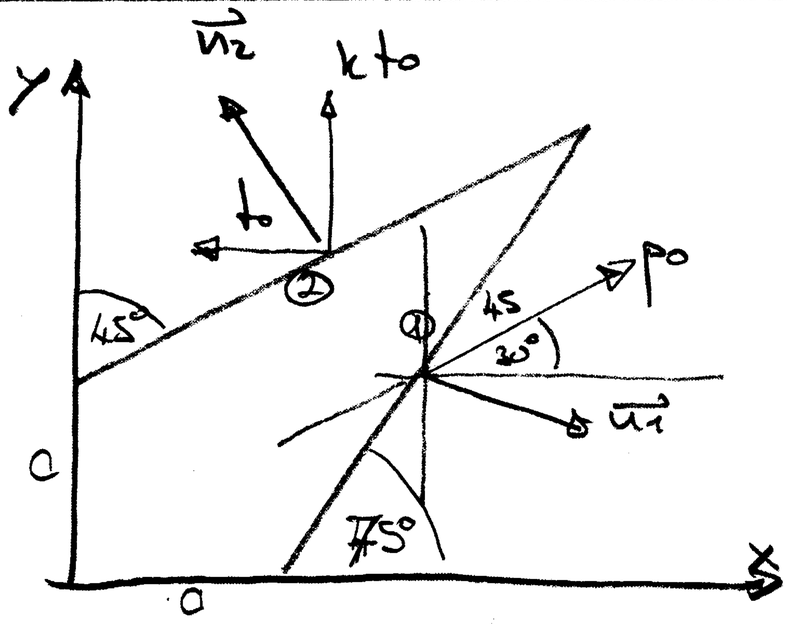


$\nu = 0,3$

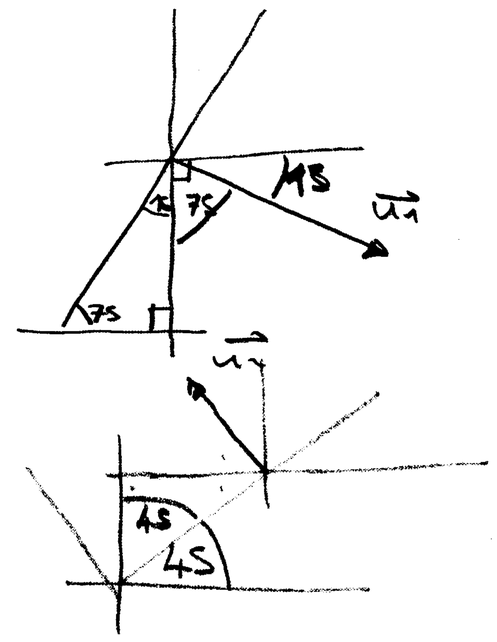
$a = 0,2$

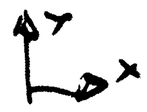
$p_o = 100$ MPa

$t_o = 50$ MPa

$E = 2,1 \cdot 10^5$ MPa

$AB = 2$





$\vec{u_1} = (\overset{x}{\cos 15}; \overset{y}{\sin 15})$

$\vec{u_2} = (-\cos 45; \sin 45)$

"p_1" $= (p_o \cos 30, p_o \sin 30)$

"p_2" $= (-t_o, k t_o)$

$p_{1x} = \sigma_x u_{x1} + \tau_{xy} u_{y1}$

$p_{1y} = \tau_{yx} u_{x1} + \sigma_y u_{y1}$

$p_o \cos 30 = \sigma_x \cos 15 + -\tau_{xy} \sin 15 \rightarrow 86,602 = 0,9659\, \sigma_x - 0,2588\, \tau_{xy}$

$p_o \sin 30 = \tau_{yx} \cos 15 + -\sigma_y \sin 15 \rightarrow 50 = 0,9659\, \tau_{xy} - 0,2588\, \sigma_y$

"p_{2x}" $= \sigma_x u_{x2} + \tau_{xy} u_{y2}$

"p_{2y}" $= \tau_{yx} u_{x2} + \sigma_y u_{y2}$

$-t_o = -\sigma_x \cos 45 + \tau_{xy} \sin 45 \rightarrow -50 = -0,7071\, \sigma_x + 0,7071\, \tau_{xy}$

$k t_o = -\tau_{yx} \cos 45 + \sigma_y \sin 45 \rightarrow k\, 50 = --0,7071\, \tau_{yx} + 0,7071\, \sigma_y$

$p_1 u_2 = p_2 u_1$

$p_o \cos 30 \cos 15 - p_o \sin 30 \sin 15 = +t_o \cos 45 + k t_o \sin 45$

$83,65 - 12,94 = 35,35 + k\, 35,35$

$$k = \frac{83,65 - 12,94 - 35,35}{35,35} = 1 \quad ; k = 1$$

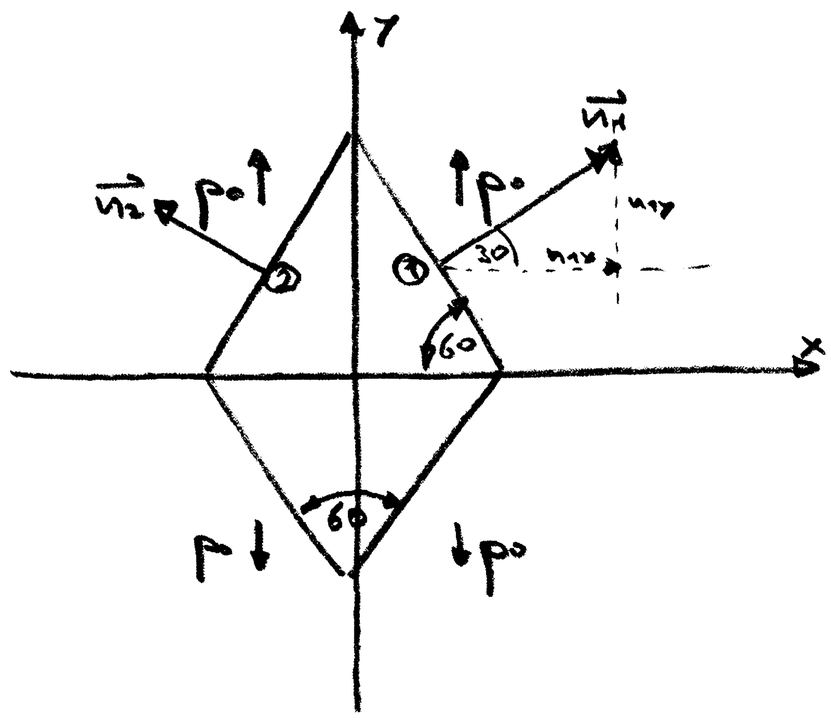
- $-50 + 0,7071\, \sigma_x = 0,7071\, \tau_{xy} \quad / : 0,7071$

  $-\frac{50}{0,7071} + \sigma_x = \tau_{xy}$

- $86,602 = 0,9659\, \sigma_x - 0,2588\, \sigma_x + 0,2588 \cdot \frac{50}{0,7071}$

$0,7071\, \sigma_x = 68,3019$

$\sigma_x = 96,59$



$p_0 = \checkmark$

$\sigma_{ij} = ?$

$p_i = \sigma_{ij} n_j \quad ; \quad \vec{n_1} = (\vec{n_{1x}}, \vec{n_{1y}}) = (\cancel{n_1}^{1} \cos 30 \; ; \; \cancel{n_1}^{1} \sin 30) = (\sqrt{3}/2, 1/2)$

$p_{1x} = \sigma_x n_{1x} + \tau_{xy} n_{1y}$

$p_{1y} = \tau_{yx} n_{1x} + \sigma_y n_{1y}$

$\emptyset = \sigma_x \sqrt{3}/2 + \tau_{xy} 1/2$

$p_0 = \tau_{yx} \sqrt{3}/2 + \sigma_y 1/2$

$p_i = \sigma_{ij} n_j \quad ; \quad \vec{n_2} = (\vec{n_{2x}}, \vec{n_{2y}}) = (-\sqrt{3}/2, 1/2)$

$p_{2x} = \sigma_x n_{2x} + \tau_{xy} n_{2y}$

$p_{2y} = \tau_{yx} n_{2x} + \sigma_y n_{2y}$

$\emptyset = -\sigma_x \sqrt{3}/2 + \tau_{xy} 1/2$

$p_0 = -\tau_{yx} \sqrt{3}/2 + \sigma_y 1/2$

$$\sigma_{ij} = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$$

- $\sigma_x \sqrt{3}/2 + \cancel{\tau_{xy} 1/2} = -\sigma_x \sqrt{3}/2 + \cancel{\tau_{xy} 1/2}$

  $\cancel{2} \sigma_x \sqrt{3}/\cancel{2} = \emptyset \rightarrow \sigma_x = \emptyset$

