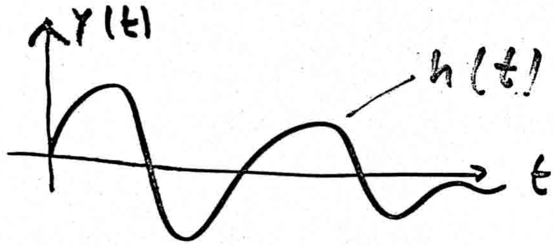
$$x(t) \longrightarrow \boxed{\text{FILTER}} \longrightarrow y(t)$$

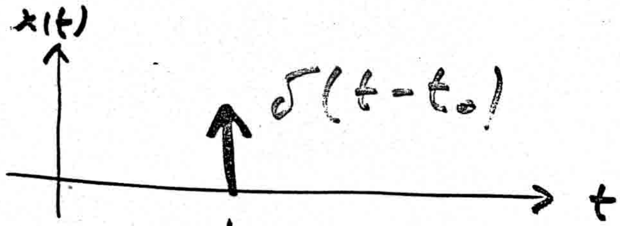
OMEJITEV:
SAMO LINEARNI FILTRI!

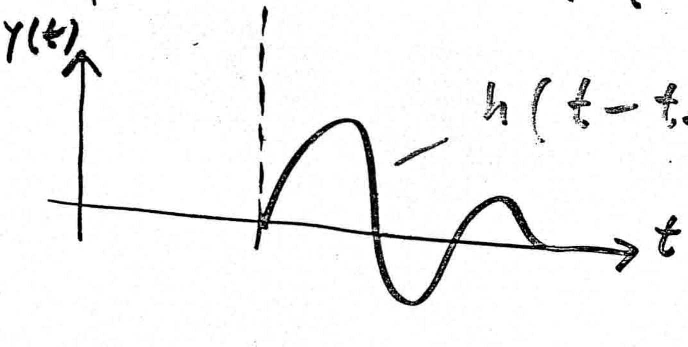




$h(t)$ - IMPULZNA PRENOSNA FUNKCIJA (IMPULSE RESPONSE FUNCTION)







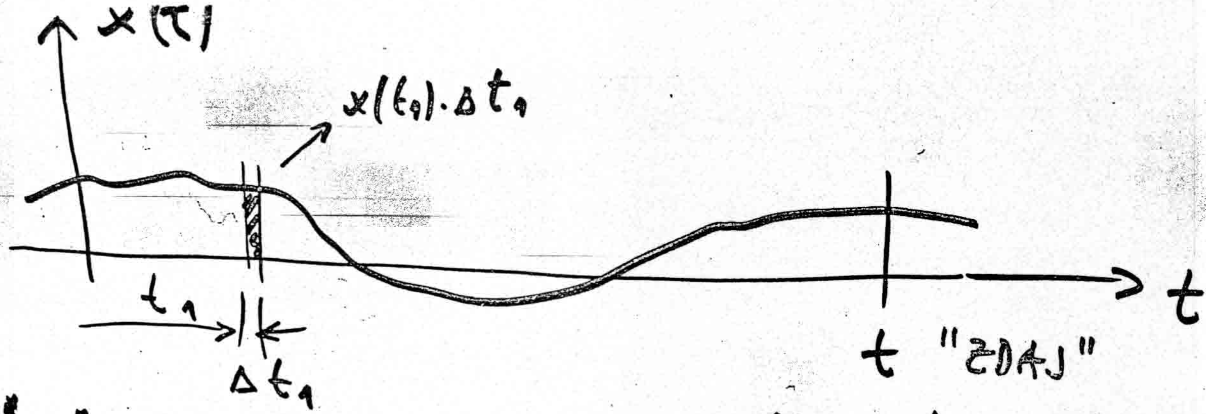
→ V PRIMERU INVARIANTNEGA SISTEMA.

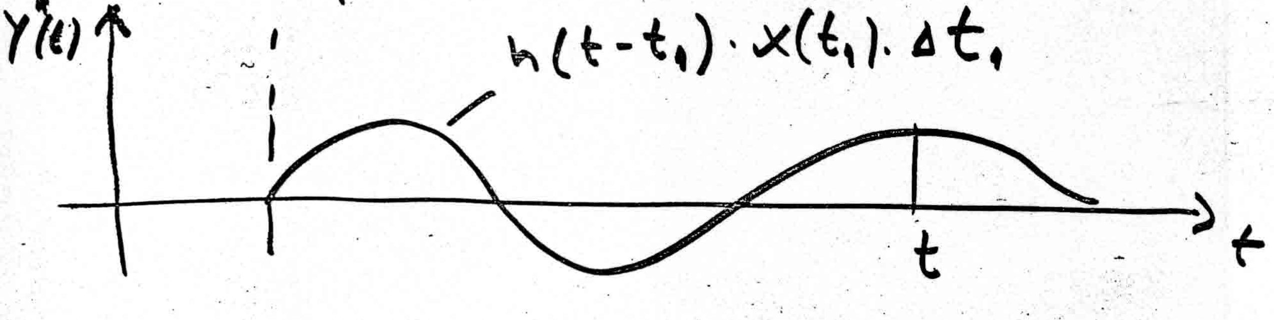
PREDPOSTAVIMO TUDI KAVZALNE SISTEME! (CAUSAL SYSTEMS)

ČE IMAMO IMPULZ

$k \cdot \delta(t-t_0)$, DOBIMO ODGOVOR

$k \cdot h(t-t_0)$





TOREJ

$$y(t) \sim \sum_{t_1=-\infty}^{t} h(t-t_1)\cdot x(t_1)\cdot \Delta t_1$$

TER V LIMITNEM PROCESU

(A) $$\boxed{y(t) = \int_{-\infty}^{t} h(t-t_1)\cdot x(t_1)\cdot dt_1}$$

KONVOLUCIJSKI
SUPERPOZICIJSKI
DUHAMEL-OV
INTEGRAL

ČE $t - t_1 = \tau$

$-dt_1 = d\tau \quad \Rightarrow$

(B) $$\boxed{y(t) = -\int_{+\infty}^{0} h(\tau)\cdot x(t-\tau)\,d\tau = \int_{0}^{\infty} h(\tau)\cdot x(t-\tau)\cdot d\tau}$$

OZNAKE

$$h * x = x * h$$

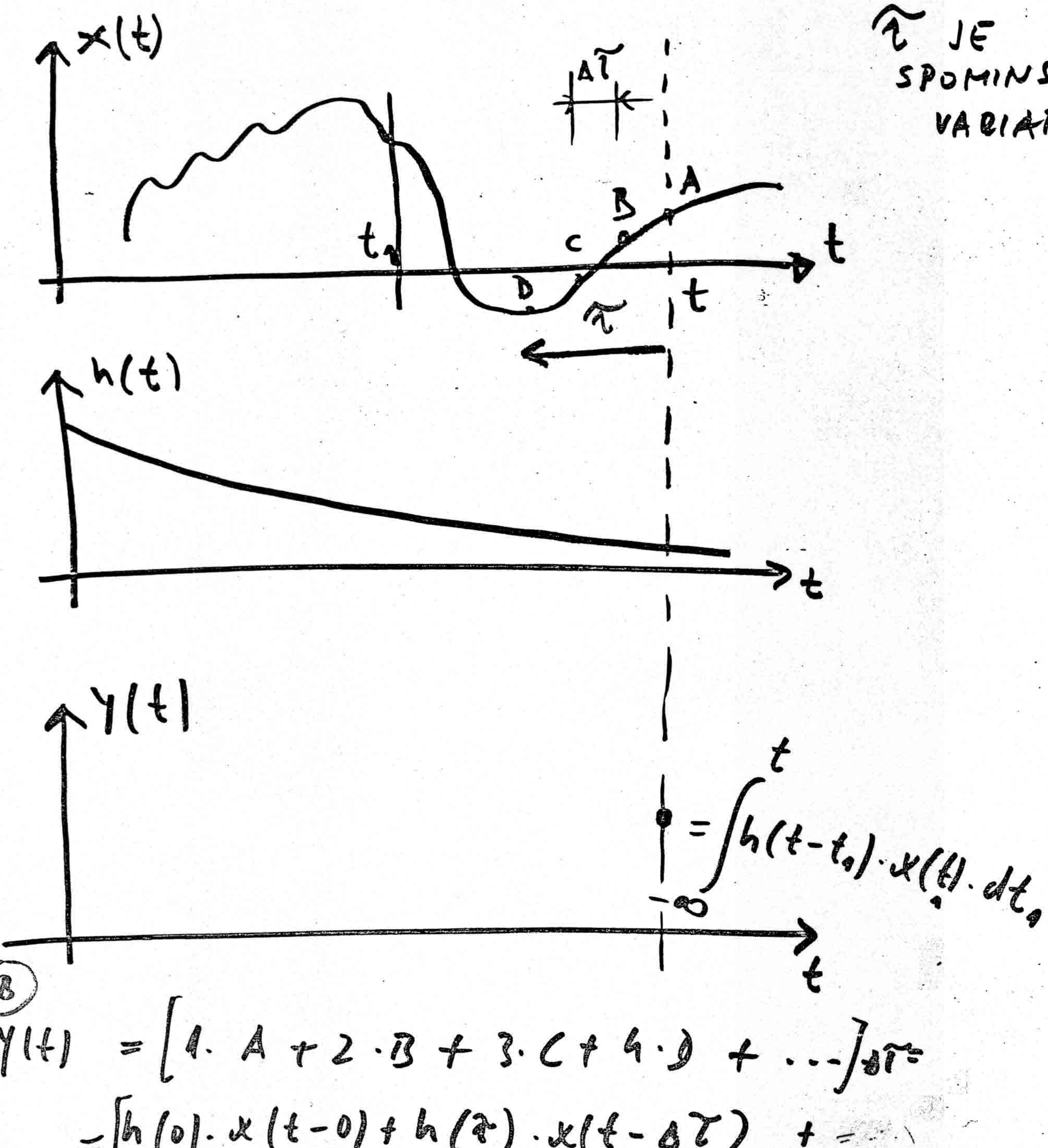
ČE $h(\tau)=1$ – FILTER KI NIKOLI NE POZABI → INTEGRATOR

