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Visja trdnost - Teorija April 2002

1. Dokaz enakosti tangencialnih napetosti
2. Misesov pogoj tecenja
3. Navier-Lame-jeve enacbe

Visja trdnost - Teorija 17.06.2002

1. Navier-Lame-jeve enacbe
2. Dokaz enakosti tangencialnih napetosti
3. Vpliv temperaturnih obremenitev na elasticno telo (Hookov zakon)

Visja trdnost - Teorija 01.07.2002

1. Dokaz enakosti tangencialnih napetosti
2. Navier-Lame-jeve enacbe
3. Dolocitev modula plasticnosti

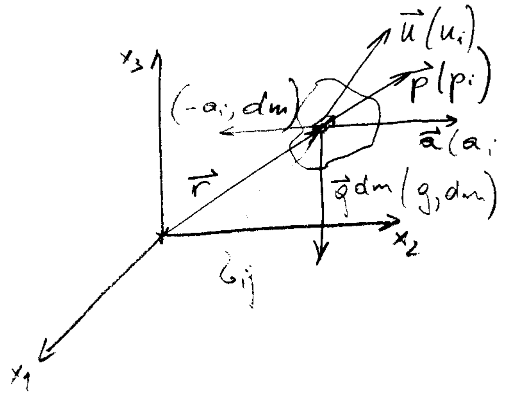
Visja trdnost - Teorija November 2002

1. Dokaz enakosti tangencialnih napetosti
2. Navier-Lame-jeve enacbe
3. Misesov pogoj tecenja

Visja trdnost - Teorija 17.04.2003

1. Navier-Lame-jeve enacbe
2. Misesov pogoj tecenja
3. Dolocitev modula plasticnosti

Ravnotežne enačbe mehanike



① $\sum \vec{F}_i = 0$
② $\sum \vec{M}_i = 0$
pogoja

Moment nadomestimo z dvojico sil

$dm = \rho\, dV$

$\rho = konst.$

① $\int_A \vec{p}\, dA + \int_V \vec{g}\, \rho\, dV - \int_V \vec{a}\, \rho\, dV = \vec{0}$

② $\int_A \vec{r} \times \vec{p}\, dA + \int_V \vec{r} \times \vec{g}\, \rho\, dV - \int_V \vec{r} \times \vec{a}\, \rho\, dV = \vec{0}$

① $\int_A p_i\, dA + \int_V g_i\, \rho\, dV - \int_V a_i\, \rho\, dV = 0$

② $\int_A \epsilon_{ijk}\, r_j p_k\, dA + \int_V \epsilon_{ijk}\, r_j g_k\, \rho\, dV - \int_V \epsilon_{ijk}\, r_j a_k\, \rho\, dV = 0$

$p_i = \sigma_{ij} n_j$

$$\boxed{\int_V \sigma_{ij,j}\, dV = \int_A p_i\, dA}$$

$\int_V \sigma_{ij,j}\, dV + \int_V g_i\, \rho\, dV - \int_V a_i\, \rho \cdot dV = 0$

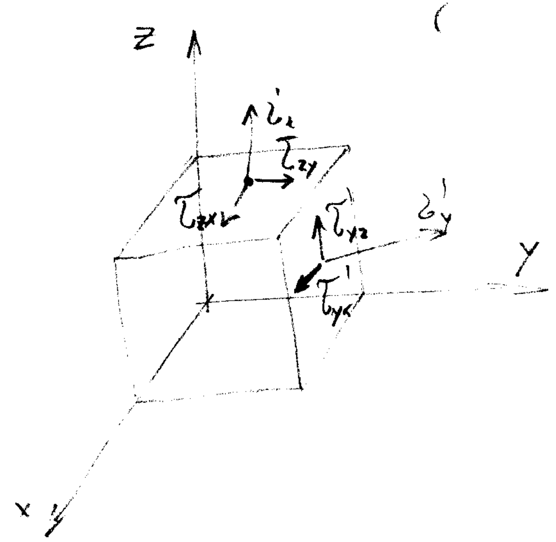
$\int_V \underbrace{(\sigma_{ij,j} + g_i \rho - a_i \rho)}\, dV = 0$; $\int_V [\;]\, dV = \lim_{\Delta V \to 0} \ldots = 0$

