- Navier - Lame-jeve enačbe ✓ (1)
- Določitev modula plastičnosti ✓ (6)
- Misesov pogoj tečenja ✓ (7)
- Dokaz enakosti tangencialnih napetosti ✓ (2)

- Hookov zakon / vpliv temp. obremenitev na elastično telo ✓ (8)

- kompresijski modul ✓ (5)
- Reološki modeli ✓ (4)
- Lega in ~~napetost~~ velikost glavnih normalnih napetosti ✓ (3)
- Airijeva funkcija ✓ (9)

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- Navier - Lame

$\sigma_{ij,j} + X_i = 0$; $\sigma_{ij,j}[\varepsilon_{ij}(u_i)] + X_i = 0$; $\sigma_{ij} = 2\mu \underset{(u_{i,j} + u_{j,i})}{\varepsilon_{ij}} + \lambda \varepsilon_{kk}\delta_{ij} - (2\mu + 3\lambda)\alpha T \delta_{ij}$; $\sigma_{ij,j}$

- Dokaz enakosti tangencialnih napetosti

Sliko, q_i, $-\ddot{u}_i \rho$, $\ddot{u}_i$, $\int_V \rho r_{ijk}(-\ddot{u}_j + q_j) x_k \, dV + \int_A r_{ijkp} n_j x_k \, dA$

- Lega in velikost glavnih normalnih napetosti

GN-NN-T ; $\vec{T} = \emptyset$; $\vec{\sigma} \equiv \vec{p}$; $\vec{p} = (\sigma_{nx}, \sigma_{ny}, \sigma_{nz})$, Matrika $p_x = \sigma_{nx} \ldots$; $\sigma_{nx} = \sigma_x \ldots n_x$
izračunamo, ustavimo, dobimo, homogena enačba

- Reološki modeli (5 grafov)
- Kompresijski modul

hidrostatični pritisk kontinuum $\sigma_x = \sigma_y = \sigma_z = -p$; $T_{\ldots} = 0$, $\sigma_{kk} = \frac{E}{1-2\nu} \varepsilon_{kk}$

- Določitev modela plastičnosti

$s_{ij} s_{ij}$, $e_{ij} e_{ij}$; $d\varepsilon^p_{ij} = de^p_{ij} = d\lambda \, s_{ij}$; $\tau_{oct} = 1/3 \, [(\sigma_1 - \sigma_2) \ldots]^{1/2}$, Efektivna nap. ; $\sigma_e = 3/\sqrt{2} \, \tau_{oct}$

- Misesov pogoj tečenja

$\boxed{\Phi}$, Predpostavki, krog z enačbo, k, lin. nap. stanje, čeče, Tenzor

- Airyjeva funkcija

- rav. nap. st ; $F = F(x,y)$, $\sigma_x, \sigma_y, \tau_{xy}$; $\Delta\Delta F = 0$ enačba, kompatibilitetna enačba
- skalarna f

$\varepsilon_x = 1/E \, [\sigma_x - \nu\sigma_y] + \alpha T$; $\gamma_{xy} = 1/G \, \tau_{xy}$

- Vpliv temperature na elastični material

ΔT, Slika, $\varepsilon_x^t \, \varepsilon_y^t \, \varepsilon_z^t$, $\gamma_{xy} = 0$, $T = T(x,y,z,t)$, Prenos en $k\nabla^2 T + \frac{Q}{\rho c} = \frac{\partial T}{\partial t}$
Vezane en $kT_{,ii} + Q = \rho c_E \dot{T} + m T_0 \varepsilon_{kk}$

Hookeov zakon z upoštevanjem temp. izraznega

$\varepsilon_{ij} = 1/2G \, \sigma_{ij} - \nu/E \, \sigma_{kk}\delta_{ij}$

$\sigma_{kk} = \frac{E}{1-2\nu} \varepsilon_{kk}$

$\sigma_{ij} = 2\mu \, \varepsilon_{ij} + \lambda \varepsilon_{kk} \delta_{ij}$

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Navier - Lamé-jeve enačbe

$$\sigma_{ij,j} + X_i = \emptyset$$

$$\sigma_{ij,j}[\varepsilon_{ij}(u_i)] + X_i = 0$$

$$\sigma_{ij} = 2\mu\,\varepsilon_{ij} + \lambda\,\varepsilon_{kk}\,\delta_{ij} - (2\mu + 3\lambda)\,\alpha T\,\delta_{ij}$$