POGOJ ZA STABILNOST

$$\int_0^\infty |h(\tilde{\tau})|\, d\tilde{\tau} < \infty$$

POMISLIMO GRAFIČNO:

$\tilde{\tau}$ JE SPOMINSKA VARIABLA



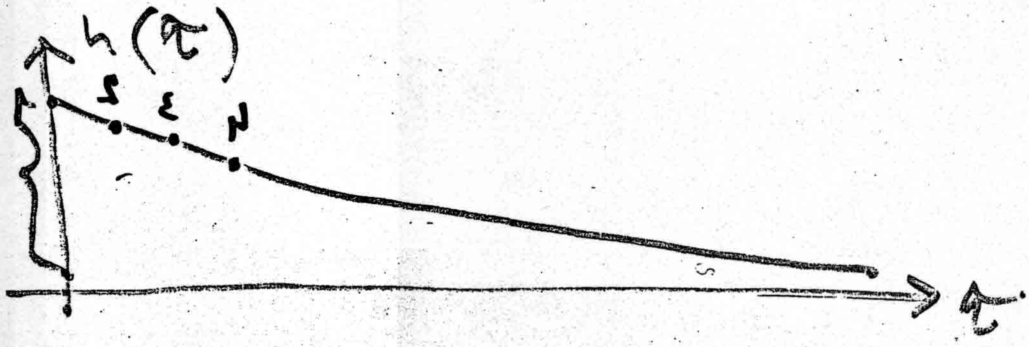
iz (B)

$$y(t) = [1\cdot A + 2\cdot B + 3\cdot C + 4\cdot D + \dots]\Delta\tilde{\tau} =$$

$$= [h(0)\cdot x(t-0) + h(\tilde{\tau})\cdot x(t-\Delta\tilde{\tau}) + \dots$$
$$+ h(2\Delta\tilde{\tau})\cdot x(t-2\Delta\tilde{\tau}) + h(3\Delta\tilde{\tau})\cdot x(t-3\Delta\tilde{\tau}) + \dots]\Delta\tilde{\tau}$$

$h(\tilde{\tau})$ POMENI, KAKO SI FILTER ZAPOMNI ZGODOVINO = DINAMIČNI SPOMIN FILTRA.

$$y(t) = \int_0^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$



NAPREJ

$$y(t) = \int_0^\infty h(\tau) \cdot x(t-\tau)\, d\tau \quad \Big/ \ \mathcal{F}$$

$$\mathcal{F}[y(t)] = \int_{-\infty}^{\infty} \Big[\int_{-\infty}^{\infty} h(\tau) \cdot x(t-\tau)\, d\tau \Big] \cdot e^{-i2\pi f t} \cdot dt =$$

$-\infty$, $h(\tau) = 0$ če $\tau < 0$

$$= \int_{-\infty}^{\infty} h(\tau) \underbrace{\Big[\int_{-\infty}^{\infty} x(t-\tau) \cdot e^{-i2\pi f t} \cdot dt \Big]}_{e^{-i2\pi f \tau} \cdot \underline{X}(f)} \cdot d\tau =$$

sedaj $t - \tau = u$

$$= \int_{-\infty}^{\infty} h(\tau) \cdot e^{-i2\pi f \tau} \cdot \underline{X}(f)\, d\tau = H(f) \cdot \underline{X}(f)$$

torej

$$\boxed{\underline{Y}(f) = H(f) \cdot \underline{X}(f)}$$

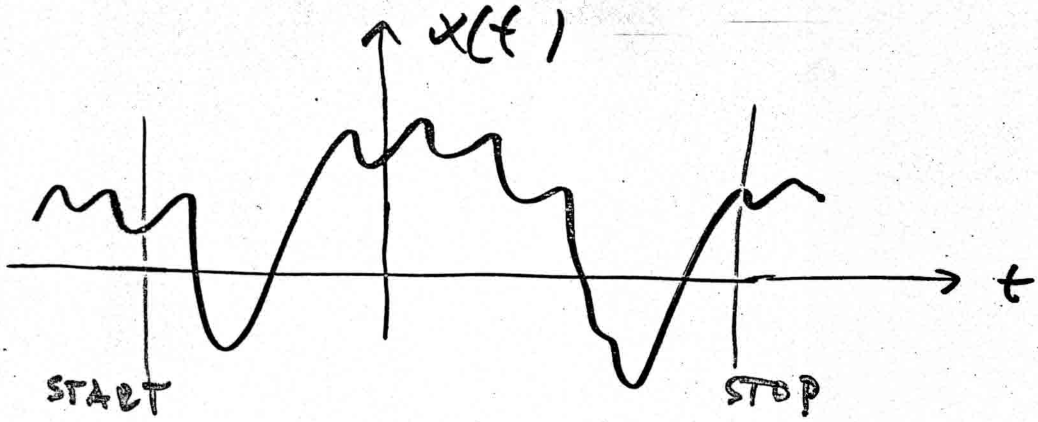
kjer

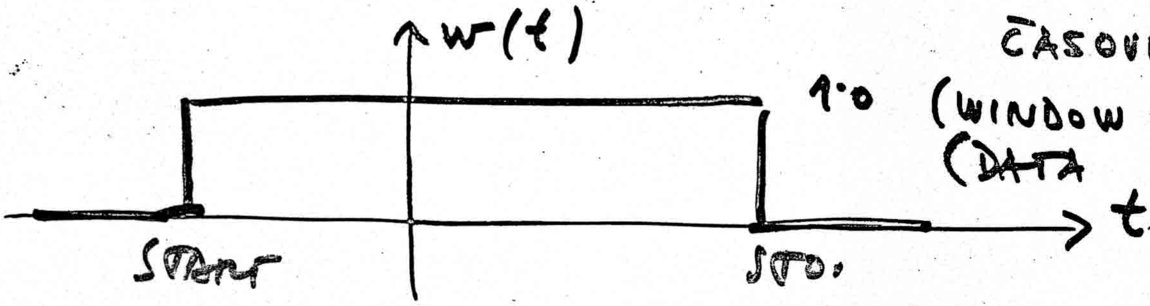
$H(f)$ - FREKVENČNA PRENOSNA FUNKCIJA

(FREQUENCY RESPONSE FUNCTION)

VPLIV ČASOVNIH OKEN

(DATA TRUNCATION, WINDOWING)