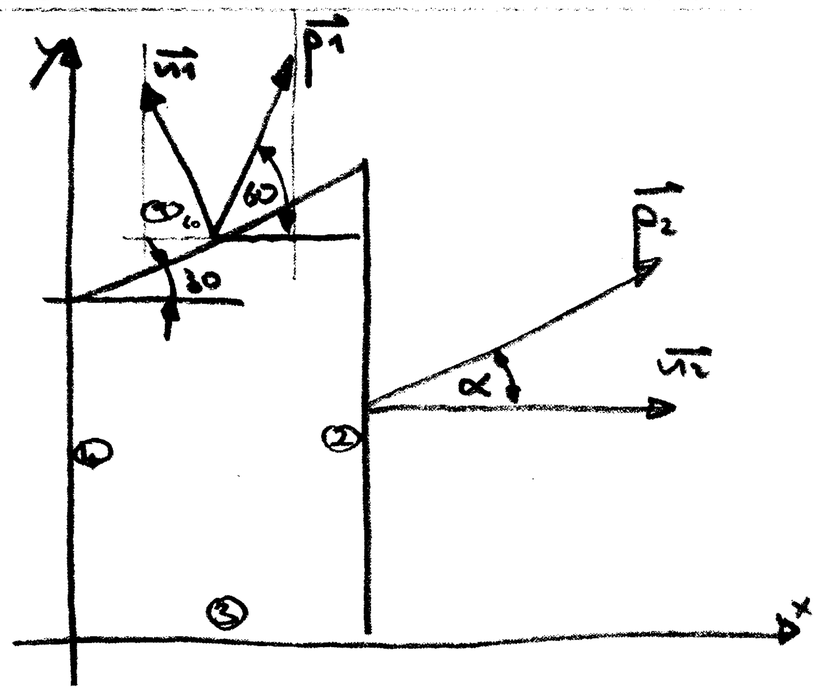
- $\tau_{yx} \sqrt{3}/2 + \cancel{\sigma_y 1/2} = -\tau_{yx} \sqrt{3}/2 + \cancel{\sigma_y 1/2}$

  $\cancel{2} \tau_{yx} \sqrt{3}/\cancel{2} = \emptyset \rightarrow \tau_{yx} = \emptyset$

- $\tau_{xy} = \emptyset$
- $2 p_0 = \cancel{2} \sigma_y 1/\cancel{2} \rightarrow \sigma_y = 2 p_0$

2D - 3D

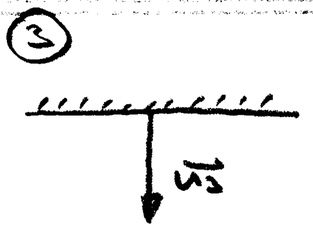
$$\sigma_{ij} = \begin{pmatrix} \emptyset & \emptyset & \emptyset \\ \emptyset & 2p_0 & \emptyset \\ \emptyset & \emptyset & \emptyset \end{pmatrix}$$



$p_1 = 80$ MPa

$p_2 = 50$ MPa

$\alpha,\ \sigma_{ij},\ p_3,\ p_4 = ?$

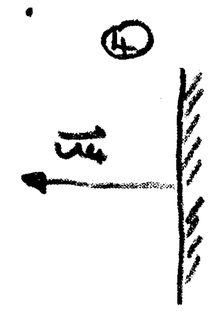


$\vec{n_3} = (0, -1)$

$p_{3x} = \tau_{xy}\, n_{1y} = -49{,}64$

$p_{3y} = \sigma_y\, n_{1y} = -108{,}659$

$p_3 = \underline{114{,}61 \text{ MPa}}$



$\vec{n_4} = (-1, 0)$

$p_{4x} = \sigma_x\, n_{1x} = -5{,}98$

$p_{4y} = \tau_{yx}\, n_{1x} = -49{,}64$

$p_4 = \underline{50 \text{ MPa}}$

$\vec{p_i}\,\vec{n_j} = \vec{p_j}\,\vec{n_i}$

$\vec{p_1}\,\vec{n_2} = \vec{p_2}\,\vec{n_1}$

$\vec{n_1} = (-\cos 60\ ;\ \sin 60) = (-0{,}5,\ \sqrt{3}/2)$

$\vec{n_2} = (1\ ;\ 0)$

$\vec{p_1} = (p_1 \cos 60\ ;\ p_1 \sin 60)$

$\vec{p_2} = (p_2 \cos\alpha\ ;\ p_2 \sin\alpha)$

$$(p_1 \cos 60,\ p_1 \sin 60)(1, 0) = (p_2 \cos\alpha,\ p_2 \sin\alpha)(\underbrace{-\cos 60}_{1/2},\ \underbrace{\sin 60}_{\sqrt{3}/2})$$

$$80 \cos 60 = -50 \cos\alpha \cos 60 + 50 \sin\alpha \sin 60$$

$$40 = -25 \cos\alpha + 25\sqrt{3} \sin\alpha \quad (*)$$

$$(40 + 25 \cos\alpha) = 25\sqrt{3} \sin\alpha \quad |^2$$

$$40^2 + 2 \cdot 40 \cdot 25 \cos\alpha + 25^2 \cos^2\alpha = 25^2 \cdot 3 \sin^2\alpha$$

$$1600 + \cancel{2000 \cos\alpha} + 625 \cos^2\alpha - 1875 + \cancel{1875 \cos^2\alpha} = \emptyset$$

$$2500 \cos^2\alpha + 2000 \cos\alpha - 275 = \emptyset \quad ;\ \cos\alpha = u$$

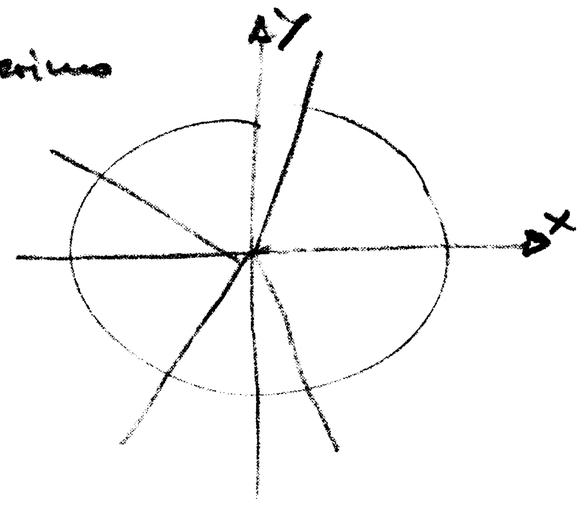
$$2500\, u^2 + 2000\, u - 275 = \emptyset$$

$$u_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Vstavimo in preverimo $(*)$

$u_1 = 0{,}1196 \rightarrow \alpha_1 = \underline{83{,}13}$ ✓ ; $\alpha_2 = \cancel{276{,}87}$

$u_2 = -0{,}9196 \rightarrow \alpha_3 = 156{,}86$ ✓ ; $\alpha_4 = \cancel{203{,}14}$



$p_i = \sigma_{ij}\, n_j$

$p_{2x} = \sigma_x \cancel{n_{2x}}^{1} + \cancel{\tau_{xy} n_{2y}}^{0} \Rightarrow \sigma_x = p_2 \cos\alpha_1 \rightarrow \sigma_x = 5{,}98$ MPa

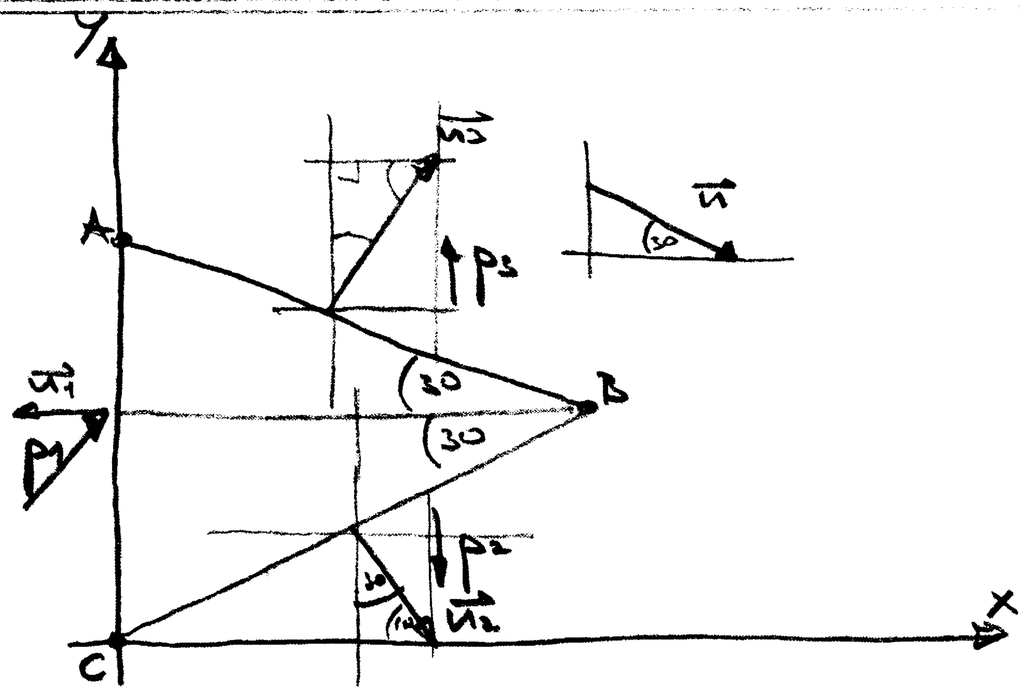
$p_{2y} = \tau_{yx} \cancel{n_{2x}}^{0} + \cancel{\sigma_y n_{2y}}^{0} \Rightarrow \tau_{yx} = p_2 \sin\alpha_1 \rightarrow \tau_{yx} = 49{,}64$ MPa

$p_{1x} = \sigma_x n_{1x} + \tau_{xy} n_{1y} \rightarrow 40 = -2{,}99 + \sqrt{3}/2\, \tau_{xy} \rightarrow \tau_{xy} = 49{,}64$ MPa } enako !!!!