$$\sigma_{ij} = 2\mu\,\tfrac{1}{2}(u_{i,j} + u_{j,i}) + \lambda\,\varepsilon_{kk}\,\delta_{ij} - (2\mu + 3\lambda)\,\alpha T\,\delta_{ij}$$

$$\sigma_{ij,j} = \mu\,(u_{i,jj} + u_{j,ij}) + \lambda\,\varepsilon_{kk,j}\,\delta_{ij} - (2\mu + 3\lambda)\,\alpha T_{,j}\,\delta_{ij}$$

$$u_{i,jj} = u_{i,kk} = \Delta u_i$$

$$u_{j,ij} = u_{k,ik} = \frac{\partial^2 u_k}{\partial x_i \partial x_k} = \frac{\partial^2 u_k}{\partial x_k \partial x_i} = u_{k,ki} = \frac{\partial u_{k,k}}{\partial x_i} = \frac{\partial \varepsilon_{kk}}{\partial x_i} = \varepsilon_{kk,i}$$

$$u_{k,k} = \varepsilon_{kk} = \varepsilon_v$$

$$\varepsilon_{kk,j}\,\delta_{ij} = \varepsilon_{kk,i}$$

$$T_{,j}\,\delta_{ij} = T_{,i}$$

$$\mu(\Delta u_i + \varepsilon_{kk,i}) + \lambda\,\varepsilon_{kk,i} - (2\mu + 3\lambda)\,\alpha T_{,i} + X_i = \emptyset$$

Konačno oblik NLE

$$\boxed{\mu\left(\Delta u_i + \frac{\mu+\lambda}{\mu}\,\varepsilon_{kk,i}\right) - (2\mu + 3\lambda)\,\alpha T_{,i} + X_i = \emptyset}$$

$$\varepsilon_{kk} = \varepsilon_v = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

$$\mu\left(\Delta u + \frac{\mu+\lambda}{\mu}\,\frac{\partial \varepsilon_v}{\partial x}\right) - (2\mu + 3\lambda)\,\alpha\,\frac{\partial T}{\partial x} + X = \emptyset$$

$$\mu\left(\Delta v + \frac{\mu+\lambda}{\mu}\,\frac{\partial \varepsilon_v}{\partial y}\right) - (2\mu + 3\lambda)\,\alpha\,\frac{\partial T}{\partial y} + Y = \emptyset$$

$$\mu\left(\Delta w + \frac{\mu+\lambda}{\mu}\,\frac{\partial \varepsilon_v}{\partial z}\right) - (2\mu + 3\lambda)\,\alpha\,\frac{\partial T}{\partial z} + Z = \emptyset$$

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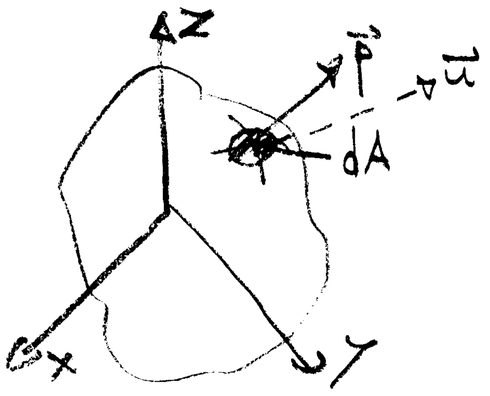
Dokaz enakosti tangencialnih napetosti



g_i ... masna sila
$-\ddot{u}_i \rho$ .. vztrajnostna sila
$\ddot{u}_i$ .. pospešek

$\Sigma F_i = 0$; $\Sigma M_i = 0$

$p\,dA$, $-\ddot{u}_i \rho\, dV$, $\rho g_i\, dV$

$$\int_V \rho\, \varepsilon_{ijk} (-\ddot{u}_j + g_j)\, x_k\, dV + \int_A \varepsilon_{ijk} p_j x_k\, dA = 0 \quad ; \quad p_i = \sigma_{ij} n_j$$

$$\int_A \varepsilon_{ijk} p_j x_k\, dA = \int_A \varepsilon_{ijk} \sigma_{jm} n_m x_k\, dA = \int_V \varepsilon_{ijk} \sigma_{jm} x_{k,m}\, dV =$$

$$= \int (\varepsilon_{ijk} \sigma_{jm,m} x_k + \varepsilon_{ijk} \sigma_{jm} x_{k,m})\, dV$$