ČASOVNO OKNO (WINDOW FUNCTION) (~~DATA~~ WINDOW)

TOREJ, KAR OPAZUJEMO, JE

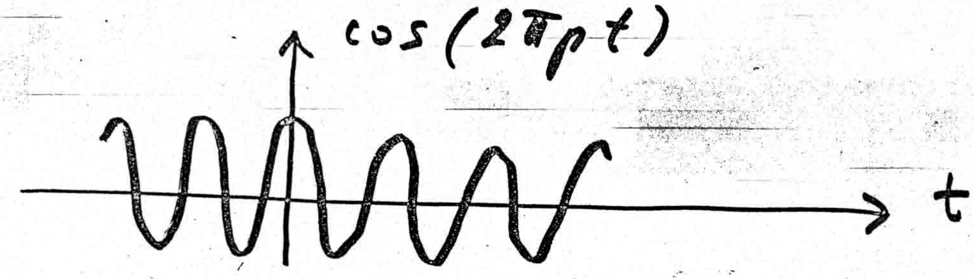
$x_T(t) = x(t) \cdot w(t)$ (T - TRUNCATED)

- $\underbrace{\mathcal{F}[x(t)\cdot w(t)] = X(f) * W(f)}$

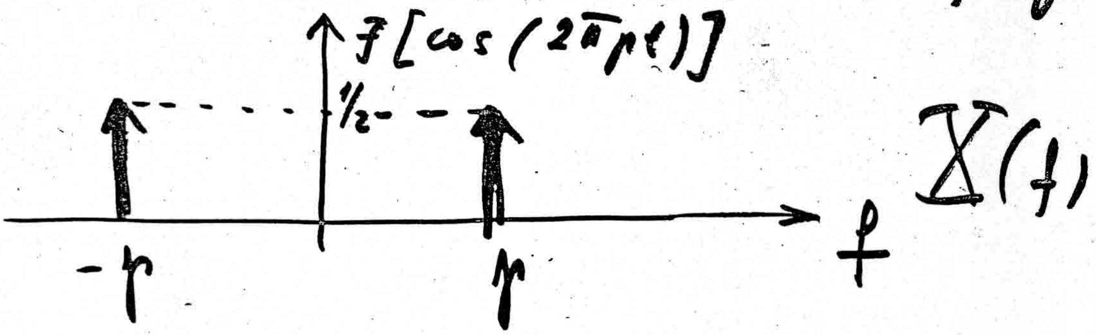
JE KAKŠNA LASTNOST $\mathcal{F}[\;]$

- $\mathcal{F}[x(t-t_0)] = e^{-i2\pi f t_0} \cdot X(f)$
- $\mathcal{F}[h(t) * x(t)] = H(f) \cdot X(f)$

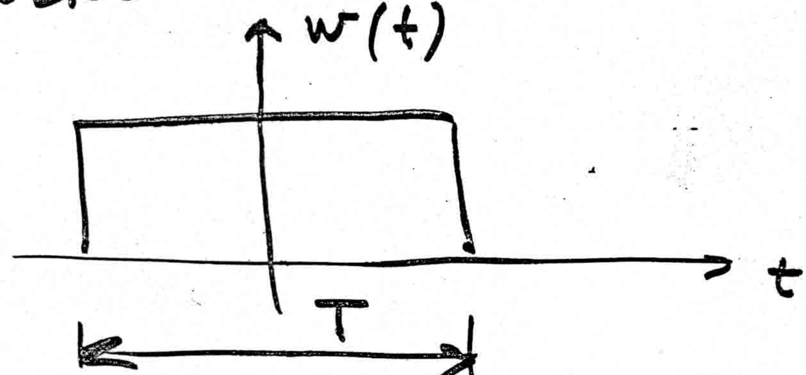
VZEMIMO COS SIGNAL !

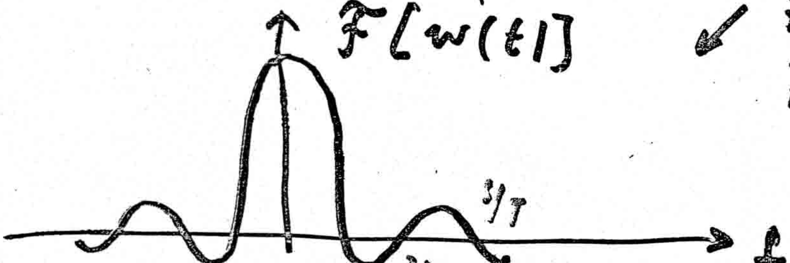


$$\mathcal{F}[\cos(2\pi p t)] = \frac{1}{2}\left[\delta(f-p)+\delta(f+p)\right]$$

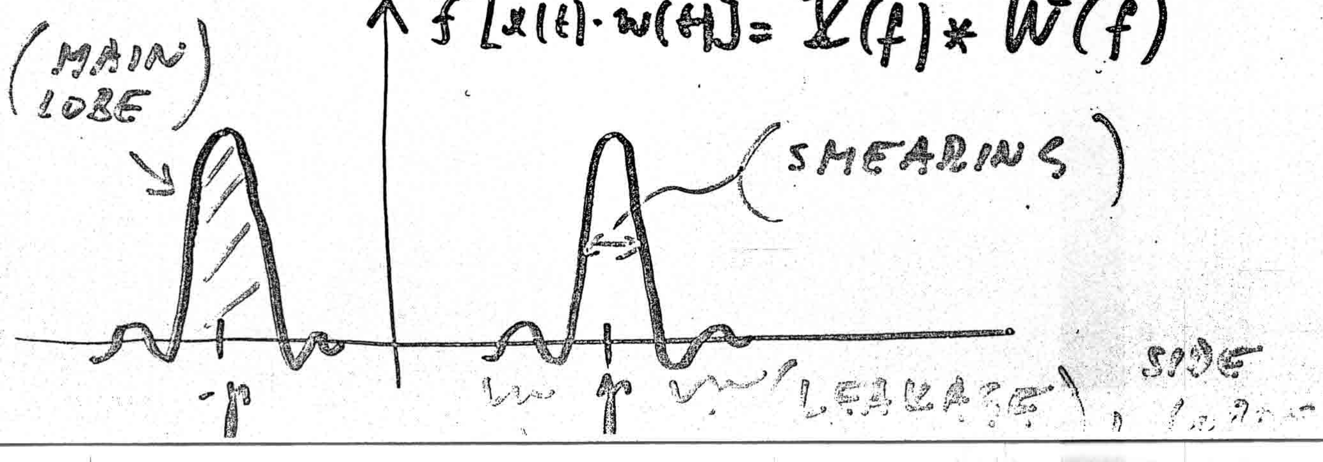


VZEMIMO (BOXCAR, RECTANGULAR) PRAVOKOTNO OKNO





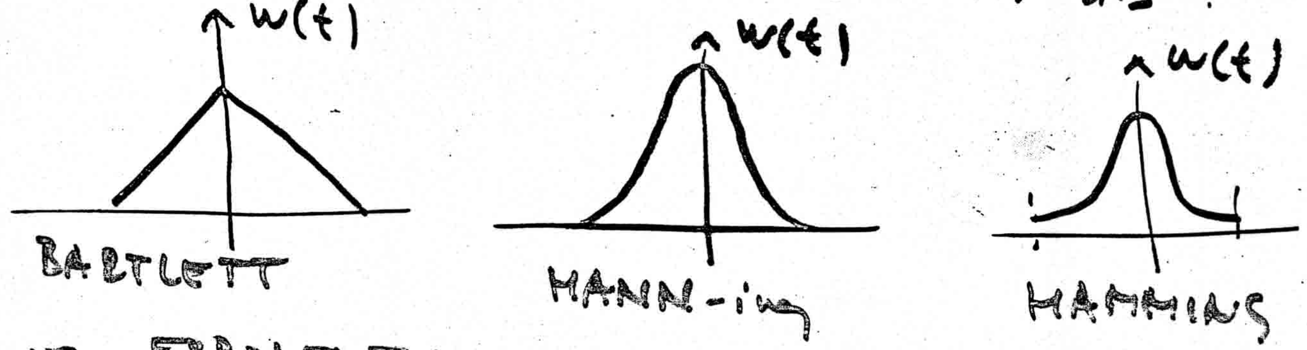
FOURIER TRANS OD PRAVOKOTNEGA OKNA $W(f)$



