HOOKOV ZAKON

$$\varepsilon_{ij} = \frac{1}{2\cdot\mu}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij} + \alpha\cdot\Delta T\cdot\delta_{ij}$$

$$\mu = G = \frac{E}{2\cdot(1+\nu)}$$

$$\varepsilon_{xx} = \frac{1+\nu}{E}\sigma_x - \frac{\nu}{E}(\sigma_x+\sigma_y+\sigma_z) + \underset{\text{če ni }\Delta T}{\emptyset}$$

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y+\sigma_z)\right]$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_y - \nu(\sigma_z+\sigma_x)\right]$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x+\sigma_y)\right]$$

$$\varepsilon_{xy} = \frac{1+\nu}{E}\tau_{xy} = \varepsilon_{yx}$$

$$\varepsilon_{yz} = \frac{1+\nu}{E}\tau_{yz} = \varepsilon_{zy}$$

$$\varepsilon_{zx} = \frac{1+\nu}{E}\tau_{zx} = \varepsilon_{xz}$$

RAVNINSKO NAPETOSTNO STANJE

RAVNINSKO DEFORMACIJSKO STANJE

$$\varepsilon_z = \emptyset \Rightarrow \sigma_z \neq \emptyset$$

$$\sigma_r = A + Br^{-2} - \frac{3-2\nu}{8(1-\nu)}\rho\omega^2r^2$$

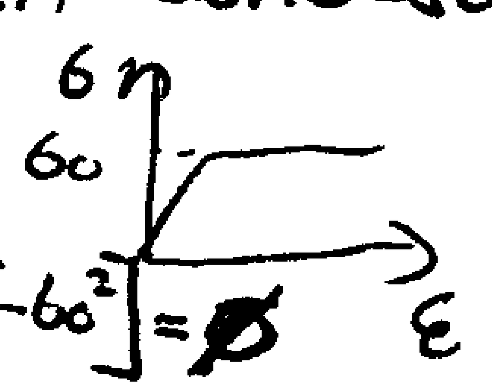
$$\sigma_\varphi = A - Br^{-2} - \frac{1+2\nu}{8(1-\nu)}\rho\omega^2r^2$$

$$\sigma_z = 2\nu A - \frac{\nu}{2(1-\nu)}\rho\omega^2r^2 - \alpha\Delta T E$$

$$u_r = \frac{(1+\nu)\cdot r}{E}\left[(1-2\nu)A - Br^{-2} - \frac{1-2\nu}{8(1-\nu)}\rho\omega^2r^2\right] + (1+\nu)\alpha\Delta T\cdot r$$

KROŽNE STENE V ELASTOPLASTIČNEM OBMOČJU

TRESCA:



$$\left[(\sigma_1-\sigma_2)^2-\sigma_0^2\right]\left[(\sigma_2-\sigma_3)^2-\sigma_0^2\right]\left[(\sigma_3-\sigma_1)^2-\sigma_0^2\right] = \emptyset$$

$$\left[(\sigma_r-\sigma_\varphi)^2-\sigma_0^2\right]\left[\sigma_\varphi^2-\sigma_0^2\right]\left[\sigma_r^2-\sigma_0^2\right] = \emptyset$$

1) $\sigma_r - \sigma_\varphi = \pm\sigma_0 \rightarrow \sigma_r = \mp\sigma_0 \ln\left(\frac{c}{r}\right)$

$\rightarrow \sigma_\varphi = \sigma_r \mp \sigma_0 = \mp\sigma_0\left[\ln\left(\frac{c}{r}\right)+1\right]$

$$u_r = \mp\frac{(1-\nu)\sigma_0}{E} r\cdot\ln\left(\frac{c}{r}\right) + \frac{D}{r} + \alpha\Delta T\cdot r$$

2) $\sigma_\varphi = \pm\sigma_0$; $\sigma_r = \pm\sigma_0 + \frac{C}{r}$

$$u_r = \frac{1-\nu}{E}(\pm\sigma_0 r + C) + \frac{D}{r} + \alpha\cdot\Delta T\cdot r$$

3) $\sigma_\varphi = \pm\sigma_0$; $\sigma_r = \pm\sigma_0 = \sigma_\varphi \rightarrow u_r = \pm\frac{r\cdot\sigma_0(1-\nu)}{E} + \frac{D}{r} + \alpha\cdot\Delta T\cdot r$