$x_{k,m} = \delta_{km}$

$$\int [\rho \varepsilon_{ijk} (-\ddot{u} + g_i) x_k + \varepsilon_{ijk} \sigma_{jm,m} x_k + \varepsilon_{ijk} \sigma_{jm} x_{k,m}]\, dV = 0$$

splošne rav. za elastomehanike

$$\int [\varepsilon_{ijk} x_k (\cancel{-\rho\ddot{u} + \rho g_i} + \sigma_{jm,m}) + \varepsilon_{ijk} \sigma_{jm} x_{k,m}]\, dV = 0$$

$\sigma_{jm} \delta_{km} = \sigma_{jk}$

$\int \varepsilon_{ijk} \sigma_{jk} = 0$

ε_{ijk} lastnosti alternirajoče tenzorje

$\sigma_{kj} - \sigma_{jk} = 0$

$\sigma_{kj} = \sigma_{jk}$

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Lega in velikost glavnih normalnih napetosti

Glavne napetosti so normalne nap ki se pojavijo na ravninah brez tan nap

$\vec{\tau} = 0 \rightarrow \vec{\sigma} \equiv \vec{p}$

$\vec{p} = (\sigma_{nx}, \sigma_{ny}, \sigma_{nz})$

$$\begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix} = \begin{Bmatrix} \sigma_x n_x + \tau_{yx} n_y + \tau_{zx} n_z \\ \tau_{xy} n_x + \sigma_y n_y + \tau_{zy} n_z \\ \tau_{xz} n_x + \tau_{yz} n_y + \sigma_z n_z \end{Bmatrix} \rightarrow \begin{Bmatrix} \sigma_{nx} \\ \sigma_{ny} \\ \sigma_{nz} \end{Bmatrix} = \begin{Bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{Bmatrix} \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix}$$

$$\rightarrow \begin{Bmatrix} \sigma_x - \sigma & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y - \sigma & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \sigma \end{Bmatrix} \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix} (*) \; ; \; (\sigma_{ij} - \delta_{ij}\sigma) n_j = 0$$

n_i .. normalni kosti

Netriviale rešitve le če:

$$\begin{vmatrix} \sigma_x - \sigma & \tau_{xy} & \tau_{xy} \\ \tau_{yx} & \sigma_y - \sigma & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \sigma \end{vmatrix} = 0 \; , \; (\circ) \; |\sigma_{ij} - \delta_{ij}\sigma| = 0 \quad ; \text{ izračunamo } \sigma_{123}$$

Vrednosti σ v $(*)$ dajejo 3 ravnine brez τ

Homogene enačbe. Za vsaki σ_i le 2 cosinusa neodvisna kota
3 kot iz $1 = n_x^2 + n_y^2 + n_z^2$

Npr: ravnina 1

$(\sigma_x - \sigma_1) n_x + \tau_{xy} n_y + \tau_{xz} n_z$
$\tau_{yx} n_x + (\sigma_y - \sigma_1) n_y + \tau_{yz} n_z$
$n_x^2 + n_y^2 + n_z^2 = 1$

Karakteristična enačba $(\circ)$ $\sigma^3 - I_1\sigma^2 - I_2\sigma - I_3 = 0$

$I_1 =$
$I_2 =$ ostrenje karakteristika — napetostne invariante
$I_3 =$

$\sigma_1 > \sigma_2 > \sigma_3$

Napetostni elipsoid .. koordinatni sist. v smeri glavn. nap.

$$\sigma_{ij} = \begin{Bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{Bmatrix}$$

$I_1 = \sigma_1 + \sigma_2 + \sigma_3$
$I_2 = -(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)$
$I_3 = \sigma_1\sigma_2\sigma_3$

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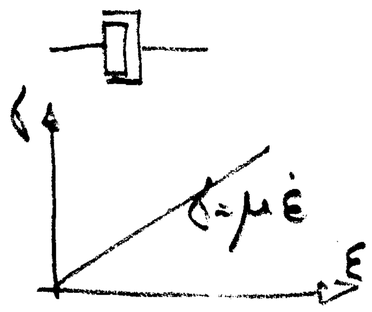
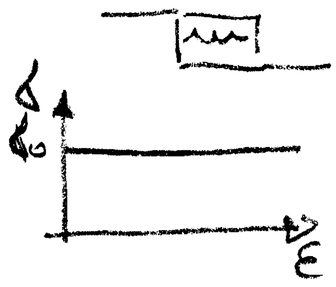
Reološki modeli