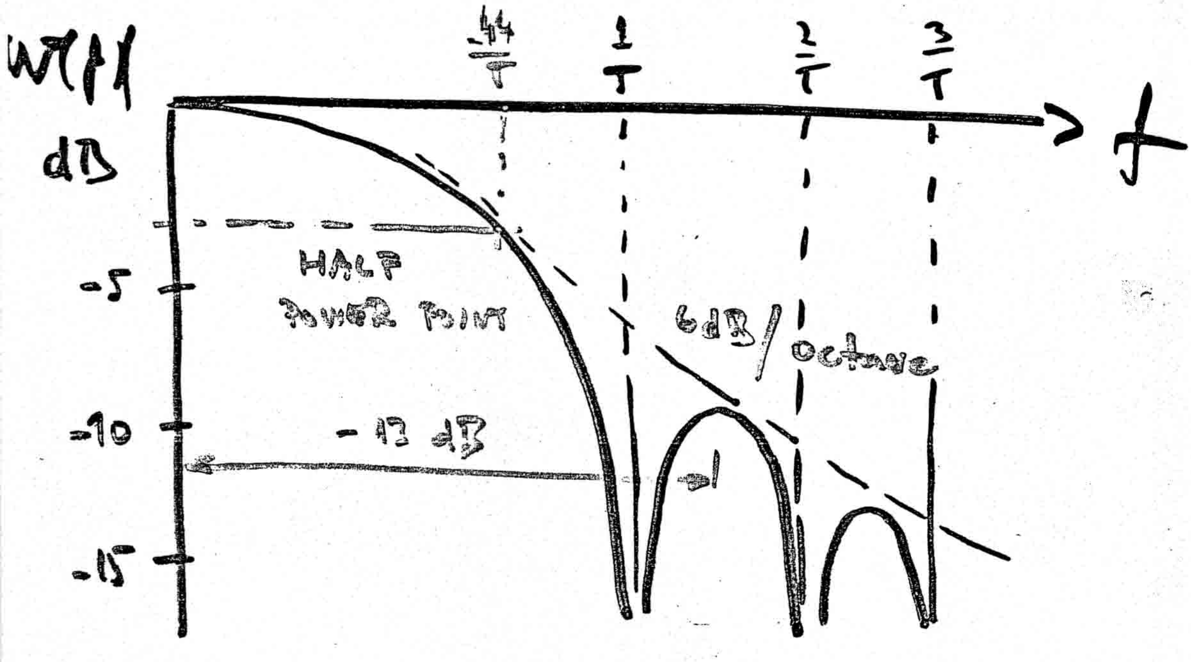
IZBIRA ČASOVNEGA OKNA JE "TRGOVANJE" MED LEAKAGE IN SMEARING.

BISTVENO: Z OPERACIJO - IZBIRO ČASOVNIH OKEN IZGUBLJAMO LOČLJIVOST!

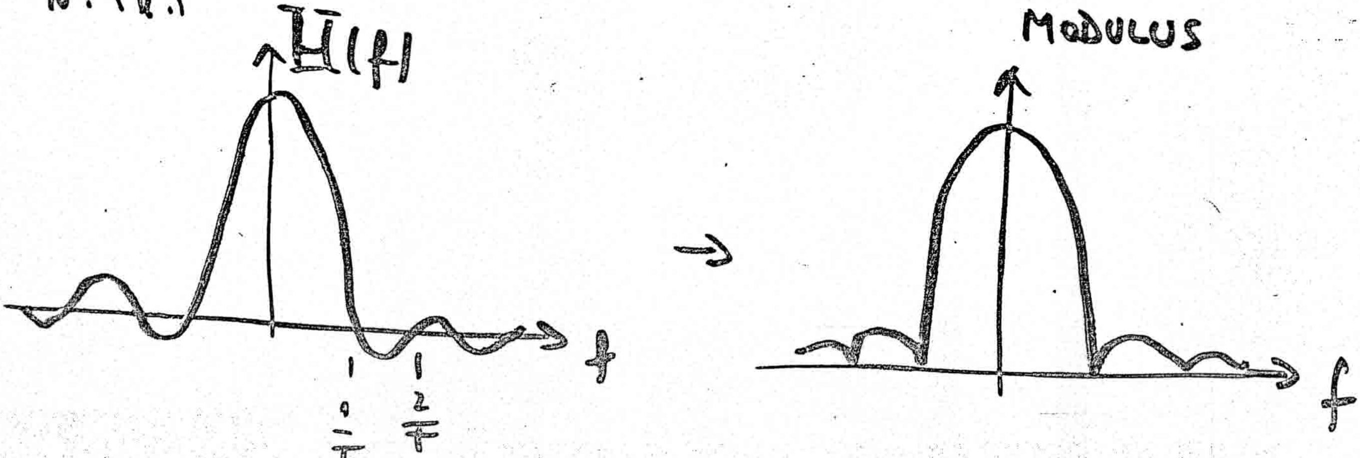
KARAKTERISTIKA OKEN SE PONAVADI PRIKAZUJE V FR. PROSTORU V dB.



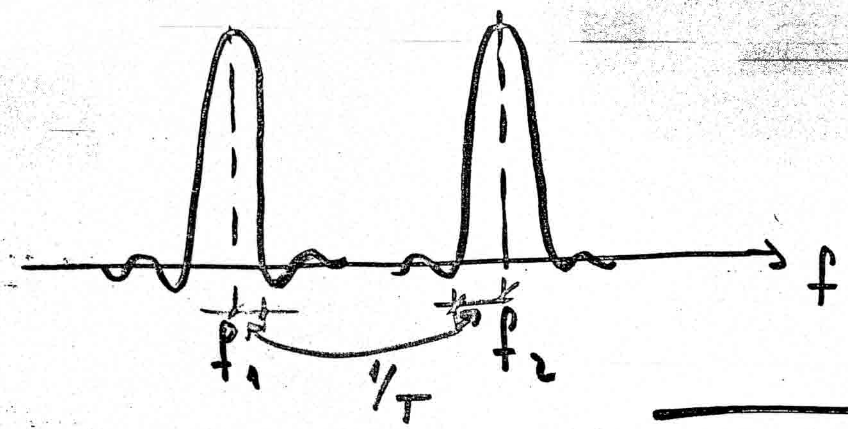
..., EXPONENTIAL, KAISER-BESSEL



N. PR.:



LOČEVANJE BLIŽNJIH FREKVENC



ŽELIMO LOČITI DVA SINUSNA SIGNALA FREKVENC f_1 IN f_2

$$f_2 - f_1 > \frac{2}{T} \Rightarrow \boxed{T > \frac{2}{f_2 - f_1}}$$

PRIMERI

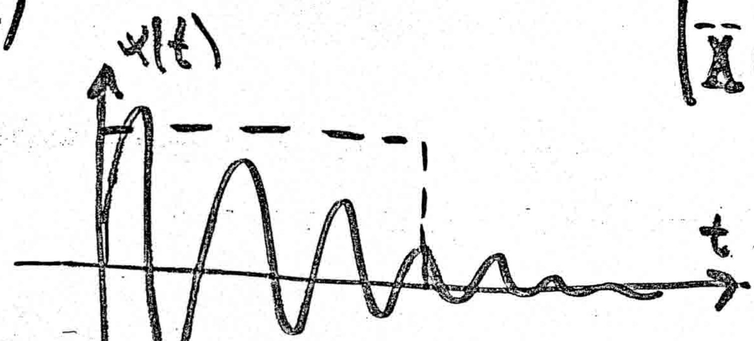
1) a) $\left.\begin{array}{l} f_1 = 100 \text{ Hz} \\ f_2 = 101 \text{ Hz} \end{array}\right\} \Rightarrow T > 2\,s$

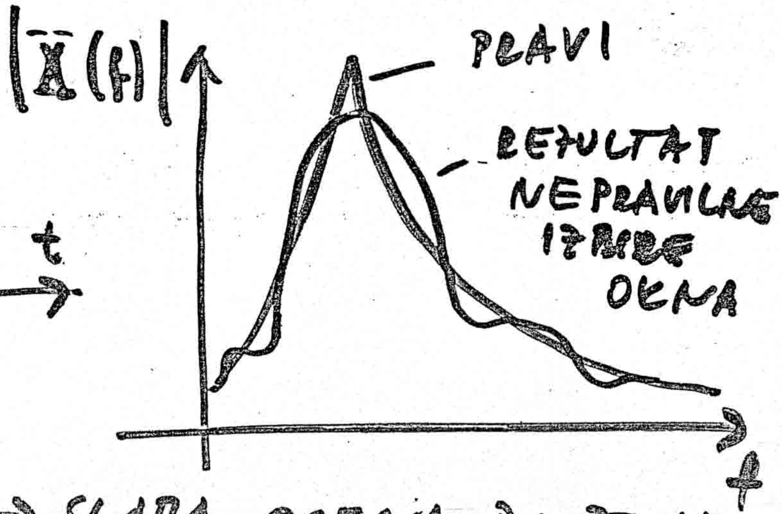
b) $\left.\begin{array}{l} f_1 = 1000 \text{ Hz} \\ f_2 = 1001 \text{ Hz} \end{array}\right\} \Rightarrow T > 2\,s$

FREKVENČNA LOČLJIVOST DOLOČI DOLŽINO SIGNALA T!