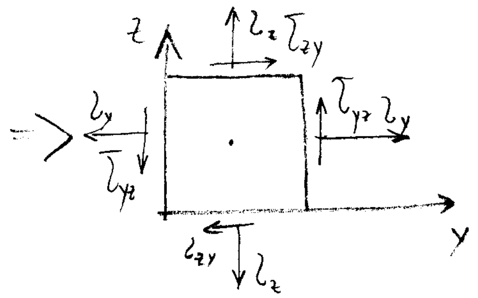
$$\boxed{\sigma_{ij,j} + \rho(g_i - a_i) = 0}$$ ravnotežna enačba sil na telesu

$\sigma_{ij,j} + \rho g_i = \rho a_i$ ravnotežne enačbe mehanike v masni točki

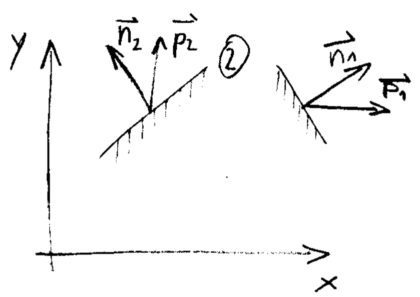
$\sigma_{ij,j} + \rho g_i = 0$ ravnotežne enačbe statike

Določi enakost tangencialnih napetosti





Da je sistem v ravnotežnem stanju mora biti $\tau_{zy} = \tau_{yz}$



$$p_i = \sigma_{ij} \cdot n_j$$

$$\sigma_{ij} = \begin{pmatrix} \sigma_x, & \tau_{xy}, & 0 \\ \tau_{yx}, & \sigma_y, & 0 \\ 0, & 0, & 0 \end{pmatrix}$$

$$p_i = \sigma_{ij} \cdot n_j$$

$$\vec{p} = (p_1, p_2, 0)$$
$$\vec{n} = (n_1, n_2, 0)$$

$$p_{1x} = \sigma_{xx} \cdot n_{x1} + \tau_{xy} n_{y1}$$
$$p_{1y} = \tau_{yx} n_{x1} + \sigma_{yy} n_{y1}$$

$$p_{2x} = \sigma_{xx} n_{x2} + \tau_{xy} n_{y2}$$
$$p_{2y} = \tau_{yx} n_{x2} + \sigma_{yy} n_{y2}$$

$$p_1 \cdot n_{x2} = \sigma_x n_{x1} n_{x2} + \tau_{xy} n_{y1} n_{x2}$$
$$p_{1y} n_{y2} = \tau_{yx} n_{x1} n_{y2} + \sigma_y n_{y1} n_{y2}$$
$$-p_{2x} n_{x1} = -\sigma_x n_{y2} n_{x1} - \tau_{xy} n_{y2} n_{x1}$$
$$-p_{2y} n_{y1} = -\tau_{yx} n_{x2} n_{y1} - \sigma_y n_{y2} n_{y1}$$

(+)

(+)

Kompatibilnostne enačbe

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \varepsilon_z = \frac{\partial w}{\partial z}$$

$$\varepsilon_{xy} = \frac{1}{2}\gamma_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\varepsilon_{yz} = \frac{1}{2}\gamma_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

$$\varepsilon_{zx} = \frac{1}{2}\gamma_{zx} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$$

tangencialne z normalnimi:

$$2\frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y} = \frac{\partial^2}{\partial x \partial y}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \frac{\partial^2}{\partial y^2}\varepsilon_x + \frac{\partial^2}{\partial x^2}\varepsilon_y$$

$$2\frac{\partial^2 \varepsilon_{yz}}{\partial y \partial z} = \frac{\partial^2}{\partial y \partial z}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \frac{\partial^2}{\partial z^2}\varepsilon_y + \frac{\partial^2}{\partial y^2}\varepsilon_z$$

$$2\frac{\partial^2 \varepsilon_{zx}}{\partial z \partial x} = \frac{\partial^2}{\partial z \partial x}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) = \frac{\partial^2}{\partial x^2}\varepsilon_z + \frac{\partial^2}{\partial z^2}\varepsilon_x$$

normalne s tangencialnimi:

$$\frac{\partial^2 \varepsilon_x}{\partial y \partial z} = \frac{\partial^3 u}{\partial x \partial y \partial z}$$