Newtonovo telo



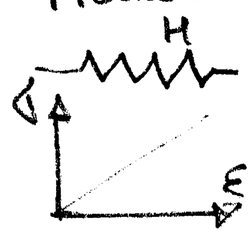
$s_{ij} = 2\mu e_{ij}$

Saint Venantovo telo



$s_{ij} = s_{ij0}$

Hookovo telo

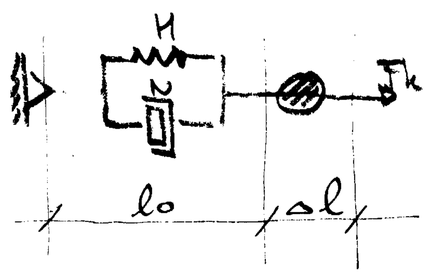


$\sigma = E\varepsilon$

$s_{ij} = 2\mu e_{ij}$

μ .. din. kož viskoz

Kelvinovo telo



$F_K = F_H + F_N \quad /A$

$\Delta l_K = \Delta l_H + \Delta l_N \quad /l_0$

$\hookrightarrow \sigma_K = \sigma_H + \sigma_N$

$\hookrightarrow \varepsilon_K = \varepsilon_H = \varepsilon_N$

$\sigma_N = \mu \dot{\varepsilon}_N$

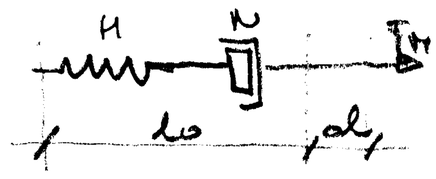
$\sigma_H = E \varepsilon_H$

$\sigma_K = \mu \dot{\varepsilon}_N + E \varepsilon_H$

$\sigma_K = (E + \mu \frac{1}{\Delta t}) \varepsilon_K$

$\varepsilon_K = \sigma_K / (E + \mu \frac{1}{\Delta t})$

Maxwellovo telo



$F_M = F_H = F_N \quad /A$

$\Delta l_M = \Delta l_H + \Delta l_N \quad /l_0$

$s_{ijM} = s_{ijH} = s_{ijN}$

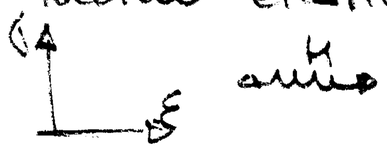
$e_{ijM} = e_{ijH} + e_{ijN}$

$\frac{1}{\Delta t} e_{ijM} = \{ \frac{1}{2\mu} \frac{1}{\Delta t} + \frac{1}{2\mu} \} s_{ijM}$