---

2)

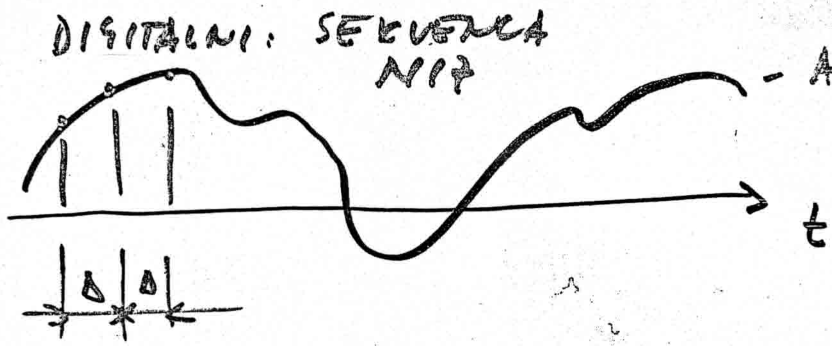




⇒ SLABA OCENA DUŠENJA

ANALOGNO - DIGITALNA PRETVORBA

(ANALOG TO DIGITAL CONVERSION)



• PREDPOSTAVKA : Δ je VES ČAS VZORČENJA ENAK. (UNIFORM SAMPLING)

$t = n \cdot \Delta$, n - INTEGER

DOGOVOR O OZNAKAH

$x(n \cdot \Delta) = x(n) = x_n$ - NIZ ŠTEVIL

1/ ? KOLIKO INFORMACIJE IZGUBIMO Z DISKRETIZACIJO

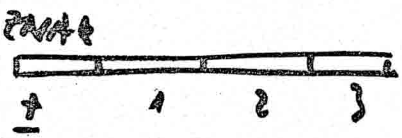
2/ ? KAKO NATANČNO LAHKO PODATKE SHRANJUJEMO

V RAČUNALNIKU SMO SHRANILI :

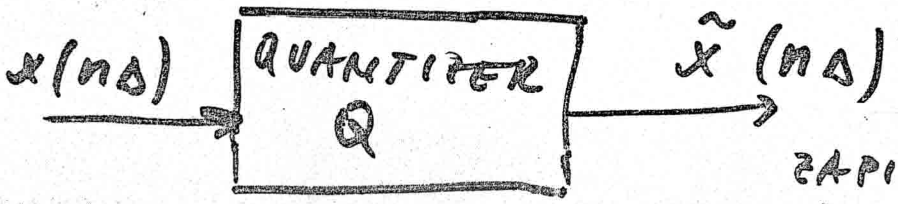
$\tilde{x}(n\Delta) = x(n\Delta) + e(n\Delta)$

↳ NAPAKA

N. PR. : IMAMO 3 BITNO DIG. BESEDO



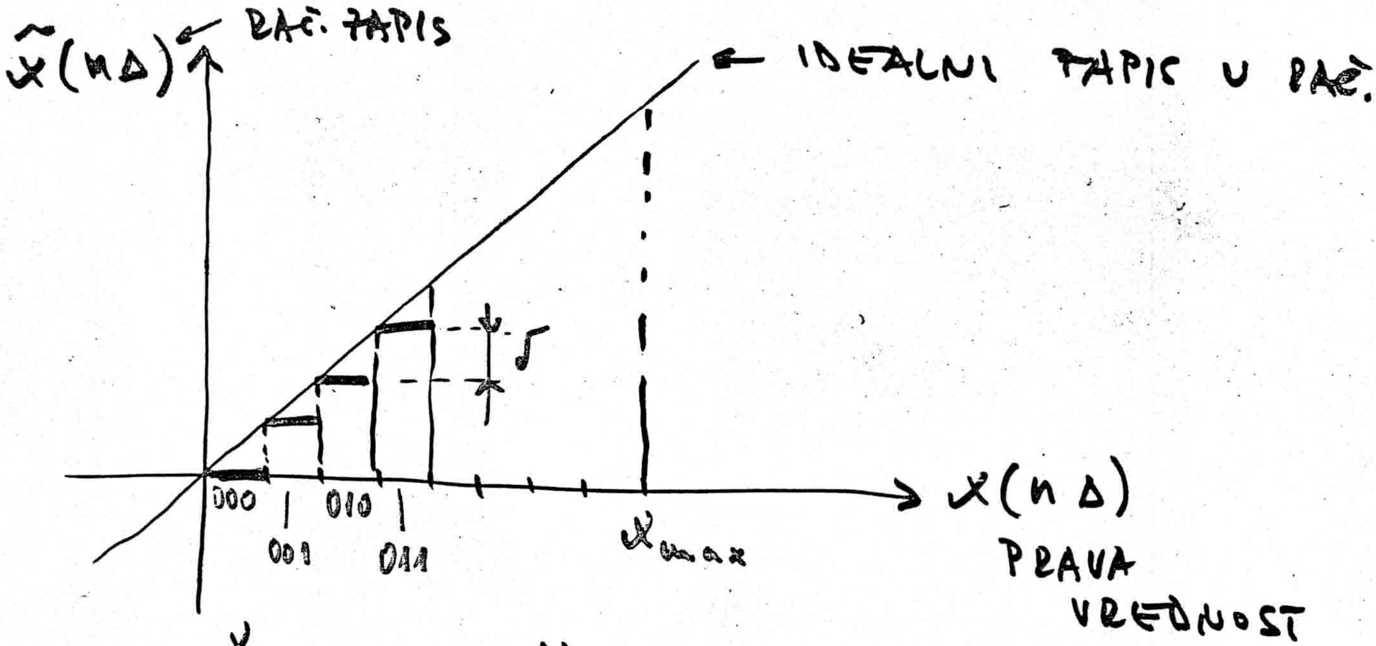
$\Rightarrow 2^3 = 8$ MOŽNIH STANJ



ZAPIS V RAČUNALNIKU

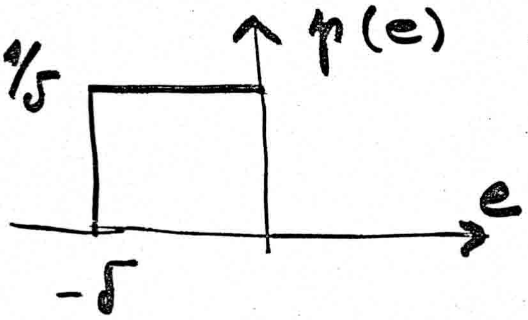
ŠE ENKRAT : A/D PRETVORBA

$x(t) \rightarrow$ VZORČENJE $\rightarrow x(n\Delta) \rightarrow$ KVANTIZACIJA $\rightarrow \tilde{x}(n\Delta) \rightarrow$



$$\delta = \frac{x_{max}}{8} = \frac{x_{max}}{2^b}\,,$$ b – ŠT. BITOV

DINAMIČNI OBSEG (DYNAMIC RANGE)



$p(e)$ – VERJETNOSTNA GOSTOTA NAPAK (PROBABILITY DENSITY FUNCTION)

$$\int_{-\infty}^{\infty} p(e)\cdot de = 1$$

• PREDPOSTAVKA ENAKOMERNE PORAZDELITVE.