$$2\frac{\varepsilon_{yz}}{\partial x} = \frac{\partial^2 v}{\partial x \partial z} + \frac{\partial^2 w}{\partial x \partial y}$$

$$2\frac{\varepsilon_{zx}}{\partial y} = \frac{\partial^2 w}{\partial y \partial x} + \frac{\partial^2 u}{\partial y \partial z}$$

$$2\frac{\varepsilon_{xy}}{\partial z} = \frac{\partial^2 u}{\partial y \partial z} + \frac{\partial^2 v}{\partial x \partial z}$$

seštejemo $-\frac{2\varepsilon_{yz}}{\partial x} + \frac{2\varepsilon_{zx}}{\partial y} + \frac{2\varepsilon_{xy}}{\partial z}$

odvajamo po x in delimo z 2

x
z

$$\frac{\partial^2 \varepsilon_x}{\partial y \partial z} = \frac{\partial}{\partial x}\left(-\frac{\partial \varepsilon_{yz}}{\partial x} + \frac{\partial \varepsilon_{zx}}{\partial y} + \frac{\partial \varepsilon_{xy}}{\partial z}\right)$$

$$\frac{\partial^2 \varepsilon_y}{\partial x \partial z} = \frac{\partial}{\partial y}\left(-\frac{\partial \varepsilon_{zx}}{\partial y} + \frac{\partial \varepsilon_{xy}}{\partial z} + \frac{\partial \varepsilon_{yz}}{\partial x}\right)$$

$$\frac{\partial^2 \varepsilon_z}{\partial x \partial y} = \frac{\partial}{\partial z}\left(-\frac{\partial \varepsilon_{xy}}{\partial z} + \frac{\partial \varepsilon_{yz}}{\partial x} + \frac{\partial \varepsilon_{zx}}{\partial y}\right)$$

Vpliv temperaturnih sprememb na elastično telo

$$\varepsilon = \frac{\sigma}{E} \; ; \; \gamma = \frac{\tau}{G}$$

$$\underbrace{G}_{\text{strižni modul}} = \frac{E}{2(1+\nu)}$$ modul elastičnosti, poissonovo število

$$\sigma_{ij} = \underbrace{C_{ijkl}}_{\text{tenzor elastičnih konstant}} \cdot \varepsilon_{kl}$$

Sledi:

$$\sigma_{ij} = 2\mu\,\varepsilon_{ij} + \lambda\,\varepsilon_{kk}\,\delta_{ij} \quad \text{in} \quad \varepsilon_{ij} = \frac{1}{2G}\,\sigma_{ij} - \frac{\nu}{E}\cdot\frac{E}{1-2\nu}\,\varepsilon_{kk}\,\delta_{ij}$$

$$\underbrace{\mu = G = \frac{E}{2(1+\nu)} \qquad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}}_{\text{Lamejevi konstanti}}$$

Dodamo še vpliv temperature:

$$\varepsilon_{ij}^{T} = \alpha \cdot T\,\delta_{ij} \quad \ldots \quad \alpha\,[K^{-1}]$$ temp.

$$\alpha = \alpha\,[T] \qquad T\,[K] \ldots \text{sprememba temp.}$$

$$\varepsilon_{ij} = \varepsilon_{ij}^{?} - \varepsilon_{ij}^{T}$$

$$\varepsilon_{ij} = \frac{1}{2\mu}\,\sigma_{ij} - \left(\frac{\nu}{E}\,\sigma_{kk} - \alpha T\right)\delta_{ij}$$

Kompresijski modul

Če obremenimo kontinuum s hidrostatičnim pritiskom, potem je:

$$\sigma_x = \sigma_y = \sigma_z = -p$$

$$\tau_{xy} = \tau_{xz} = \tau_{yz} = 0$$

$$\sigma_{kk} = \frac{E}{1-2\nu}\,\varepsilon_{kk} - \frac{3E}{1-2\nu}\,\alpha T$$

$$-3p = \frac{E}{1-2\nu}\,\varepsilon_{kk} = \frac{E}{1-2\nu}\left(\varepsilon_x + \varepsilon_y + \varepsilon_z\right)$$