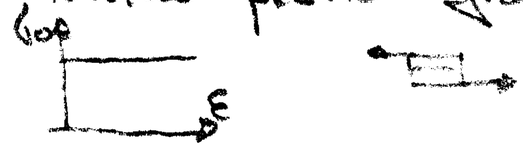
$s_{ijM} = \ldots$

Reološki modeli najbolj klasičnih materialov

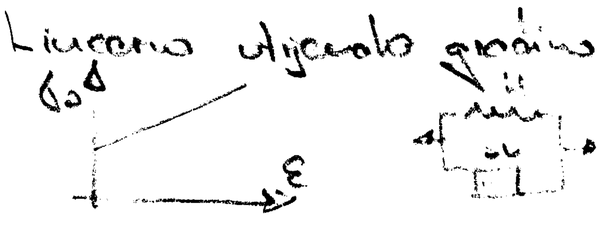
- Idealno elastično gradivo



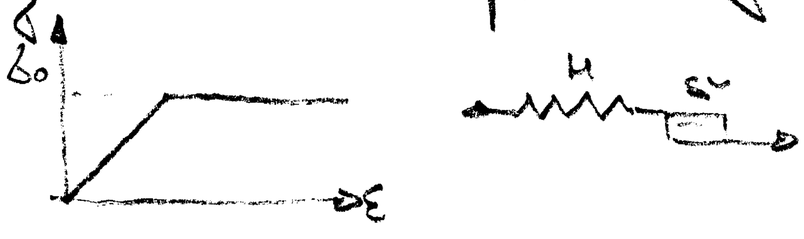
- Idealno plastično gradivo



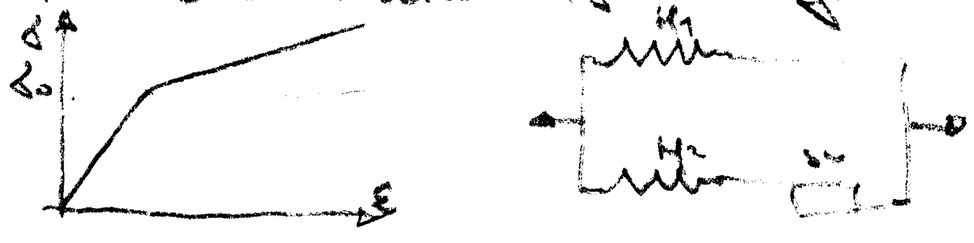
- Linearno utrjevalo gradivo



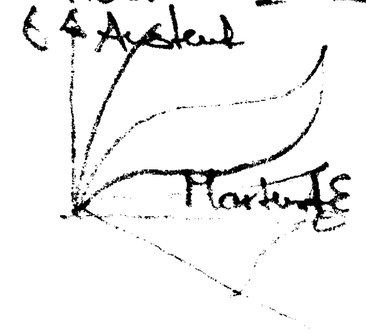
- Elastično - idealnoplastično gradivo



- Elastično linearno utrjevalno gradivo



- Gradivo z oblikovnim spominom



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Kompresijski modul

Če obremenimo kontinuum z hidrostatičnim pritiskom, potem je

$\sigma_x = \sigma_y = \sigma_z = -p$

$\tau_{xy} = \tau_{xz} = \tau_{yz} = 0$

$\sigma_{kk} = \frac{E}{1-2\nu} \varepsilon_{kk}$

$\sigma_{kk} = \sigma_{11} + \sigma_{22} + \sigma_{33} = -p - p - p = -3p$

$-3p = \frac{E}{1-2\nu} \varepsilon_{kk} = \frac{E}{1-2\nu} (\varepsilon_x + \varepsilon_y + \varepsilon_z)$; $\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$

$-3p = \frac{E}{1-2\nu} \varepsilon_v$; $\varepsilon_v = \frac{(-3p)(1-2\nu)}{E} = \frac{3(1-2\nu)}{E}(-p)$

kompresijski modul $K = \frac{E}{3(1-2\nu)}$ $\qquad \varepsilon_v = \frac{3(1-2\nu)}{E}(-p) = -\frac{p}{K}$

$\lim_{K \to \infty} \varepsilon_v = \lim_{K \to \infty} -\frac{p}{K} = 0$

$\lim_{\nu \to 1/2} \frac{E}{3(1-2\nu)} = \lim_{\nu \to 1/2} K = \infty \implies \varepsilon_v^{pl} = 0 \Rightarrow \nu^{pl} = 1/2$

Volumen se v plastičnem območju ne spremeni več

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Določitev modula plastičnosti

$s_{ij} s_{ij} = 3 \tau_{okt}^2$

$e_{ij} e_{ij} = 3 \gamma_{okt}^2$

$$d\varepsilon_{ij}^{pl} = de_{ij}^{pl} = d\lambda\, s_{ij}$$

$$de_{ij}^{pl} de_{ij}^{pl} = d\lambda^2 s_{ij} s_{ij} = d\lambda^2\, 3\, \tau_{okt}^2 = 3 (d\gamma_{okt}^{pl})^2$$

$$d\lambda = \frac{d\gamma_{okt}^{pl}}{\tau_{okt}}$$

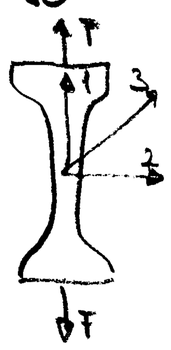
$$\tau_{okt} = 1/3 \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2} = \tau_{okt}(\sigma_{ij})$$

$$\gamma_{okt} = 1/3 \left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2 \right]^{1/2}$$