$p_{1y} = \tau_{yx} n_{1x} + \sigma_y n_{1y} \rightarrow 69{,}282 = -24{,}82 + \sqrt{3}/2\, \sigma_y$

$\hookrightarrow \sigma_y = 108{,}659$ MPa

$$\sigma_{ij} = \begin{pmatrix} 5{,}98 & 49{,}64 & \emptyset \\ 49{,}64 & 108{,}659 & \emptyset \\ \emptyset & \emptyset & \emptyset \end{pmatrix}$$



$p_2 = 40$ MPa

$E = 200$ GPa $= 200 \cdot 10^9$ Pa $= 200.000$ MPa

$\overline{AB} = 100$ mm

$\Delta \overline{AB} = ?$ $\qquad \nu = 0,3$

$\vec{u_2} = (u_{2x}, u_{2y}) = (\cancel{u_2} \cos 60, -\cancel{u_2} \sin 60) = (0,5 ; -\sqrt{3}/2)$

$p_{2x} = \sigma_x u_{2x} + \tau_{xy} u_{2y}$

$p_{2y} = \tau_{yx} u_{2x} + \sigma_y u_{2y}$

$\emptyset = \sigma_x \cdot 0,5 + -\tau_{xy} \sqrt{3}/2$

$-p_2 = \tau_{yx} 0,5 - \sigma_y \sqrt{3}/2$

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}$$

$\Rightarrow \vec{u_3} = (u_3 \cos 60 ; u_3 \sin 60) = (0,5 ; \sqrt{3}/2)$

$\emptyset = \sigma_x 0,5 + \tau_{xy} \sqrt{3}/2 \quad \rightarrow \quad \tau_{xy} = \emptyset$

$p_3 = \tau_{yx} 0,5 + \sigma_y \sqrt{3}/2$

$$\sigma_{ij} = \begin{pmatrix} \emptyset & \emptyset & \emptyset \\ \emptyset & 46,188 & \emptyset \\ \emptyset & \emptyset & \emptyset \end{pmatrix}$$

$\vec{p_i} \vec{u_j} = \vec{p_j} \vec{u_i}$

$\vec{p_2} \vec{u_3} = \vec{p_3} \vec{u_2}$

$(0 ; p_2)(0,5 ; \sqrt{3}/2) = (0 ; p_3)(0,5 ; -\sqrt{3}/2)$

$-p_2 \sqrt{3}/2 = -p_3 \sqrt{3}/2$

$p_2 = p_3$

$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$

$\varepsilon_{xx} = (-\nu \sigma_y) 1/E = -6,92 \cdot 10^{-5}$

$\varepsilon_{yy} = \sigma_y 1/E = 2,3 \cdot 10^{-4}$

$\varepsilon_{zz} = (-\nu \sigma_y) 1/E = -6,92 \cdot 10^{-5}$

$\varepsilon_{xy, yz, zx} \Rightarrow$ usi $\emptyset$ zaradi $\tau_{xy,z} = \emptyset$

$\Rightarrow \emptyset = 2 \tau_{yx} 0,5 \quad \rightarrow \quad \tau_{yx} = \emptyset$

$\Rightarrow + p_2 = + \sigma_y \sqrt{3}/2 \quad \rightarrow \quad \sigma_y = 46,188$ MPa

$\Rightarrow \sigma_x = \emptyset$

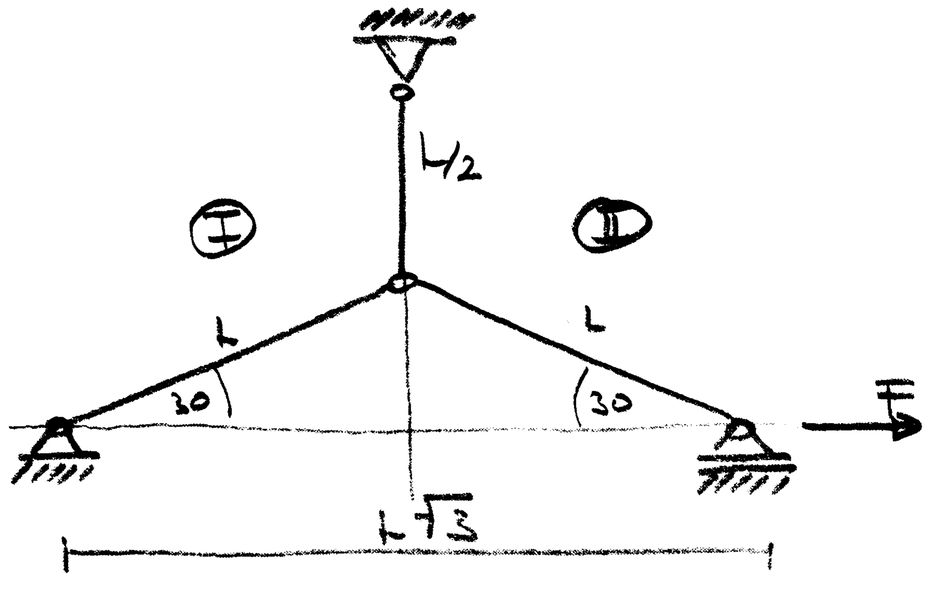
$$\varepsilon_{ij} = \begin{pmatrix} -6,92 \cdot 10^{-5} & \emptyset & \emptyset \\ \emptyset & 2,3 \cdot 10^{-4} & \emptyset \\ \emptyset & \emptyset & -6,92 \cdot 10^{-5} \end{pmatrix}$$

$\varepsilon_{u,AB} = \varepsilon_{xx} \cos^2 \varphi + \varepsilon_{yy} \sin^2 \varphi + \cancel{\varepsilon_{xy} \sin(2\varphi)} \rightarrow \emptyset$; $\vec{u}(u_x, u_y) = (\cos 30 ; -\sin 30)$

$\vec{u} = (\sqrt{3}/2 ; -0,5)$

$\varepsilon_{uAB} = -6,92 \cdot 10^{-5} \cos^2 30 + 2,3 \cdot 10^{-4} \sin^2 30$

$\varepsilon_{uAB} = 0,056 \cdot 10^{-4}$; $\Delta \overline{AB} = \varepsilon_{uAB} \overline{AB} = \underline{5,6 \cdot 10^{-4} \text{ mm}}$



E, I, A, F

N: sila u polici = ?