RNS $\sigma_z = \emptyset$; $\varepsilon_{zz} \neq \emptyset$

za polarne koordinate:

$$\varepsilon_{rr} = \frac{1}{E}\left[\sigma_r - \nu(\sigma_\varphi+\sigma_z)\right] + \alpha\Delta T = \frac{\partial u_r}{\partial r}$$

$$\varepsilon_{\varphi\varphi} = \frac{1}{E}\left[\sigma_\varphi - \nu(\sigma_z+\sigma_r)\right] + \alpha\Delta T = \frac{u_r}{r}$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_z - \nu(\sigma_r+\sigma_\varphi)\right] + \alpha\Delta T = \frac{\partial u_z}{\partial z}$$

KROŽNE STENE:

$$\sigma_\varphi = \sigma_r + \frac{d\sigma_r}{dr}\cdot r + r^2\cdot\omega^2\cdot\rho$$

$$-\sigma_r(1+\nu) + \sigma_\varphi(1+\nu) + \frac{d\sigma_\varphi}{dr} - \frac{d\sigma_r}{dr}\cdot r = \emptyset$$

DOBIMO:

$$\sigma_r = A + Br^{-2} - \frac{3+\nu}{8}\rho\omega^2r^2$$

$$\sigma_\varphi = A - Br^{-2} - \frac{1+3\nu}{8}\rho\omega^2r^2$$

$$u_r = \frac{r}{E}\left[(1-\nu)A - (1+\nu)Br^{-2} - \frac{1-\nu^2}{8}\rho\omega^2r^2\right] + \alpha\Delta T\cdot r$$

$$u_z = \frac{\nu\cdot h\cdot z}{E}\left[\frac{1+\nu}{2}\rho\omega^2r^2 - 2A\right] + \alpha\Delta T z$$

HUBERJEVA PORUŠITVENA HIPOTEZA

$$\sigma_p = \sqrt{\sigma_r^2 + \sigma_\varphi^2 - \sigma_r\sigma_\varphi + 3\tau_{r\varphi}^2} \leq \sigma_{DOP}$$

AIRY-jeva funkcija

RNS: $\Delta\Delta F - (1-\nu)\Delta V = \emptyset$

RDS: $\Delta\Delta F - \frac{1-2\nu}{1-\nu}\Delta V = \emptyset$

Če je $\Delta V = \emptyset$, potem velja:

$$\Delta\Delta F = \frac{\partial^4 F}{\partial x^4} + \frac{\partial^4 F\cdot 2}{\partial x^2\partial y^2} + \frac{\partial^4 F}{\partial y^4} = \emptyset$$

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} - V$$

$$\sigma_y = \frac{\partial^2 F}{\partial x^2} - V$$

$$\tau_{xy} = -\frac{\partial^2 F}{\partial x\partial y}$$

MISESOV POGOJ:

$$(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 \leq 2\sigma_0^2$$

$$(\sigma_r-\sigma_\varphi)^2 + \sigma_\varphi^2 + \sigma_r^2 \leq 2\sigma_0^2$$

$$\int_A \sigma_x\cdot dA = \text{sila}$$

$$\int y\cdot\sigma_x\cdot dA = M$$

MOMENT

$a_{ij} \cdot b_{ikl} = c_{jkl}$

$\delta_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ - ENOTSKI TENZOR

$\delta_{ij} \cdot a_{jk} = a_{ik}$

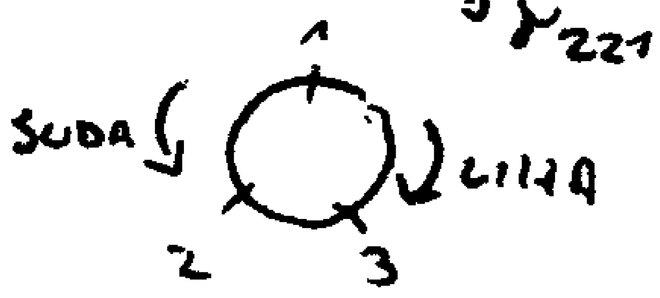
$\delta_{ij} \cdot a_{jik} = a_{iik} = a_{jjk}$

SKALARNI PRODUKT

$\vec{a} \cdot \vec{b} = a_i b_i$

VEKTORSKI PRODUKT

$\gamma_{ijk} = \begin{cases} 1 \to \gamma_{123} = 1 \\ -1 \to \gamma_{132} = -1 \\ 0 \to \gamma_{221} = \emptyset \end{cases}$