$\sigma_1 \neq 0$; $\sigma_2 = \sigma_3 = 0$

Efektivna napetost (lahko izrazimo poljubno napetostno stanje)

$$\sigma_e = 3/\sqrt{2}\ \tau_{okt}(\sigma_{ij})$$



$\varepsilon - \nu\varepsilon = \varepsilon_g$ ... specifične kontrakcije

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix} = \begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & -\nu\varepsilon & 0 \\ 0 & 0 & -\nu\varepsilon \end{pmatrix}$$

$$\gamma_{okt}(\varepsilon) = 1/3 \left[(\varepsilon + \nu\varepsilon)^2 + (-\nu\varepsilon - \varepsilon)^2 \right]^{1/2} = 1/3 \left[2\varepsilon^2 (1+\nu)^2 \right]^{1/2}$$

$$\gamma_{okt}(\varepsilon) = \varepsilon/3\, \sqrt{2}\, (1+\nu)$$

$$\gamma_{okt}^{pl}(\varepsilon) = \varepsilon^{pl}/3\, (1+\nu^{pl})\sqrt{2} \quad ; \quad \nu^{pl} = 1/2$$

$$\gamma_{okt}^{pl} = \sqrt{2}/2\ \varepsilon^{pl} = \varepsilon^{pl}/\sqrt{2} = \gamma_{okt}^{pl}$$

$$\varepsilon_e^{pl} = \sqrt{2}\ \gamma_{okt}^{pl}(\varepsilon_{ij}) = \sqrt{2}\ \gamma_{okt}^{pl}(\sigma_{ij}) \Rightarrow d\varepsilon^{pl} = \sqrt{2}\, d\gamma_{okt}^{pl}(\sigma_{ij})$$

$$d\lambda = \frac{d\gamma_{okt}^{pl}}{\tau_{okt}} = \frac{d\varepsilon^{pl}\ 3}{\sqrt{2}\sqrt{2}\,\sigma_e} = \frac{3\, d\varepsilon^{pl}}{2\ \sigma_e}$$

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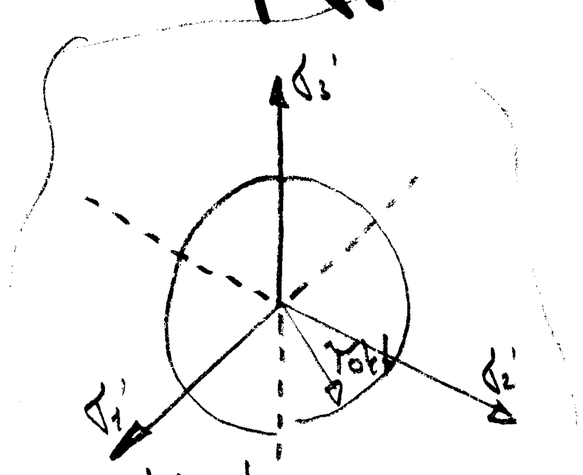
Misesov pogoj tečenja



π.. oktaedrično ravnino

k... radij kroga

- Predpostavke: - izotropni material
  - kriterij, ki ga opiše τ_{okt} simetrično na osi $\sigma_1, \sigma_2, \sigma_3$ + simetrično na simetrala $\sigma_{12,23,31}$

Mises predpostavi krog z enačbo

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2=k^2$$

Enoosno napetostno stanje

$\sigma_1=\sigma_0$; $\sigma_2=\sigma_3=0$

Ustavimo $\rightarrow$ $\sigma_0^2+\sigma_0^2=k^2$ $\rightarrow$ $k^2=2\sigma_0^2$

$$\rightarrow \boxed{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2=2\sigma_0^2}$$ Misesov pogoj

če

$$\tau_{okt}^2=\frac{1}{9}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right] \rightarrow \tau_{okt}^2=\frac{2}{9}\sigma_0^2$$

$$\tau_{okt}=\frac{\sqrt{2}}{3}\sigma_0^2$$

Pogoj preoblikovalnega dela

če

$$W_p=\frac{3}{4G}\tau_{okt}^2$$

$$W_p=\frac{1}{6G}\sigma_0^2$$

Material začne teči, ko doseže preoblikovalno delo vrednost na desni strani enačbe

Tenzorsko obliko

$$s_{ij}s_{ij}=\frac{2}{3}\sigma_0^2=3\cdot\frac{2}{9}\sigma_0^2$$

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Vpliv temperature na elastični material