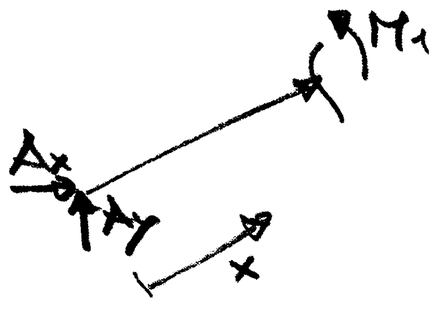
Glavni sistem

$A_x = -F \rightarrow A_x = -F$

$A_y = -B_y \rightarrow A_y = \emptyset$

$B_y L\sqrt{3} = \emptyset \rightarrow B_y = \emptyset$

I

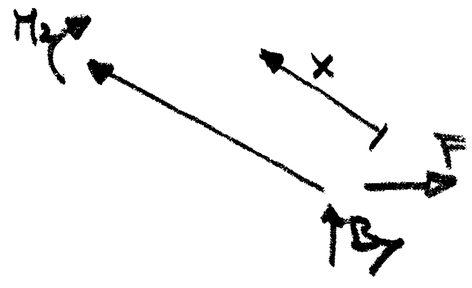


$M_1 + A_x \sin 30\, x = \emptyset$

$M_1 + A_x\, 0{,}5\, x = \emptyset$

$M_1 = +0{,}5\, F x$

II

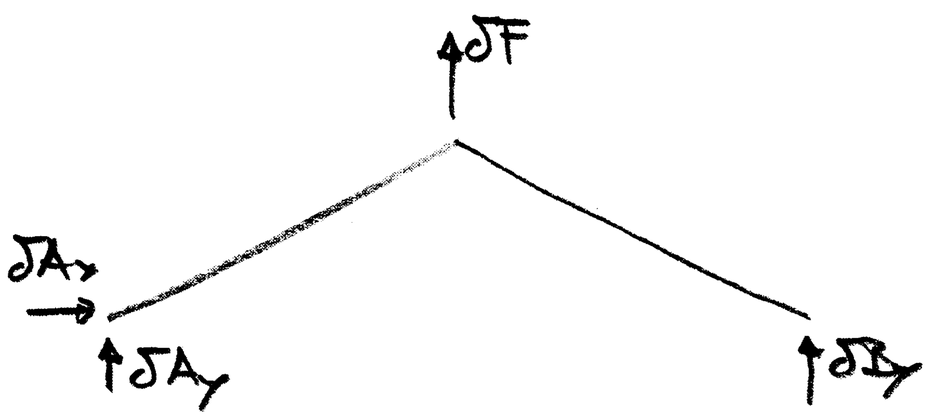


$M_2 - F x \sin 30 = \emptyset$

$M_2 - F x\, 0{,}5 = \emptyset$

$M_2 = +0{,}5\, F x$

Virtualni sistem



$\delta A_x = \emptyset$

$\delta A_y + \delta B_y + \delta F = \emptyset$

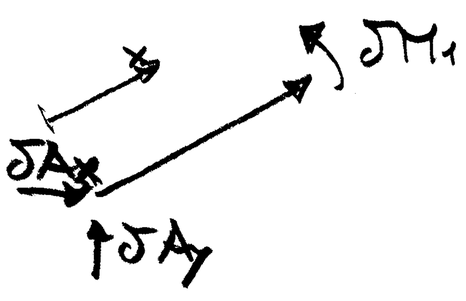
$\delta F\, \cancel{L} \cos 30 + \delta B_y\, 2\cancel{L} \cos 30 = \emptyset$

$\delta F = -2\delta B_y \;;\; \delta A_y + \delta B_y - 2\delta B_y = \emptyset$

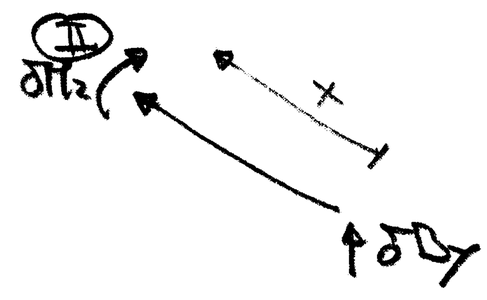
to $\delta A_y = \delta B_y$

to $\delta B_y = \delta A_y = -\frac{\delta F}{2}$

I



II



$\delta M_1 - \delta A_y\, x \cancel{\cos} \overset{\sqrt{3}\cdot 0{,}5}{30} = \emptyset$

$\delta M_1 = \sqrt{3}\, 0{,}5\, x\, \delta A_y = -\frac{\delta F x \sqrt{3}}{4}$

$\delta M_2 = \delta B_y\, x \cos 30 = \delta B_y\, x \sqrt{3}\, 0{,}5$

$\delta M_2 = -\frac{\delta F x \sqrt{3}}{4}$

$$\delta W_N^a = -\int_0^L 0{,}5\,Fx \cdot \frac{\delta F x \sqrt{3}}{4EI}\,dx + \int_0^L 0{,}5\,Fx\,\frac{\delta F x \sqrt{3}}{4EI}\,dx =$$

$$= -\frac{\delta F\, F \sqrt{3}}{8EI}\,\frac{L^3}{3}\cdot 2 = \frac{\sqrt{3}\,F\,\delta F\,L^3}{12\,EI}$$

$$\delta W_N^a = \delta W_2^a = u_a\,\cancel{\delta F} = \frac{\sqrt{3}\,F\,\cancel{\delta F}\,L^3}{12\,EI}$$

$$\boxed{u_a = \frac{F L^3 \sqrt{3}}{12\,EI}}$$

$$\Delta L = \frac{\delta F\, L/2}{EA}$$

$$\delta W_N^v = \int_0^l + \frac{\delta F x \sqrt{3}}{4\,EI}\,\frac{\delta F x \sqrt{3}}{4}\,dx + \int_0^l -\text{''}-\,dx =$$

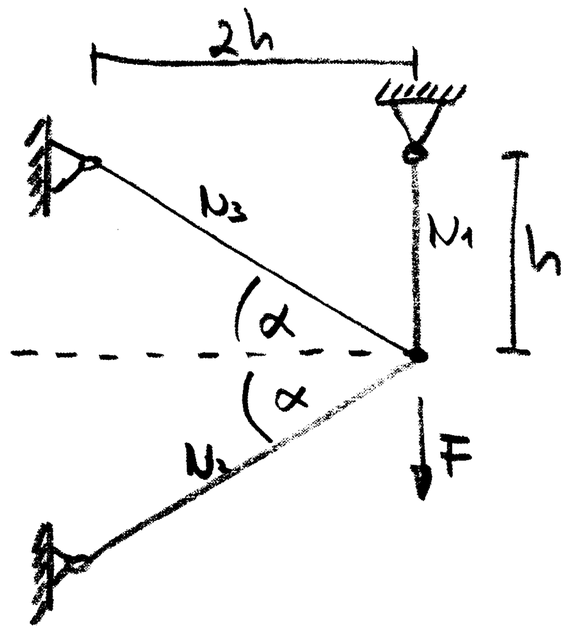
$$= \frac{\delta F^2\,\cancel{\sqrt{3}^2}}{16\,EI}\,\frac{L^3}{\cancel{3}}\,2 = \frac{\delta^2 F\,L^3}{8EI}$$

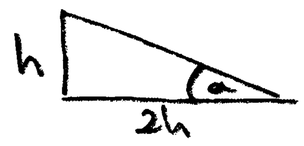
$$\delta W_N^v = \delta W_2^v = u_v\,\cancel{\delta F} = \frac{\delta F^2\,L^3}{8EI}$$

$$\boxed{u_v = \frac{\delta F\,L^3}{8EI}}$$

$$u_a + u_v + \Delta L = \emptyset$$

$$\frac{F L^3 \sqrt{3}}{12EI} + \frac{\delta F L^3}{8EI} + \frac{\delta F L}{2EA} = \emptyset \rightarrow \boxed{\delta F = -\frac{\frac{F L^3 \sqrt{3}}{12EI}}{\left[\frac{L^3 A}{8EIA} + \frac{L\,4I}{8EIA}\right]}}$$



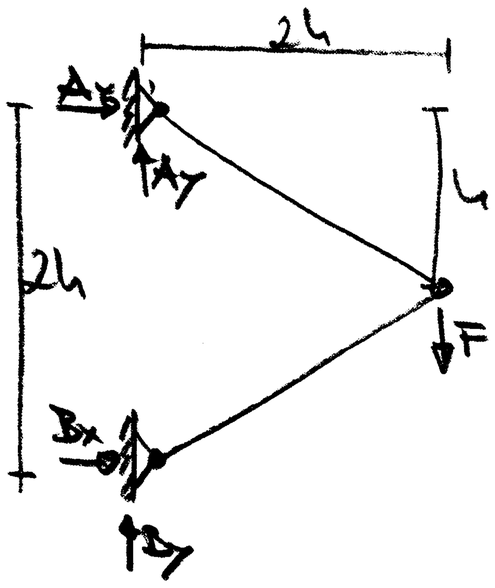


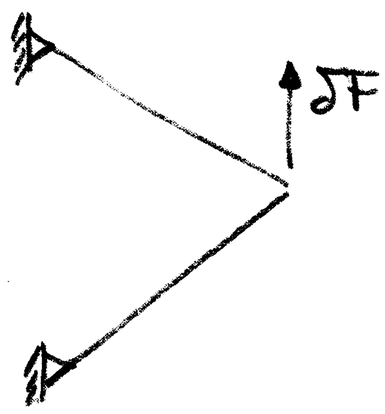
$tg\alpha = \frac{h}{2h} = 0{,}5$

$\alpha = 26{,}56°$ → uznam iz navodil $\alpha = 30°$ in ne 26...