SNR – (SIGNAL TO NOISE RATIO)

$$S_N = 10\cdot\log_{10}\frac{\sigma_x^2}{\sigma_e^2}$$

KJER

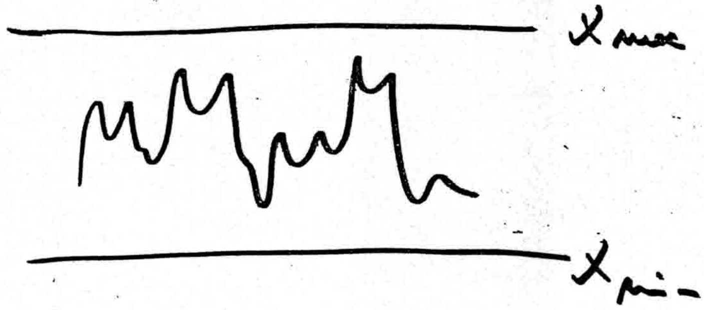
σ_x – VARIANCA SIGNALA
σ_e – VARIANCA NAPAKE

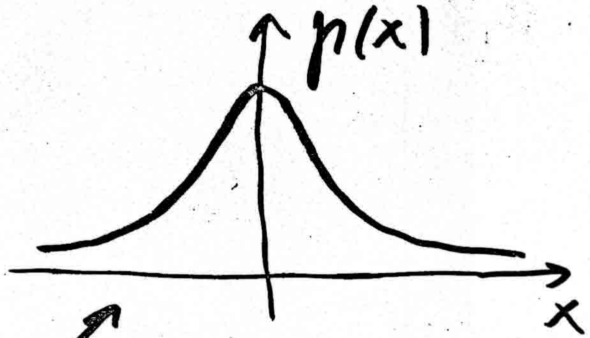
$\Big\}$ MOČ PO DEFINICIJI

$\sigma_e^2 = \int (e-\mu_e)^2 \cdot p(e)\, de = \frac{\delta^2}{12}$ POKAŽITE !

$\downarrow$ SR. VREDNOST

KAJ PA MOČ SIGNALA ?





• PREDPOSTAVKA GAUSSOVE PORAZDELITVE SIGNALA

$\Rightarrow \sigma_x \sim \frac{X_{max}}{4}$ (NI TEO. DOKAZANO)

SLEDI

$$\boxed{S_N = 6 \cdot b - 1'24 \quad [dB]}$$

• NEODVISNO OD X_{max}

• LINEARNO V b

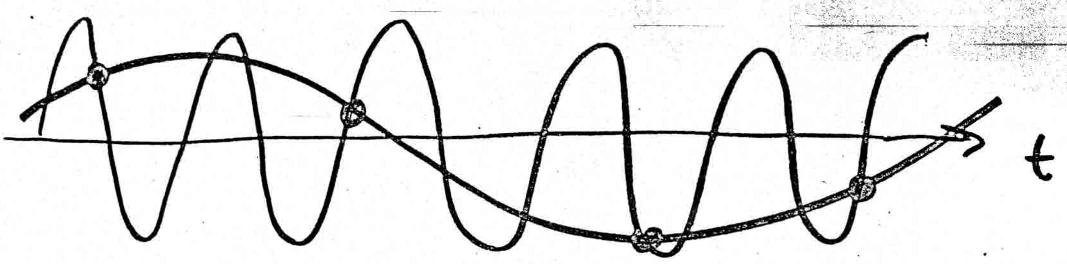
$S_N \sim 6 \cdot b$

ČE IMAMO 12 bit A/D CONVERTER

$\Rightarrow S_N \sim 70$ dB.

$\Rightarrow$ V TEM PRIMERU NE MOREMO OPAZOVATI DELA SIGNALA KI JE VEČ KOT 70 dB NIŽJE OD MAKSIMALNE VREDNOSTI.

PROBLEMATIKA VZORČENJA (ALIASING)



IMEJMO SIGNAL $\cos(2\pi p t)$. VZORČIMO S ČASOM Δ, $t = n \cdot \Delta$; $f_s = 1/\Delta$ VZORČNA FREKVENCA

IZRAČUNAJMO

$$S = \cos\left[2\pi(p \pm k \cdot f_s)\cdot t\right] =$$

$$= \cos\left[2\pi\left(p \pm \frac{k}{\Delta}\right)\cdot n\Delta\right] = \cos\left[2\pi(pn\Delta \pm kn)\right]$$

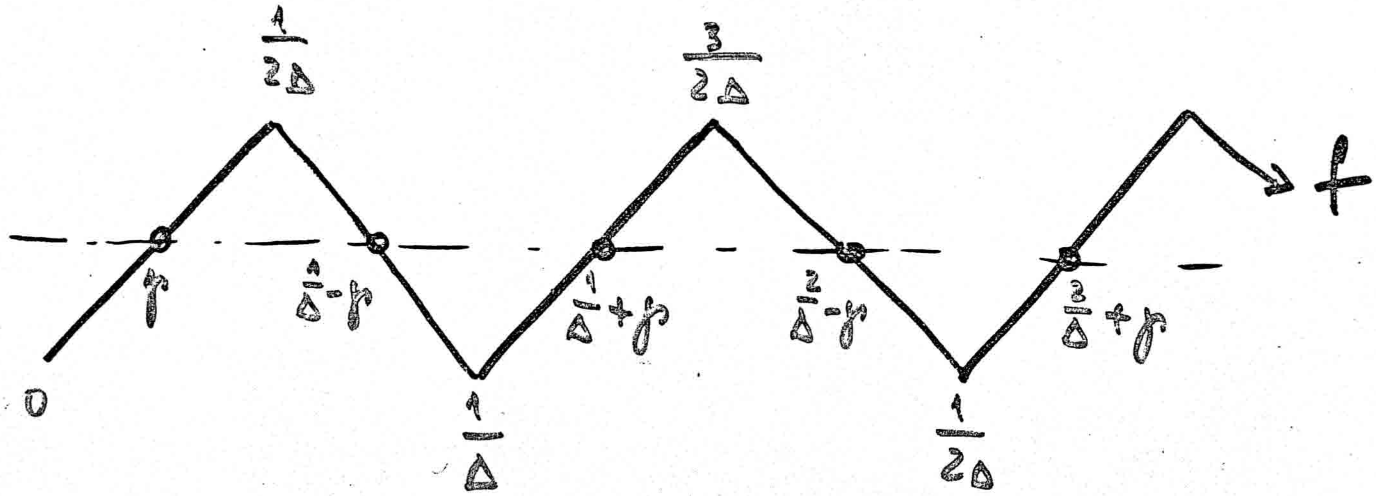
$$\cos(x \pm y) = \cos x \cdot \cos y \mp \sin x \cdot \sin y$$

$$S = \cos 2\pi p n\Delta \cdot \underbrace{\cos 2\pi kn}_{1} \mp \sin 2\pi p n \Delta \cdot \underbrace{\sin 2\pi kn}_{0}$$

$$S = \cos(2\pi p t)$$

$\Rightarrow$ VELIKO FREKVENC ZAVZAME ISTE VREDNOSTI.

GRAFIČNI PRIKAZ

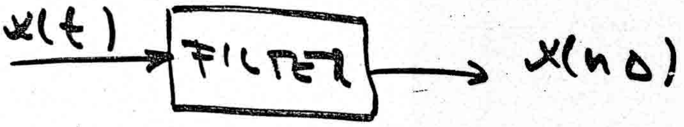


NAPREJ

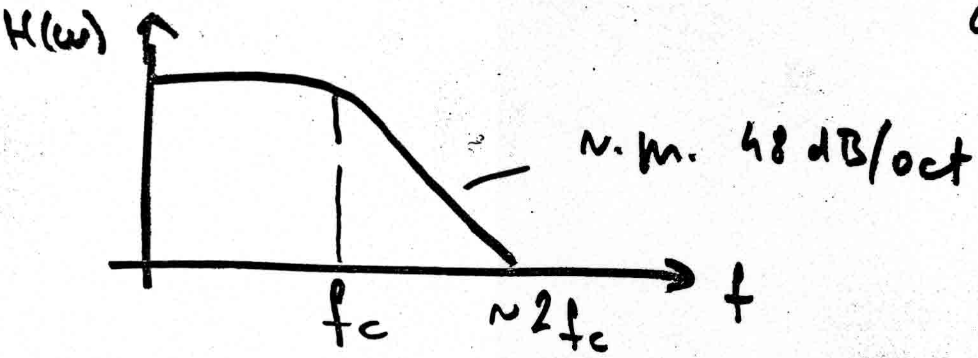
IMEJMO SIGNAL

$s = a \cdot \sin(100 \cdot 2\pi t) + b \cdot \sin(1000 \cdot 2\pi t)$

ČE SLEDIMO NYQUIST-OVEMU KRITERIJU, POTEM $f_s > 2000$ Hz. ZANIMA PA NAS LE FREKVENCA 100 Hz.