$\vec{c} = \vec{a} \times \vec{b} = \gamma_{ijk} a_j b_k =$

$= \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$

$\delta_{ij} \gamma_{ijk} = c_k = \emptyset = (0,0,0)$

$\delta_{ij} \delta_{ij} = 3$

$\gamma_{ijk} a_j a_k = \emptyset$

SIMETRIČNOST in ASIMETRIČNOST

$a_{ij} = a_{ij}^S + a_{ij}^A$

$a_{ij}^S = \frac{1}{2}(a_{ij} + a_{ji})$

$a_{ij} = \frac{1}{2}(a_{ij} - a_{ji})$

za simetričnost: $a_{ij}^S = a_{ji}^S$

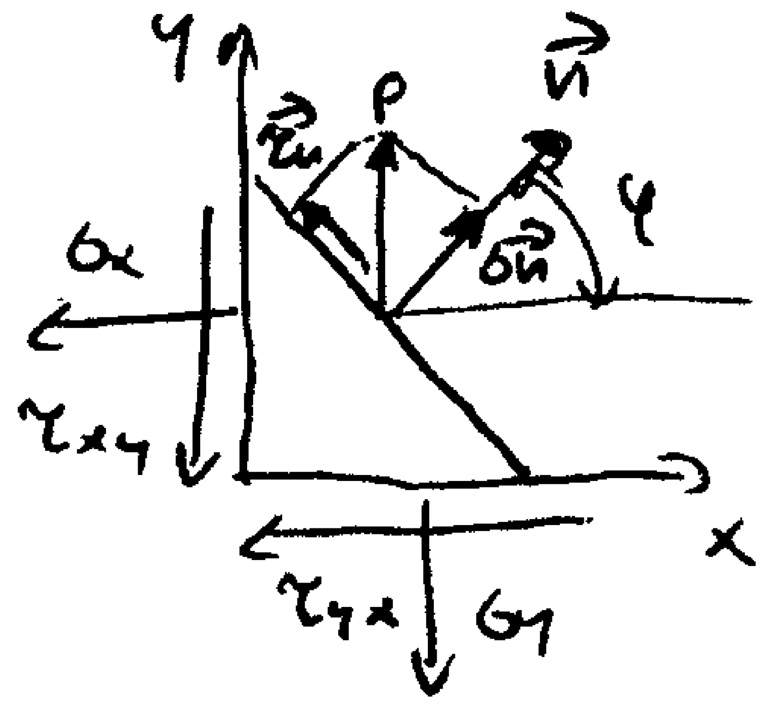
za asimetričnost: $a_{ij}^A = -a_{ji}^A$

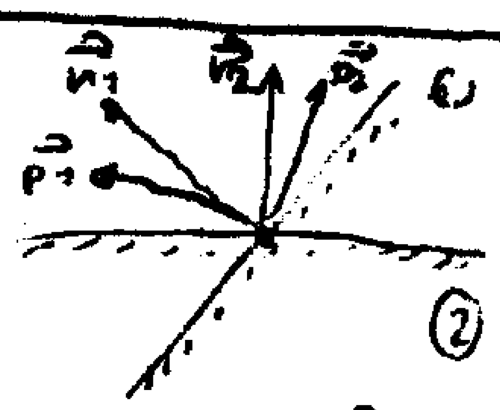
TEORIJA NAPETOSTI

$p_i = \sigma_{ij} \cdot n_j = \sigma_{i1} n_1 + \sigma_{i2} n_2$

$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\varphi + \tau_{xy} \sin 2\varphi$

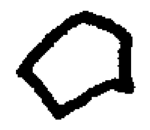
$\tau_n = \frac{\sigma_y - \sigma_x}{2} \sin 2\varphi + \tau_{xy} \cos 2\varphi$





$\vec{p_1} \vec{n_2} = \vec{p_2} \vec{n_1}$

$\sigma_{ij1} = \sigma_{ij2} = $ konst., ko:

(1)  - zunanji robovi ravni

(2) obremenitev po posameznih kosih = konst.

Za ravnotežje mora veljati ravnovesje sil in ne napetost.

GLAVNE NORMALNE NAPETOSTI

$(\sigma_{ij} - \sigma \delta_{ij}) n_j = \emptyset$

$\sigma_{12} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

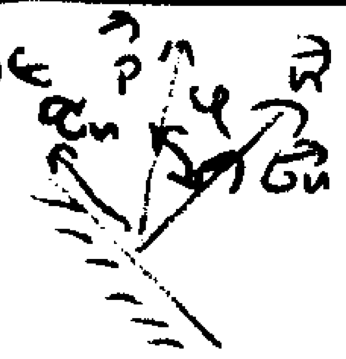
$\det(\sigma_{ij} - \sigma \delta_{ij}) = \emptyset$

$(\sigma_{i1} - \sigma \delta_{i1}) n_1 + (\sigma_{i2} - \sigma \delta_{i2}) \cdot n_2 = \emptyset$