$F = 130\ kN$

A_1, A_2, A_3

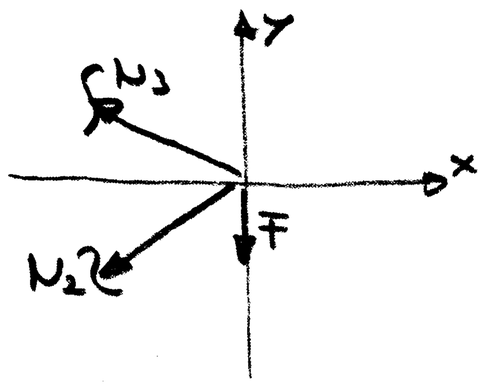


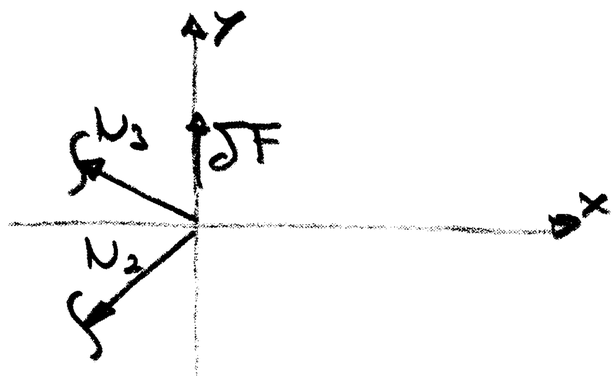


$A_x + B_x = 0$

$A_y + B_y = F$

$A_x h - B_x h + A_y 2h + B_y 2h = 0$





$-N_{3x} - N_{2x} = 0$

$N_{3y} - N_{2y} - F = 0$

$-N_3 \cos\alpha - N_2 \cos\alpha = 0 \quad /:\cos$

$N_3 \sin\alpha - N_2 \sin\alpha - F = 0 \quad /:\cos$

$N_3 \cos\alpha = -N_2 \cos\alpha$

$N_3 = -N_2$

$N_3\, tg\alpha - N_2\, tg\alpha - F/\cos\alpha = 0$

$-2 N_2\, tg\alpha - F/\cos\alpha = 0$

$+2 N_2 \sin\alpha + F = 0$

$$\boxed{N_2 = -\frac{F}{2\sin\alpha}}$$

$$\boxed{N_3 = \frac{F}{2\sin\alpha}}$$

→ $N_2 = -F$

$N_3 = F$

$-N_{3x} - N_{2x} = 0$

$N_{3y} - N_{2y} + \delta F = 0$

$-N_3 \cos\alpha - N_2 \cos\alpha = 0$

$N_3 \sin\alpha - N_2 \sin\alpha + \delta F = 0$

$N_3 = -N_2$

$-2 N_2 \sin\alpha + \delta F = 0$

$$\boxed{\delta N_2 = \frac{\delta F}{2\sin\alpha}}$$

$$\boxed{\delta N_3 = -\frac{\delta F}{2\sin\alpha}}$$

$\delta N_2 = \delta F$

$\delta N_3 = -\delta F$

$$\boxed{\Delta L = \frac{\delta F\, h}{E A_1}}$$

$$\delta W_N^G = \int_0^{2{,}236h} \frac{-F\,\delta F}{E A_2}\, dN + \int^{2{,}236h} \frac{F(-\delta F)}{E A_3}\, dN = -\frac{F\,\delta F}{E A_2}\, 2{,}2h - \frac{F\,\delta F}{E A_3}\, 2{,}2h =$$

$$= -\frac{F\,\delta F\, 2{,}2h}{E}\left[\frac{1}{A_2} + \frac{1}{A_3}\right]$$

$$\delta W_z^G = u_G\,\delta F = \delta W_N^G = \uparrow$$

$$\boxed{u_G = -\frac{F\, 2{,}2h}{E}\left[\frac{1}{A_2} + \frac{1}{A_3}\right]}$$

$$\delta W_N^v = \int_0^{2{,}2h} \frac{\delta F\,\delta F}{E A_2}\, dN + \int_0^{2{,}2h} \frac{(-\delta F)(-\delta F)}{E A_3}\, dN = \frac{\delta F^2\, 2{,}2h}{E}\left[\frac{1}{A_2} + \frac{1}{A_3}\right]$$

$$\delta W_z^v = u_v\,\delta F = \nearrow = \delta W_N^v$$

$$\boxed{u_v = \frac{\delta F\, 2{,}2h}{E}\left[\frac{1}{A_2} + \frac{1}{A_3}\right]}$$

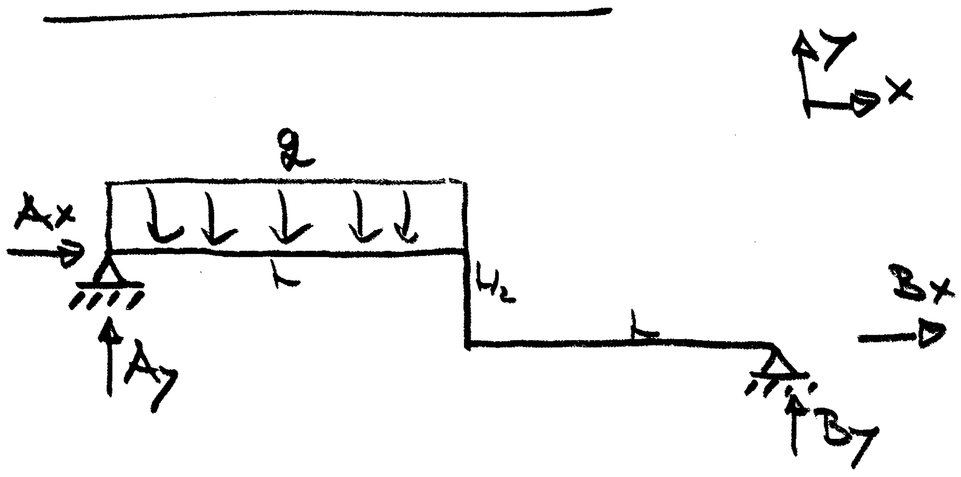
$$\Rightarrow u_G + u_v + \Delta L = 0$$

$$\left[-\frac{F\, 2{,}2h}{E} + \frac{\delta F\, 2{,}2h}{E}\right]\left[\frac{1}{A_2} + \frac{1}{A_3}\right] + \frac{\delta F\, h}{E A_1} = 0$$

$$\delta F = \frac{\cancel{E}\; F\, 2{,}2\cancel{h}}{\cancel{h}\,\cancel{E}\left[2{,}2\left[1/A_2 + 1/A_3\right] + \left[1/A_1\right]\right]} \quad \ldots$$

Vprašanje je kako tečejo meje integrala $\int_V ds$ za N v tem primeru?

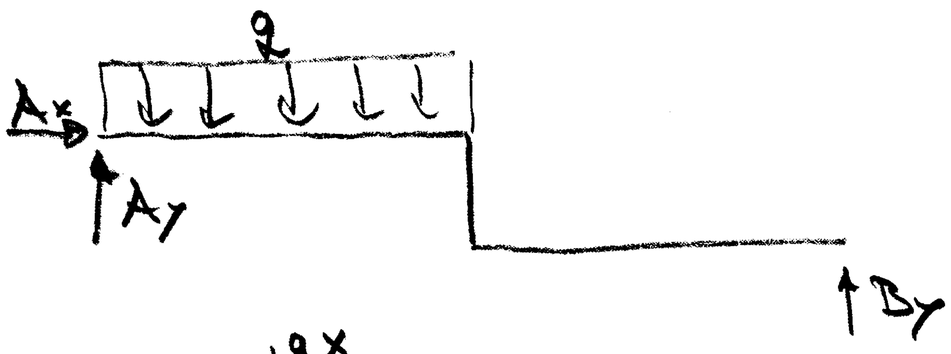
Inomo sistem



$B_x = ?$

$EI = konst$

- Glavni sistem

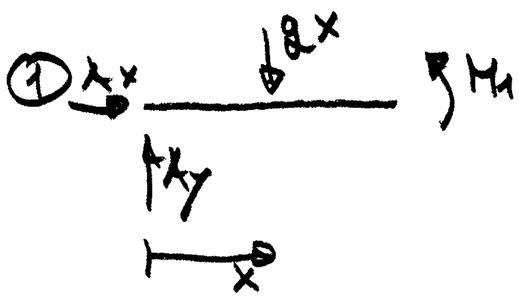


Reakcije:

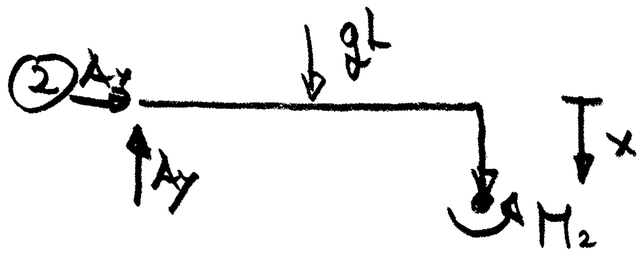
$A_y + B_y - qL = 0$; $A_y = qL - B_y = \frac{3qL}{4} = A_y$