- ΔT - raztezanje, krčenje
- oblika se ne spremeni, le volumen zato normalne dilatacije

$$\varepsilon_x^t = \alpha \Delta T$$
$$\varepsilon_y^t = \alpha \Delta T \qquad \gamma_{xy}^t = \gamma_{yz}^t = \gamma_{zx}^t = \emptyset$$
$$\varepsilon_z^t = \alpha \Delta T$$

α - lin. raztezostni koeficient

$T = T(x,y,z,t)$

Prenosna enačba $\varkappa \nabla^2 T + \frac{Q}{\rho c} = \frac{\partial T}{\partial t}$

$$\varkappa T_{,ii} + \frac{Q}{\rho c} = \dot{T} \quad \text{ali} \quad k T_{,ii} + Q = \rho c \dot{T} \quad ; \quad \varkappa = \frac{k}{\rho c}$$

Stacionarni primer $\dot{T} = 0 \rightarrow k T_{,ii} + Q = 0$

Brez notranje generacije toplote $T_{,ii} = \emptyset \quad ; Q = \emptyset$

Splošna enačba (bližnji realni procesi)

$$k T_{,ii} + Q = \rho c_v \dot{T} + m T_0 \dot{\varepsilon}_{kk}$$

Hookov zakon z upoštevanjem temp. deformacije

$$\varepsilon_{ij} = \frac{1}{2G} \sigma_{ij} - \frac{\nu}{E} \sigma_{kk} \delta_{ij}$$

$$\varepsilon_{ij} = \frac{1}{2G} \sigma_{ij} - \delta_{ij} \left(\frac{\nu}{E} \sigma_{kk} - \alpha T \right)$$

$$\sigma_{kk} = \frac{E}{1-2\nu} \varepsilon_{kk}$$

$$\sigma_{kk} = \frac{E}{1-2\nu} \varepsilon_{kk} - \frac{3E}{1-2\nu} \alpha T$$

$$\sigma_{ij} = 2\mu \varepsilon_{ij} + \lambda \varepsilon_{kk} \delta_{ij}$$

$$\sigma_{ij} = 2\mu \varepsilon_{ij} + \lambda \varepsilon_{kk} \delta_{ij} - \alpha T (2\mu + 3\lambda) \delta_{ij}$$

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Airy-jeva funkcija / Napetostna funkcija

- ravninsko napetostno stanje
- ravninski problem možno rešiti ce uvedemo skalarno funkcijo

$F = F(x,y)$, ki je v zvezi z napetostmi

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} \quad ; \quad \sigma_y = \frac{\partial^2 F}{\partial x^2} \quad ; \quad \tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} - xY - yX$$

Če $\Delta\Delta F = \emptyset$ zadostimo elastostatski ravninskega problema

Kompatibilnostna enačba za ravninsko napetostno stanje:

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu \sigma_y\right] + \alpha T = \frac{1}{E}\left[\frac{\partial^2 F}{\partial y^2} - \nu \frac{\partial^2 F}{\partial x^2}\right] + \alpha T$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu \sigma_x\right] + \alpha T = \frac{1}{E}\left[\frac{\partial^2 F}{\partial x^2} - \nu \frac{\partial^2 F}{\partial y^2}\right] + \alpha T$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} = \frac{1}{G}\left[-\frac{\partial^2 F}{\partial x \partial y} - xY - yX\right]$$

$$2(1+\nu) = 2 + 2\nu$$

$$\frac{\partial^4 F}{\partial y^4} - \nu \frac{\partial^4 F}{\partial x^2 \partial y^2} + E\alpha \frac{\partial^2 T}{\partial y^2} + \frac{\partial^4 F}{\partial x^4} - \nu \frac{\partial^4 F}{\partial x^2 \partial y^2} + \alpha E \frac{\partial^2 T}{\partial x^2} + E/G\left[\frac{\partial^4 F}{\partial x^2 \partial y^2}\right] = \emptyset \quad ; \quad \frac{E}{G} = (1+\nu)2$$

$$\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} + \alpha E\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \emptyset$$

$$\Delta\Delta F + \alpha E \Delta T$$

Če ni temperaturne obremenitve

$$\Delta\Delta F = \emptyset$$

- Robni pogoji so tesave

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