$$-3p = \frac{E}{1-2\nu}\,\varepsilon_v$$

$$\varepsilon_v = \frac{3(1-2\nu)}{E}(-p)$$

Kompresijski modul $K = \dfrac{E}{3(1-2\nu)}$

$$\lim_{K\to\infty} \varepsilon_K = \lim_{K\to\infty} \frac{p}{K} = 0$$

$$\lim_{\nu\to\frac{1}{2}} \frac{E}{3(1-2\nu)} = \lim_{\nu\to\frac{1}{2}} K = \infty \Rightarrow \varepsilon_v^{pl} = 0 \Rightarrow \nu^{pl} = \frac{1}{2}$$

volumen se v plastičnem področju ne spreminja več

$$\varepsilon_y = 0 \Rightarrow \nu^{pl} = \frac{1}{2}$$

Navier - Lamejeve enačbe

$$\sigma_{ij,j} + \rho f_i = 0$$ ravnotežne enačbe statike

$$\sigma_{ij,j} + X_i = \rho a_i \qquad / \sigma_{ij,j} + X_i = 0$$

$$\sigma_{ij} = \sigma_{ij}\left[\varepsilon_{ij}(u_i)\right]$$

$$\left(\sigma_{ij}\left[\varepsilon_{ij}(u_i)\right]\right)_{,j} + X_i = a_i \qquad \Rightarrow E = H = 3$$

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij} - (2\mu + 3\lambda)\alpha T\delta_{ij}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$$

$$\sigma_{ij} = 2\mu\frac{1}{2}\left(u_{i,j} + u_{j,i}\right) + \lambda\varepsilon_{kk}\cdot\delta_{ij} - (2\mu + 3\lambda)\alpha T\delta_{ij}$$

$$\sigma_{ij,j} = \mu\left(u_{i,jj} + u_{j,ij}\right) + \lambda\varepsilon_{kk,j}\delta_{ij} - (2\mu + 3\lambda)\alpha T_{,j}\delta_{ij}$$

$$u_{i,jj} = \frac{\partial^2 u_i}{\partial x_1\partial x_1} + \frac{\partial^2 u_i}{\partial x_2\partial x_2} + \frac{\partial^2 u_i}{\partial u_3\partial u_3} = \frac{\partial^2 u_i}{\partial x_1^2} + \frac{\partial^2 u_i}{\partial x_2^2} + \frac{\partial^2 u_i}{\partial x_3^2} = \Delta u_i$$

$$u_{j,ij} = \frac{\partial^2 u_j}{\partial x_i\partial x_j} = \frac{\partial^2 u_j}{\partial x_j\partial x_i} = u_{j,ji} = \frac{\partial u_{j,j}}{\partial x_i} = \frac{\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}}{\partial x_i} =$$

$$= \frac{\varepsilon_{jj}}{\partial x_i} = \frac{\varepsilon_{kk}}{\partial x_i} = \varepsilon_{kk,i}$$

Airy-eva funkcija

Funkcija je primerna za popis napetostnega stanja ravninskih elementov

Airy je pokazal, da je mogoče rešiti ravninski prostor, če uvedemo skalarno funkcijo : $F = F(x, y)$ ki je v naslednji zvezi

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} ; \sigma_y = \frac{\partial^2 F}{\partial x^2} ; \tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} - x Y - y X$$

Če imamo rotacijsko simetrično def. stanje

$$\sigma_r = \frac{1}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial \varphi^2} ; \sigma_\varphi = \frac{\partial^2 F}{\partial r^2}$$

$$\tau_{r\varphi} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial F}{\partial \varphi}\right) \qquad \boxed{\tau_{r\varphi} = 0}$$

Izhodišče je kompatibilnostna enačba :

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu(\sigma_y + \sigma_z)\right)$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu(\sigma_x + \sigma_z)\right)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{E} 2(1+\nu)$$

$$\frac{1}{E}\left[\frac{\partial^4 F}{\partial y^4} - \nu\frac{\partial^4 F}{\partial x^2 \partial y^2} + E\alpha\frac{\partial^2 T}{\partial y^2} + \frac{\partial^4 F}{\partial x^4} - \nu\frac{\partial^4 F}{\partial x^2 \partial y^2} + E\alpha\frac{\partial^2 T}{\partial x^2}\right] =$$

$$= -\frac{1}{E}\left[\frac{\partial^4 F}{\partial^2 x \partial^2 y}\right] \cdot 2(1+\nu)$$