$A_x = 0$

$B_y 2\cancel{L} = q\cancel{L}\frac{L}{2} \rightarrow B_y = \frac{1}{4}qL$

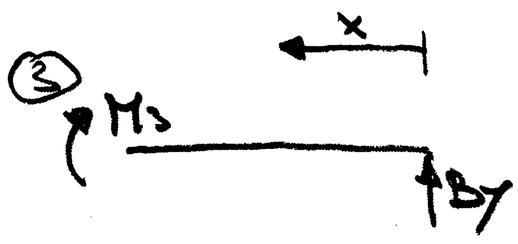


$M_1 = A_y x - qx\frac{x}{2} = \frac{3}{4}qLx - q\frac{x^2}{2}$



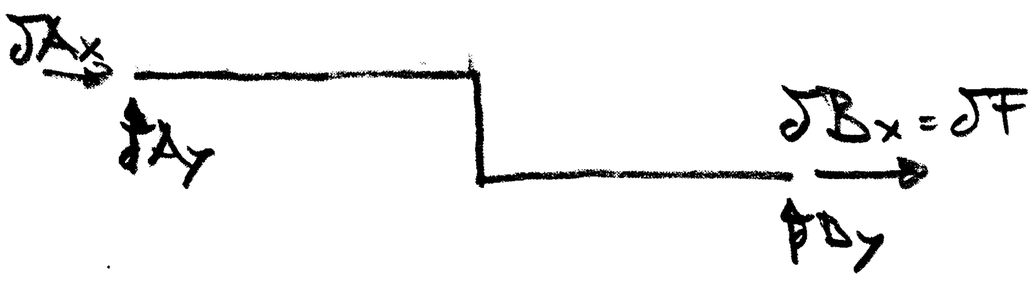
$M_2 = A_y L - qL\frac{L}{2} = \frac{3}{4}qL^2 - \frac{2}{4}qL^2 = \frac{1}{4}qL^2$

$\cancel{+A_x x \rightarrow 0}$



$M_3 = B_y x = \frac{1}{4}qLx$

- Virtualni sistem



Reakcije

$\delta A_x = -\delta F$

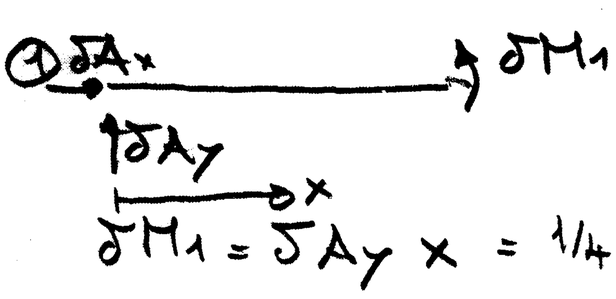
$\delta A_y + \delta B_y = 0$; $\delta A_y = -\delta B_y$

$\delta B_y 2\cancel{L} = -\delta F \frac{1}{2}\cancel{L}$

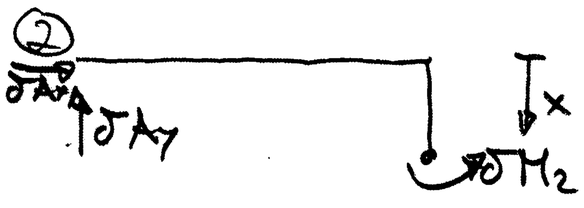
$\delta B_y = -\frac{1}{4}\delta F$

$\delta A_y = \frac{1}{4}\delta F$

$\delta A_x = -\delta F$

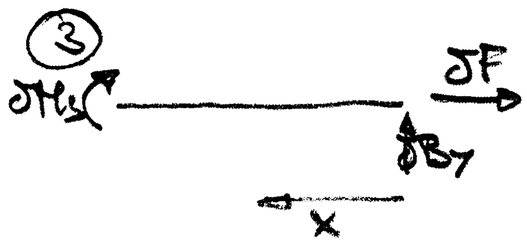


$\delta M_1 = \delta A_y x = \frac{1}{4}\delta F x$



$\delta M_2 = \delta A_y L + \delta A_x x =$

$= \frac{1}{4}\delta F L - \delta F x$



$\delta M_3 = \delta B_y x =$

$\delta M_3 = -\frac{1}{4}\delta F x$

$$u_a + u_v = \emptyset$$

$$\delta W_2^q = \delta W_v^q$$

$$\delta W_2^q = u_a \, \delta F$$

$$\delta W_{v1}^q = \frac{1}{EI}\int_0^L \left(\tfrac{3}{4} gL\, x - g\frac{x^2}{2}\right) \tfrac{1}{4}\delta F\, x \, dx =$$

$$= \frac{1}{EI}\int_0^L \frac{3\, g L\, \delta F}{4\;4} x^2 dx - \frac{1}{EI}\int_0^L \frac{g\,\delta F}{2\;4} x^3 dx =$$

$$= \frac{\cancel{3} g L \delta F}{16\, EI}\frac{L^3}{\cancel{3}} - \frac{g\,\delta F}{8 EI}\frac{L^4}{4} = \frac{g L^4 \delta F}{16\, EI} - \frac{g L^4 \delta F}{32\, EI} = \frac{g L^4 \delta F}{32\, EI}$$

$$\delta W_{v2}^q = \frac{1}{EI}\int_0^{1/2 L} \tfrac{1}{4} g L^2 \cdot \left(\tfrac{1}{4}\delta F L - \delta F\, x\right) dx =$$

$$= \frac{1}{EI}\int_0^{0{,}5L} \frac{g L^3 \delta F}{4\cdot 4} dx - \frac{1}{EI}\int_0^{0{,}5L} \frac{g L^2 \delta F}{4} x\, dx =$$

$$= \frac{g L^3 \delta F\, L}{16\, EI\cdot 2} - \frac{g L^2 \delta F\; L^2}{4\, EI\cdot 2\cdot 4} = \frac{g L^4 \delta F}{32\, EI} - \frac{g L^4 \delta F}{32\, EI} = \emptyset$$

$$\delta W_{v3}^q = \frac{1}{EI}\int_0^L -\tfrac{1}{4} g L\, x \cdot \tfrac{1}{4}\delta F x =$$

$$= -\int_0^L \frac{g L\, \delta F}{4\;4\;EI} x^2 = -\frac{g L^4 \delta F}{16\, EI\cdot 3} = \frac{g L^4 \delta F}{48\, EI}$$

$$u_a \cancel{\delta F} = \left[\frac{g L^4}{32\, EI} + \frac{g L^4}{48 EI}\right]\cancel{\delta F}$$

$$\boxed{u_a = \frac{5\, g L^4}{96\, EI}}$$

$$\delta W_{v1}^v = \frac{1}{EI}\int_0^L \left(\tfrac{1}{4}\delta F\, x\right)^2 dx = \int_0^L \frac{\delta F^2}{16\, EI} x^2 dx = \frac{\delta F^2 L^3}{48\, EI}$$

$$\delta W_{v2}^v = \frac{1}{EI}\int_0^{0{,}5L} \left(\tfrac{1}{4}\delta F L - \delta F\, x\right)\left(\tfrac{1}{4}\delta F L - \delta F\, x\right) =$$

$$= \frac{1}{EI}\int_0^{0{,}5L} \Big(\tfrac{1}{16}\delta F^2 L^2 \overbrace{- \tfrac{1}{4}\delta F^2 L x - \tfrac{1}{4}\delta F^2 L x}^{-\frac{1}{2}\delta F^2 L x} + \delta F^2 x^2\Big) dx$$

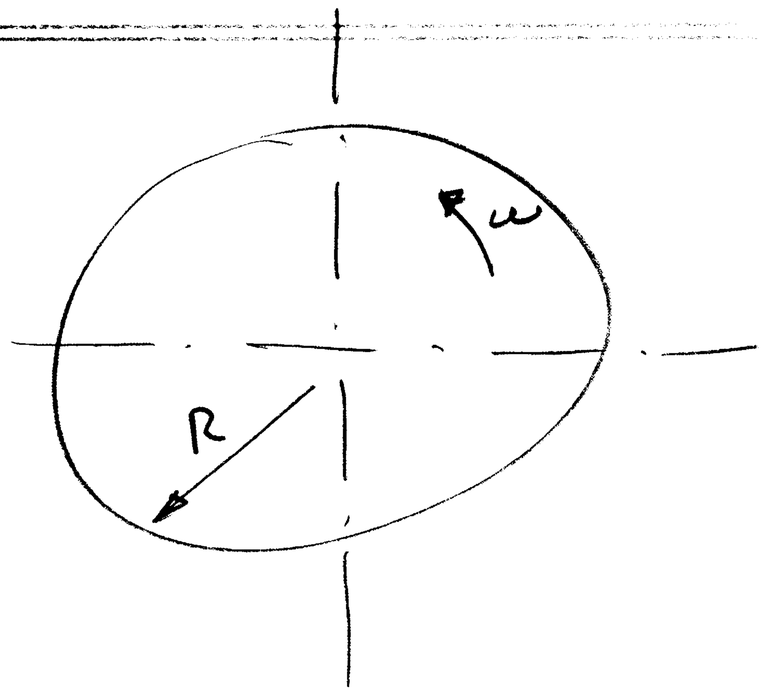
$$= \frac{\delta F^2 L^3}{32\, EI} - \frac{\delta F^2 L^3}{16\, EI} + \frac{\delta F^2 L^3}{24\, EI} = \frac{[128 - 64 + 96]\,\delta F^2 L^3}{4\, EI} = \frac{40\,\delta F^2 L^3}{EI}$$

$$\delta W_{v3}^v = \frac{1}{EI}\int_0^L \left(-\tfrac{1}{4}\delta F\, x\right)^2 dx = \frac{1}{EI}\int_0^L \tfrac{1}{16}\delta F^2 x^2 dx = \frac{\delta F^2 L^3}{48\, EI}$$

$$u_v \cancel{\delta F} = \left[\frac{L^3}{48 EI} + \frac{40 L^3}{EI} + \frac{L^3}{48 EI}\right]\delta F^{\cancel{2}} \longrightarrow \frac{L^3}{24 EI} + \frac{40 L^3}{EI}$$

$$u_v = \frac{961\, L^3}{24\, EI}\,\delta F$$

$$\hookrightarrow u_v + u_a = \emptyset \rightarrow \frac{961 L^3}{24 EI}\delta F = -\frac{5\, g L^4}{96\, EI} \rightarrow \delta F = -\frac{5\, g L^{\cancel{4}}\, 24 \cancel{EI}}{96 \cancel{EI} L^{\cancel{3}} 961} = \boxed{-\frac{120}{92256} g L = \delta F}$$