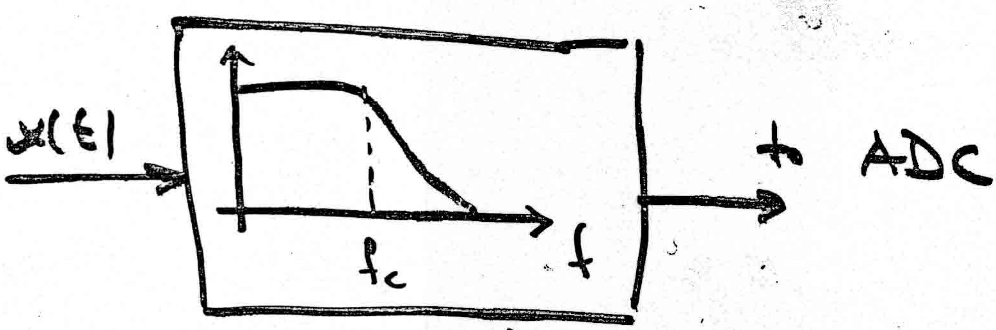
ANTIALIASING, GUARD FILTER



f_c - FREKVENCA, KI NAS ZANIMA

$\Rightarrow f_s \sim 2 \cdot 2 f_c = 4 f_c$

NAVODILA ZA IZBIRO AA FILTROV



$$\boxed{f_s \simeq 2 \cdot 10^{0.3\frac{A}{B}} \cdot f_c}\,, \text{ kjer}$$

A - DINAMIČNI OBSEG SIGNALA [dB]

B - NAKLON AA FILTRA [dB/oct]

f_c - 3 dB POINT OF FILTER [Hz]

DISKRETNA FOURIER-JEVA TRANSFORMACIJA

(DFT) (DISCRETE FOURIER TRANSFORM)

IDEALNO VZORČENJE

JE VEČ POTI OD $\mathcal{F}\{$ZVEZNIH SIGNALOV$\}$ DO DFT.

$i(t)$ „IMPULZNI VLAK"

$$i(t)=\sum_n \delta(t-n\Delta)$$

$$x_s(t) = x(t)\cdot i(t)$$

→ MAT. OPERACIJA, KI REZULTIRA VZORČENJE SIGNALA

$$X_s(f)=\mathcal{F}[x_s(t)] = \int_{-\infty}^{\infty} x(t)\cdot i(t)\cdot e^{-i2\pi f t}\cdot dt$$

$\Sigma \leftrightarrow \int$

1) IZRAČUN

$$X_s(f)=\sum_n \underline{\int_{-\infty}^{\infty} x(t)\cdot\delta(t-n\Delta)\cdot e^{-i2\pi f t}\cdot dt}$$

SIFTING PROPERTY

$$X_s(f) = \sum_{n=-\infty}^{\infty} \underline{x(n\Delta)}\cdot e^{-i2\pi f n\Delta}$$

F. TRANSFORM. SEKVENCE ŠTEVIL

2) INTERPRETACIJSKA IZVEDBA

$$i(t)=\sum_n \delta(t-n\Delta) = \sum_n C_n\cdot e^{i\frac{2\pi t n}{\Delta}}$$

F. VRSTA

$$C_n=\frac{1}{T_p}\cdot\int_{-T_p/2}^{T_p/2} x(t)\cdot e^{-\frac{i2\pi n t}{T_p}}\cdot dt = \frac{1}{\Delta}\cdot\int_{-\Delta/2}^{\Delta/2} i(t)\cdot e^{-\frac{i2\pi n t}{\Delta}}\, dt = \frac{1}{\Delta}$$

$$i(t)=\sum_n \frac{1}{\Delta}\cdot e^{\frac{i2\pi t n}{\Delta}}$$

TOREJ

$$X_s(f) = \int_{-\infty}^{\infty} x(t)\cdot \frac{1}{\Delta}\sum_n e^{\frac{j2\pi n t}{\Delta}} \cdot e^{-j2\pi f t}\cdot dt =$$

$$= \frac{1}{\Delta}\sum_n \cdot \int_{-\infty}^{\infty} x(t)\cdot e^{j\frac{2\pi n t}{\Delta}}\cdot e^{-j2\pi f t}\cdot dt$$

$$X_s(f) = \frac{1}{\Delta}\cdot\sum_n\cdot\int_{-\infty}^{\infty} x(t)\cdot e^{-j2\pi t\left(f-\frac{n}{\Delta}\right)}\cdot dt$$

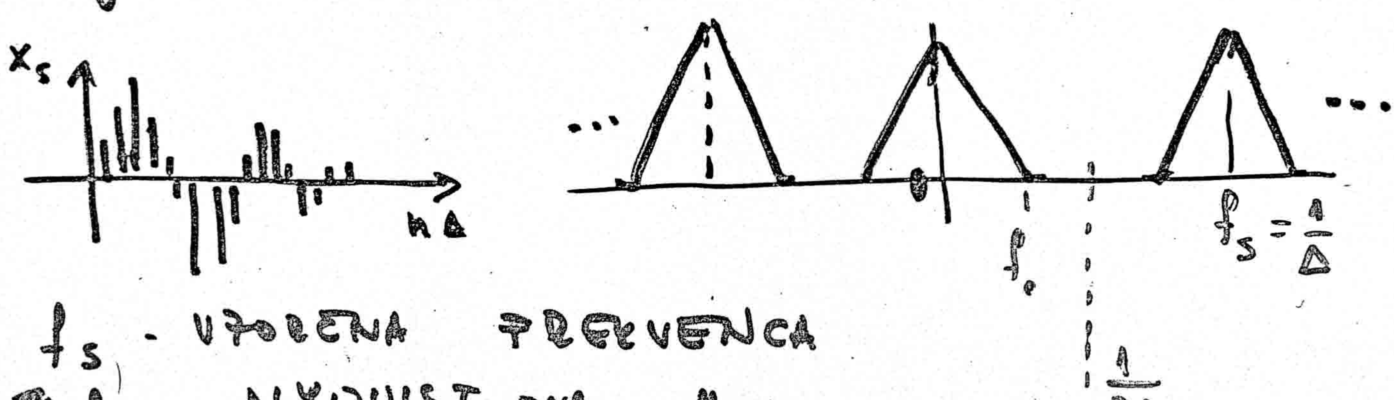
$$\boxed{X_s(f) = \frac{1}{\Delta}\cdot\sum_n X\left(f-\frac{n}{\Delta}\right)}$$

INTERPRETACIJA F. TRANSF. SEKVENCE

FOURIER-JEVA TRANSFORMACIJA VZORČENEGA SIGNALA JE VSOTA n PREMAKNJENIH F. TRANSFORMACIJ SIGNALA.