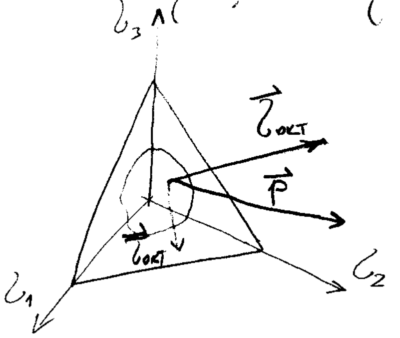
$$\underbrace{\left(\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4}\right)}_{\Delta\Delta F} + \alpha E \underbrace{\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)}_{\Delta T} = 0$$

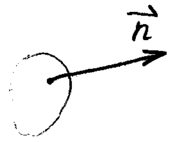
$$\boxed{\Delta\Delta F + \alpha E \Delta T = 0}$$

Kvazistatično temperaturno polje → $\Delta T = 0$

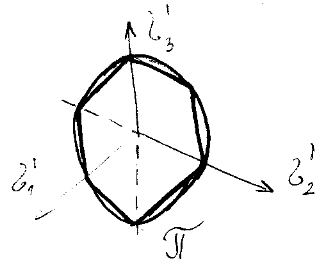
$$\boxed{\Delta\Delta F = 0}$$

Misesov pogoj tečenja





Za določitev meje plastičnosti uporabljamo hipotezi MISES-ovo ali TRESCOV



Predpostavke: · izotropni material

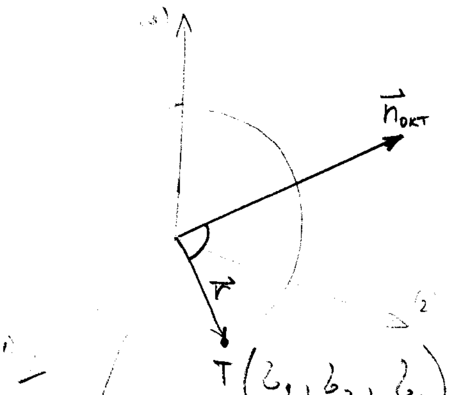
· mejna krivulje mora biti simetrične tudi glede na simetralo, ki leži med osmi, σ_1 in σ_2; σ_2 in σ_3; σ_3 in σ_1.

Pri plastifikaciji materiala se volumen ne spremeni.

$$\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z = 0$$

$$\sigma_{kk} = \frac{E}{1-2\nu}\,\varepsilon_{kk} = 0$$

$$\sigma_{okt} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = \frac{1}{3}\sigma_{kk} = 0$$ ⇒ od tod sledi, da na mejo plastičnega tečenja vpliva le τ_{okt}



$$\vec{n}_{okt} \times \vec{r} = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 1 & 1 \\ \sigma_1 & \sigma_2 & \sigma_3 \end{vmatrix} \cdot \frac{\sqrt{3}}{3} =$$

$$= \frac{\sqrt{3}}{3}\left[e_1(\sigma_3 - \sigma_2) - e_2(\sigma_3 - \sigma_1) + e_3(\sigma_2 - \sigma_1)\right]$$

$F(\tau_{okt}) = 0$

ravnina

$$|\vec{n}_{okt} \times \vec{r}| = |\vec{n}_{okt}| \cdot |\vec{r}| \cdot \sin 90° = 1 \cdot r \cdot 1 = r$$

$$r^2 = |\vec{n}_{okt} \times \vec{r}|^2 = \frac{1}{3}\left[(\sigma_3 - \sigma_2)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_2 - \sigma_1)^2\right]$$

$$3r^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

Trescin pogoj tečenja

$$\left[(\sigma_1-\sigma_2)^2-k_1^2\right]\left[(\sigma_2-\sigma_3)^2-k_1^2\right]\left[(\sigma_3-\sigma_1)^2-k_1^2\right]=0$$