$R = 0{,}1\ m$ $\qquad \rho = 7850\ kg/m^3$

$\nu = 0{,}3$

$E = 200\ GPa = 200.000\ MPa$

ω_{bp}

RP.

$u_r(R) = 0{,}01$

$\sigma_r(R) = 0$

$\sigma_r(0) \neq \infty \rightarrow B = \emptyset$

$$\emptyset = A - \frac{3+\nu}{8}\rho\omega^2 R^2 \qquad A = \frac{3+\nu}{8}\rho\omega^2 R^2 = 32{,}4\ \omega^2$$

$$A = 32{,}4\ \omega^2\ [kg/m]$$

$$u_r(R) = 0{,}01 = \frac{R}{E}\left[(1-\nu)\,32{,}4\,\omega^2 - \frac{1-\nu^2}{8}\rho\omega^2 R^2\right]$$

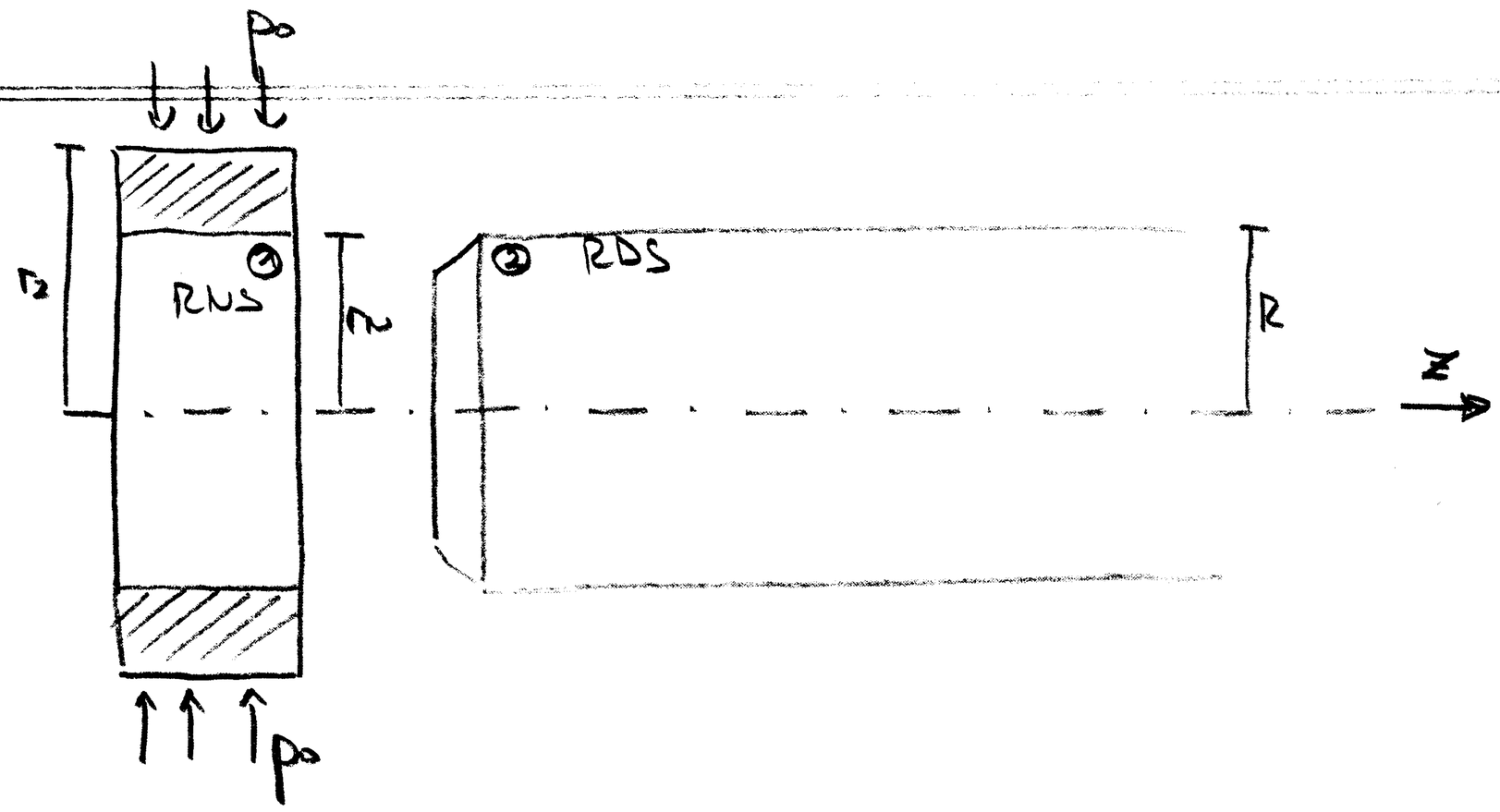
$$x = \frac{0{,}01\,E}{R} = \left[22{,}68\ \omega^2 - 8{,}92\ \omega^2\right]$$

$$\left[13{,}75\ \omega^2\right]\ [kg/m]$$

$$\omega = \sqrt{\frac{x}{13{,}75}} \qquad \frac{\frac{N}{m^2 m}}{\frac{kg}{m}} = \frac{\cancel{N}\ \cancel{m}\ \cancel{kg}\ m}{\cancel{m^3}\ \cancel{kg}\ s^2}$$

$$\omega = 1206\ s^{-1} = 2\pi n$$

$$n = 192\ s^{-1}\ rr/s = 11522\ rr/min$$



$\sigma = 120$ MPa

$p_0 = 100$ MPa

$r_N = 100$ mm

$r_2 = 250$ mm

$E = 210$ GPa

$\nu = 0{,}3$

$R = ?$

$$r_N + u_1(r_N) = R + u_2(R)$$

Rubni pogoji:

$$\sigma_{1r}(r_2) = -p_0$$
$$\sigma_{1r}(r_N) = -\sigma$$
$$\sigma_{2r}(R) = -\sigma$$
$$\sigma_{2r}(0) \neq \infty \quad ; \quad u_r(0) = 0 \rightarrow \underline{B_2 = 0}$$

$$\rightarrow -\sigma = A_2 \rightarrow A_2 = \underline{-\sigma = -120 \text{ MPa}}$$

$$\rightarrow -\sigma = A_1 + B_1 r_N^{-2} \rightarrow A_1 = -\sigma - B_1 r_N^{-2}$$

$$\rightarrow +p_0 = A_1 + B_1 r_2^{-2} \rightarrow -p_0 = -\sigma - B_1 r_N^{-2} + B_1 r_2^{-2} = +\sigma + B_1 (r_N^{-2} - r_2^{-2})$$