N. PR.

↓ VZORČIMO



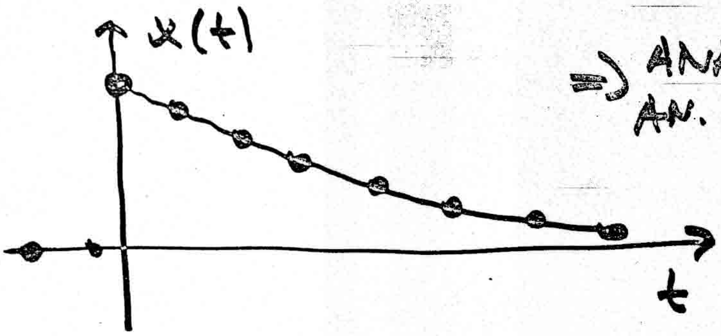
f_s - VZORČNA FREKVENCA

f_0 - NYQUIST-OVA -"-

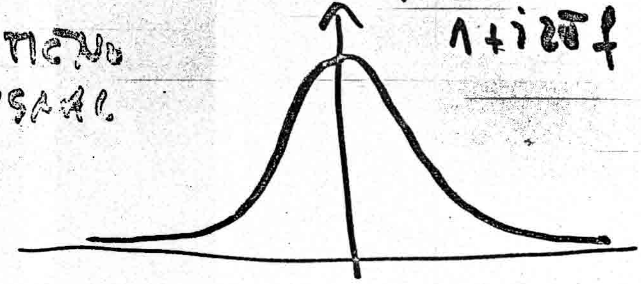
$f_s/2 = \frac{1}{2\Delta}$ - (FOLDING FREQUENCY)

$2\cdot f_0$ - NYQUIST-ova frekvenca

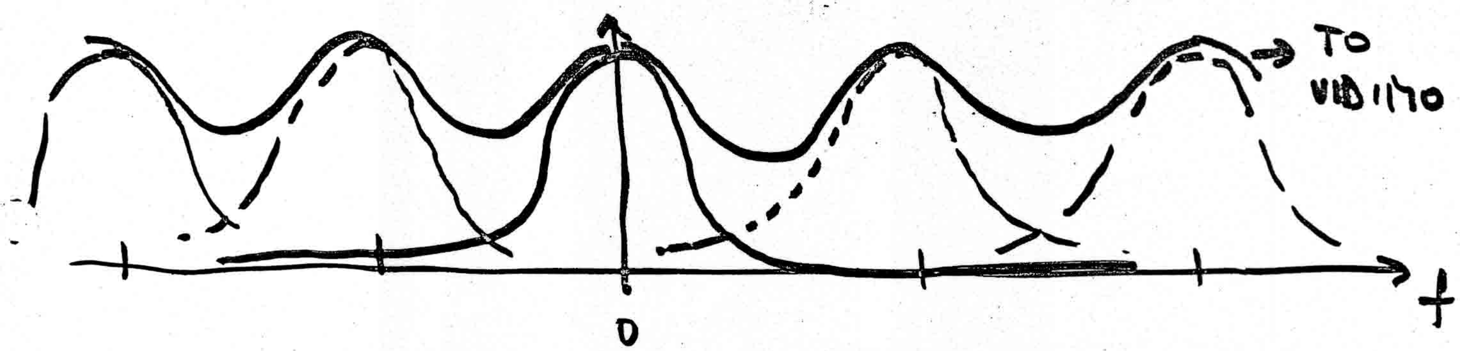
PRIMER



⇒ ANALITIČNO AN. SIGNAL



SEDAJ ZA DISKRETNA TRANSFORMACIJA

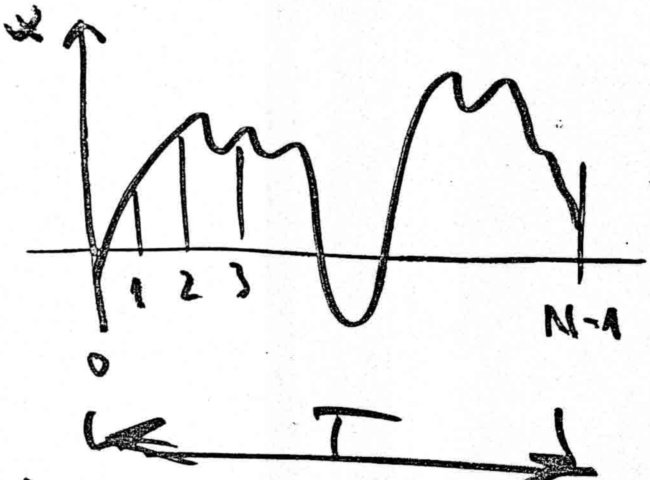


TA EFEKT SE IMENUJE ALIASING.

DFT GOES FOR EVER, IT NEVER ENDS!

SE DFT

V REALNOSTI DELAMO S KONČNO DOLGO SEKVENCO, ŠTEVILO TOČK JE N.



$n = 0, 1, 2, \ldots N-1$

INDEKS V ČASOVNEM PROSTORU

$T = N \cdot \Delta$

$f_k = \frac{k}{T} = \frac{k}{N \cdot \Delta}$

f_k - k-ta FREKVENCA V DFT.

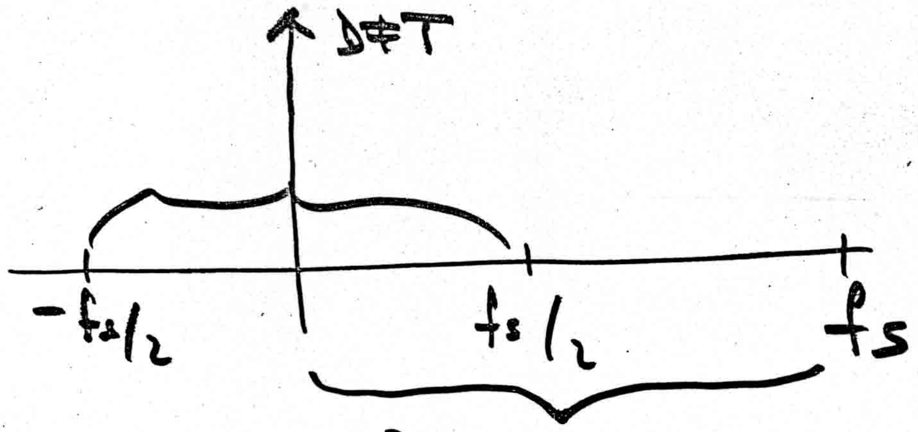
DOGOVOR O OZNAKAH

$X(e^{i 2\pi f \Delta})$ _ OZNAKA DISKRETNE F. TRANSFORMACIJE

TUDI

$$\underline{X}(k) = \underline{X}(f_k) = \underline{X}\left(\frac{k}{N\Delta}\right)$$

KAKO PRIKAZUJEMO $\mathcal{F}\{x_s(t)\}$



PONAVADI SE PRIKAZ V TEM OBSEGU

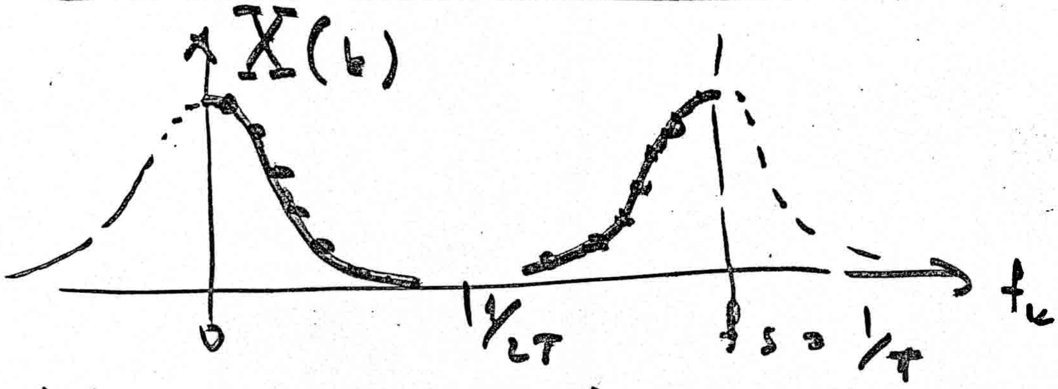
TOREJ, ČE IMAMO $x_s(n\Delta)$

$$X_s(f) = \sum_{n=-\infty}^{\infty} x(n\Delta)\cdot e^{-j2\pi f n\Delta}$$

$$f n\Delta = \frac{k\cdot n\Delta}{N\Delta}$$

$$\boxed{\underline{X}(k) = \sum_{n=0}^{N-1} x(n)\cdot e^{-j\left(\frac{2\pi}{N}\right)\cdot n\cdot k}}$$ DFT

$$\boxed{x(n) = \frac{1}{N}\cdot\sum_{k=0}^{N-1} \underline{X}(k)\cdot e^{j\left(\frac{2\pi}{N}\right)\cdot n\cdot k}}$$ IDFT



WHEN WE DO DIGITAL (DISCRETE) ANALYSIS, WE MUST THINK DIGITALLY.