$$(\sigma_0^2-k_1^2)\cdot(\sigma_0^2-k_1^2)=0$$ ... enoosno napetostno stanje

$$k_1=\sigma_0$$

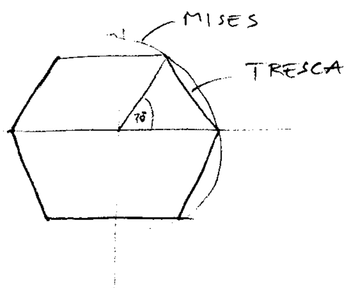
Sledi:

$$\left[(\sigma_1-\sigma_2)^2-\sigma_0^2\right]\left[(\sigma_2-\sigma_3)^2-\sigma_0^2\right]\left[(\sigma_3-\sigma_1)^2-\sigma_0^2\right]=0$$

$$\sigma_1-\sigma_2=\pm\sigma_0 \Rightarrow \tau_1=\frac{\sigma_0}{2}=\tau_2=\tau_3=\frac{\sigma_0}{2}=\frac{\sigma_1-\sigma_2}{2}$$

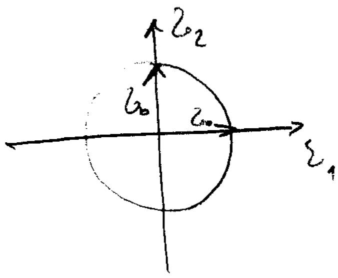
$$\sigma_2-\sigma_3=\pm\sigma_0$$

$$\sigma_3-\sigma_1=\pm\sigma_0$$ ... TRESCOV kriterij, tudi kriterij maksimalnih strižnih napetosti



$$\sigma_2=-\sigma_1$$
$$\sigma_3=0$$
$$\sigma_1=\frac{\sigma_0}{\sqrt{3}}$$

Ravninski problem



$$\sigma_3=0$$
$$(\sigma_1-\sigma_2)^2+\sigma_2^2-\sigma_1^2=2\sigma_0^2$$
$$\sigma_1^2-\sigma_1\sigma_2+\sigma_2^2=\sigma_0^2$$

$$\left.\begin{aligned}\sigma_1-\sigma_2&=\pm\sigma_0\\ \sigma_1&=\pm\sigma_0\\ \sigma_2&=\pm\sigma_0\end{aligned}\right\}$$ pogoji tečenje po TRESCU

Saint-Venantove enačbe 'Napetostne deformacijske zveze v plastičnem področju'

$$d e_{ij} = d\lambda \cdot s_{ij}$$

$$\sigma_{ij} = -p\,\delta_{ij} + 2\eta\,\dot{\varepsilon}_{ij} - \frac{2}{3}\eta\,\dot{\varepsilon}_v\,\delta_{ij}$$

$$p = -\frac{\sigma_{kk}}{3} \;;\; \varepsilon_x^{pl.} \approx 0$$ pri plastični deformaciji se volumen ne deformira nič več

$$\sigma_{ij} = \frac{\sigma_{kk}}{3}\delta_{ij} + s_{ij}$$

$$\frac{\sigma_{kk}}{3}\delta_{ij} + s_{ij} = \frac{\sigma_{kk}}{3}\delta_{ij} + 2\eta\,\dot{\varepsilon}_{ij}^{pl} - \frac{2}{3}\eta\,\underbrace{\dot{\varepsilon}_v^{pl}}_{0}\delta_{ij}$$

$$s_{ij} = 2\eta\,\dot{\varepsilon}_{ij}^{pl}$$

$$s_{ij} = 2\eta\frac{d\varepsilon_{ij}^{pl}}{dt} \Rightarrow \boxed{d\varepsilon_{ij}^{pl} = \frac{dt}{2\eta}\cdot s_{ij}}$$

$$\varepsilon_{ij}^{pl} = \frac{\varepsilon_{kk}^{pl.}}{3}\cdot\delta_{ij} + e_{ij}^{pl} \;;\; \varepsilon_{kk}^{pl} = \varepsilon_v^{pl} = 0$$

$$\varepsilon_{ij}^{pl.} = e_{ij}^{pl.}$$

$$\boxed{d\lambda = \frac{dt}{2\eta}}$$ – modul plastičnosti

$$d\varepsilon_{ij}^{pl} = de_{ij} = d\lambda\cdot s_{ij}$$ .. Saint-Venantova enačba

$$\left(d\varepsilon_k^{pl}, \frac{1}{2}d\gamma_{xy}^{pl}, \frac{1}{2}d\gamma_{xz}^{pl}\right) = d\lambda\begin{pmatrix} \frac{2\sigma_x-\sigma_y-\sigma_z}{3} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \frac{2\sigma_y-\sigma_x-\sigma_z}{3} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \frac{2\sigma_z-\sigma_x-\sigma_y}{3}\end{pmatrix}$$