$(\sigma_x - \sigma) n_x + \tau_{xy} \cdot n_y = \emptyset$

$\tau_{yx} \cdot n_x + (\sigma_y - \sigma) \cdot n_y = \emptyset$

GLAVNE TANGENCIALNE NAPETOSTI



$\tau_{max} = \frac{\sigma_2 - \sigma_1}{2}$

$\tau_n = (\sigma_2 - \sigma_1) \cos\varphi \cdot \sin\varphi$

$\tau_{ni} = \sqrt{p_i^2 - \sigma_{ni}^2}$

$\sigma_x + \sigma_y = \sigma_1 + \sigma_2$

VELIKOST IN LEGA GLAVNIH NORMALNIH NAPETOSTI

$(\sigma_{ij} - \sigma \delta_{ij}) n_{ij} = \emptyset$; $\sigma^3 - I_1 \sigma^2 + I_2 \sigma - I_3 = \emptyset$

$I_1 = \sigma_x + \sigma_y + \sigma_z$

$I_2 = \sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_z \sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2$

$I_3 = \sigma_x \sigma_y \sigma_z + 2 \tau_{xy} \tau_{yz} \tau_{zx} - \tau_{xy}^2 \sigma_z - \tau_{yz}^2 \sigma_x - \tau_{zx}^2 \sigma_y$

$\varepsilon_{12} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \varepsilon_{xy}^2}$

$\tan(2\varphi) = \frac{2\varepsilon_{xy}}{\varepsilon_x - \varepsilon_y}$

TEORIJA DEFORMACIJ

$\vec{\varepsilon} = \frac{d\vec{u}}{dr}$; $\vec{u} = (u, v, w)$

$\varepsilon_i = \frac{du_i}{dr} = u_{i,j} n_j$; $\vec{\varepsilon}_{ij} = u_{ij}$

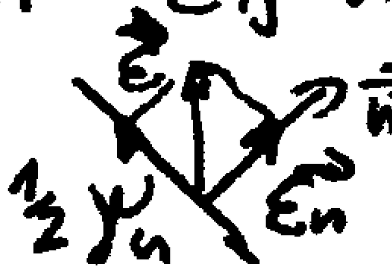
$\bar{\varepsilon}_{ij} = \varepsilon_{ij} + \omega_{ij}$; $\omega_{ij} = \emptyset$

SIMETRIČNI TENZOR — ASIMETRIČNI TENZOR

$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$

$\omega_{ij} = \frac{1}{2}(u_{i,j} - u_{j,i}) = \emptyset$

$\varepsilon_i = \varepsilon_{ij} n_j$



$\varepsilon_{xy} = \frac{1}{2} \gamma_{xy}$

$\varepsilon_n = \varepsilon_{xx} \cdot n_x^2 + \varepsilon_{yy} \cdot n_y^2 + 2 \cdot \varepsilon_{xy} \cdot n_x \cdot n_y$

$\varepsilon_{n_{AC}} = \frac{\Delta \overline{AC}}{\overline{AC}}$

$\varepsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\Delta l_x}{l_{x0}} = \frac{l_{xk} - l_{x0}}{l_{x0}}$

$\frac{1}{2} \gamma_n = \frac{\varepsilon_{yy} - \varepsilon_{xx}}{2} \sin 2\varphi + \varepsilon_{xy} \cos 2\varphi$

$\varepsilon_n = \varepsilon_i n_i = \vec{\varepsilon} \cdot \vec{n}$

$\vec{\varepsilon_n} = \varepsilon_n \cdot \vec{n}$

$\frac{1}{2} \vec{\gamma_n} = \vec{\varepsilon} - \vec{\varepsilon_n}$

$(\frac{1}{2} \gamma_n)^2 = \varepsilon^2 - \varepsilon_n^2$

STATIČNO NEDOLOČENE KONSTRUKCIJE

$U_G + U_V = \emptyset \Rightarrow \delta F = A_y$

$\delta W_N^G = \int_S \frac{M \cdot \delta M}{EI} ds$ (notranje sile)

$\delta W_Z^G = \delta F \cdot U_G$

IZENAČIMO

$\delta W_N^V = \int_S \frac{\delta M \cdot \delta M}{EI} ds$ (zunanje)

$\delta W_Z^V = \delta F \, U_V$

IZENAČIMO

$\delta W_N^G = \int_S \frac{N \cdot \delta N}{EA}$

$\Delta L = \frac{N \cdot l}{EA} + \alpha \Delta T \cdot L$

$\Delta L_2 = \frac{\delta F \cdot h}{E \cdot A_2}$

$= \emptyset$