Modul plastičnosti $d\lambda$

$s_{ij} s_{ij} = 3\tau_{okt}^2$ $\qquad d\varepsilon_{ij}^{pl} = de_{ij}^{pl} = d\lambda\, s_{ij}$

$e_{ij} e_{ij} = 3\gamma_{okt}^2$ $\qquad de_{ij}^{pl}\cdot de_{ij}^{pl} = d\lambda^2 s_{ij} s_{ij} = d\lambda^2 \cdot 3\tau_{okt}^2 = 3\left(d\gamma_{okt}^{pl}\right)^2$

$$d\lambda = \frac{d\gamma_{okt}^{pl}}{\tau_{okt}}$$

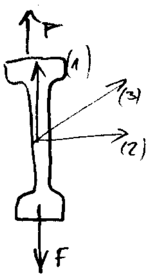
$$\tau_{okt} = \frac{1}{3}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]^{\frac{1}{2}} = \tau_{okt}(\sigma_{ij})$$

$$\gamma_{okt} = \frac{1}{3}\left[(\varepsilon_1-\varepsilon_2)^2 + (\varepsilon_2-\varepsilon_3)^2 + (\varepsilon_3-\varepsilon_1)^2\right]^{\frac{1}{2}}$$

$$\sigma_1 \neq 0,\ \sigma_2 = \sigma_3 = 0$$

Efektivna napetost naj bo tista napetost, ki je sprejemljiva z enoosnim napetostnim stanjem in preko katere lahko izrazimo poljubno napetostno stanje!

$$\sigma_e = \frac{3}{\sqrt{2}}\tau_{okt}(\sigma_{ij})$$



$\varepsilon - \nu\varepsilon = \varepsilon_2$ ... specifične kontrakcije

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_1, & 0, & 0 \\ 0, & \varepsilon_2, & 0 \\ 0, & 0, & \varepsilon_3 \end{pmatrix} = \begin{pmatrix} \varepsilon_1, & 0, & 0 \\ 0, & -\nu\varepsilon, & 0 \\ 0, & 0, & -\nu\varepsilon \end{pmatrix}$$

$$\gamma_{okt}(\varepsilon) = \frac{1}{3}\left[(\varepsilon+\nu\varepsilon)^2 + (-\nu\varepsilon-\varepsilon)^2\right]^{\frac{1}{2}} = \frac{1}{3}\left[2\varepsilon^2(1+\nu)^2\right]^{\frac{1}{2}}$$

$$\gamma_{okt}(\varepsilon) = \frac{\varepsilon}{3}(1+\nu)\sqrt{2}$$

$$\gamma_{okt}^{pl}(\varepsilon) = \frac{\varepsilon^{pl}}{3}(1+\nu^{pl})\sqrt{2} \qquad \nu^{pl} = \frac{1}{2}$$

$\gamma_{okt}^{pl} = \frac{\sqrt{2}}{2}\varepsilon^{pl} = \frac{\varepsilon^{pl}}{\sqrt{2}} = \gamma_{okt}^{pl}$ ... funkcija enoosnega napetostnega stanja

$$\varepsilon_e^{pl} = \sqrt{2}\,\gamma_{okt}^{pl}(\varepsilon_{ij}) = \sqrt{2}\,\gamma_{okt}^{pl}(\sigma_{ij}) \Rightarrow d\varepsilon_e^{pl} = \sqrt{2}\,d\gamma_{okt}^{pl}(\sigma_{ij})$$

$$d\lambda = \frac{d\gamma_{okt}^{pl}}{\tau_{okt}} = \frac{d\varepsilon_e^{pl}}{\sqrt{2}\,\frac{\sigma_e\sqrt{2}}{3}} = \frac{3}{2}\frac{d\varepsilon_e^{pl}}{\sigma_e}$$

$$\boxed{d\lambda = \frac{3}{2}\frac{d\varepsilon_e^{pl}}{\sigma_e}}$$