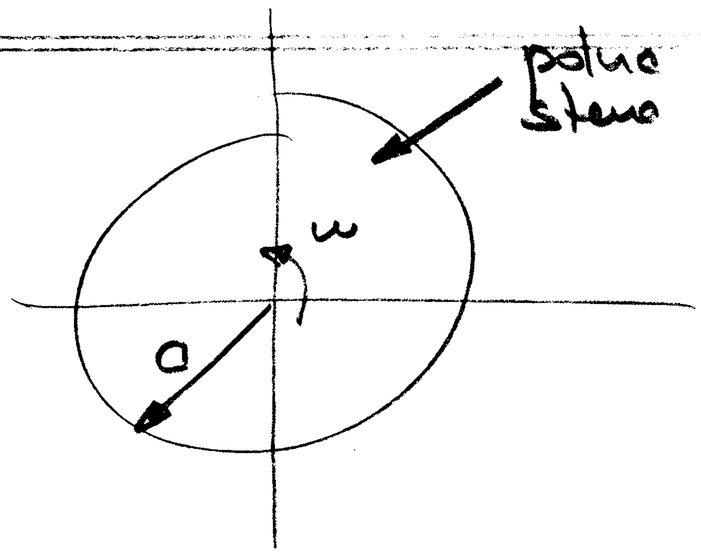
$$B_1 = \frac{p_0 - \sigma}{(r_N^{-2} - r_2^{-2})} = \underline{-0{,}2380952} \quad \left[\frac{\text{MPa}}{\text{m}^{-2}} = \text{MN}\right]$$

$$A_1 = \underline{-96{,}19 \text{ MPa}}$$

$$R + \frac{(1+\nu)R}{E}\left[(1-2\nu)A - BR^{-2}\right] = r_N + \frac{r_N}{E}\left[(1-\nu)A - (1+\nu)Br_N^{-2}\right]$$

$$R - B\frac{(1+\nu)}{ER} + \frac{(1+\nu)(1-2\nu)}{E}AR = -1727{,}37$$

$\rightarrow$ poč dobimo za R



$a = 0{,}2\ m$

$\sigma_{dop} = 120\ MPa$

$\rho = 7850\ kg/m^3$

$\nu = 0{,}3$

Huberjeva hipoteza

$\omega_{dop} = ?$

$\sigma_{dop}^2 = 2\sigma_r^2 - \sigma_r^2$

$\sigma_{dop}^2 = \sigma_r^2$

$\sigma_{dop} = \sigma_r = A_0 a^2 = 3238{,}125\,\omega^2\, 0{,}2^2$

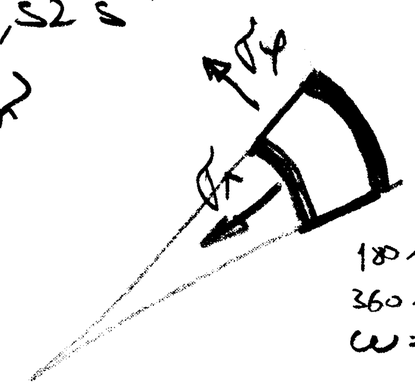
$\omega = 0{,}926$

$\omega = \dfrac{120\ MPa \cdot 10^6\ Pa\ kg\ m^2}{3238{,}125\ kg\ s^2\, 0{,}2^2\ m^2\ m^2}$

$\omega = 962{,}52\ s^{-1}$

$\omega = 2\pi\nu$

$\nu = 1/T$



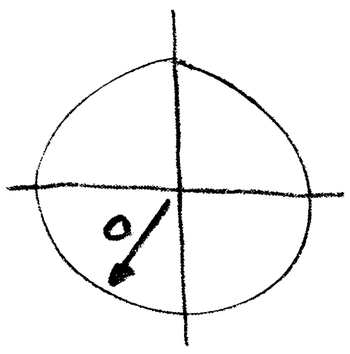
$180 \sim \pi$

$360 \sim 2\pi$

$\omega = 2\pi\nu$

$\omega = \dfrac{2\pi}{n}$

Robni pogoji



$\sigma_z = \emptyset$; $\varepsilon_{zz} \neq 0$ RNS

$u_r(0) = \emptyset$; $\sigma_r(0) \neq \infty$; $\sigma_\varphi(0) \neq \infty$

$\sigma_r(0) = \emptyset$

Rešitev

$\sigma_p^2 = 2A_0 a^2 - A_0^2 a^4$

$120^2 = 2 \cdot 3238{,}125 \cdot 0{,}2^2\, \omega^2 - 3238{,}125^2\, \omega^4\, 0{,}2^2$

$\sigma_{(r=0)} = A + B \cdot a^{-2} - \dfrac{3+\nu}{8}\rho\omega^2 a^2 = \emptyset$

$u_r(0) = \dfrac{1}{E}\left[\cancel{\dfrac{(1-\nu)A}{r}} - \dfrac{(1+\nu)B}{r^3} - \cancel{\dfrac{1-\nu^2}{8}\rho\omega^2 r}\right] \rightarrow B = \emptyset$

$-419418{,}14\,\omega^2 + 259{,}05\,\omega^2 - 14400 = \emptyset$

$\emptyset \Rightarrow \boxed{A = \dfrac{3+\nu}{8}\rho\omega^2 a^2}$; $\boxed{B = 0}$

$\omega_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Huber: $\sigma_p = \sqrt{\sigma_r^2 + \sigma_\varphi^2 - \sigma_r\sigma_\varphi + 3\tau_{r\varphi}^2} \leq \sigma_{dop}$

$A_0 = 3238{,}125\,\omega^2\ \dfrac{kg}{m^2}$

$B_0 = 1864{,}375\,\omega^2$

$\sigma_r(r) = \underbrace{\dfrac{3+\nu}{8}\rho\omega^2}_{A_0} a^2 - \dfrac{3+\nu}{8}\rho\omega^2 r^2 = A_0 a^2 - A_0 r^2$

$\rightarrow \sigma_r = \sigma_\varphi = A_0 a^2$

$\sigma_\varphi(r) = \dfrac{3+\nu}{8}\rho\omega^2 a^2 - \underbrace{\dfrac{1+3\nu}{8}\rho\omega^2}_{B_0} r^2 = A_0 a^2 - B_0 r^2$

$\sigma_p^2 = (A_0 a^2 - A_0 r^2)^2 + (A_0 a^2 - B_0 r^2)^2 - (A_0 a^2 - A_0 r^2)(A_0 a^2 - B_0 r^2)$

$\sigma_p^2 = A_0^2 a^4 - A_0^2 r^4 - 2A_0^2 a^2 r^2 + A_0^2 a^4 - B_0^2 r^4 - 2A_0 B_0 a^2 r^2 - \cancel{A_0 a^4} + \cancel{A_0 B_0 a^2 r^2} + \cancel{A_0 a^4} - A_0 B_0 r^4$

// $\sigma_p^2 = 2A_0^2 a^4 - r^4(A_0^2 + A_0 B_0 + B_0^2) - a^2 r^2(+2A_0^2 + A_0 B_0)$

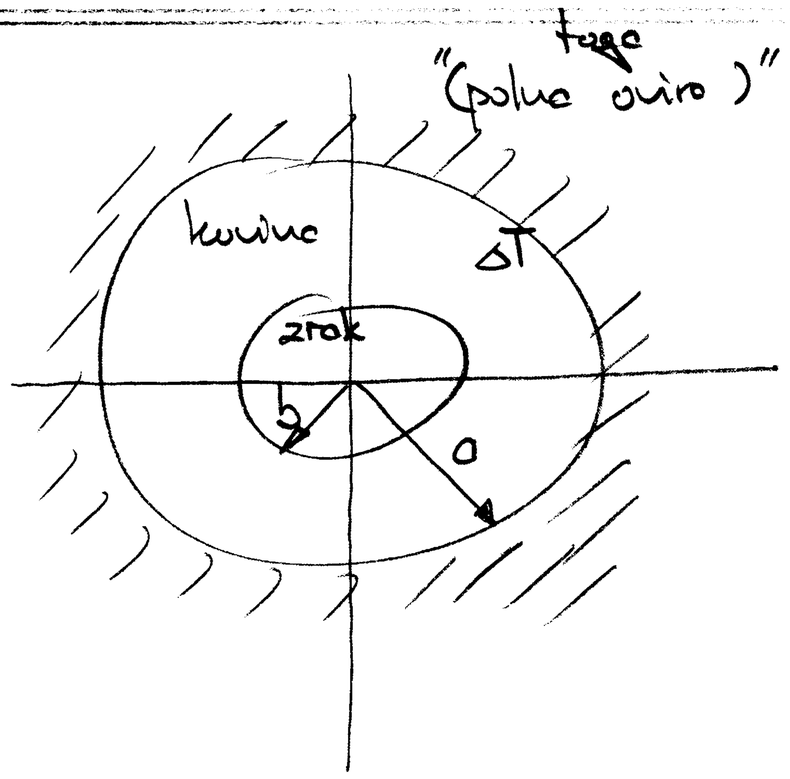
✓ $\sigma_p^2 = r^4(A_0^2 + B_0^2 - A_0 B_0) - r^2 a^2 (A_0^2 + A_0 B_0) + A_0^2 a^4$

$\dfrac{d(\sigma_p^2)}{dr} = 2\cancel{4}r^{\cancel{3}2}(A_0^2 + B_0^2 - A_0 B_0) - \cancel{2r}a^2(A_0^2 + A_0 B_0) + \emptyset = \emptyset$

$2r^2(A_0^2 + B_0^2 - A_0 B_0) = a^2(A_0^2 + A_0 B_0)$

$\dfrac{2r^2}{a^2}\left[A_0^2 + B_0^2 - A_0 B_0\right] - A_0^2 - A_0 B_0 = \emptyset$

$r_2 = \sqrt{\dfrac{a^2(A_0^2 + A_0 B_0)}{2(A_0^2 + B_0^2 - A_0 B_0)}} = \pm 0{,}2042\ m$ // ; $r_1 = \emptyset$ ✓



$a = 0{,}2\ m$

$b = 0{,}1\ m$

$E = 2 \cdot 10^5\ MPa$

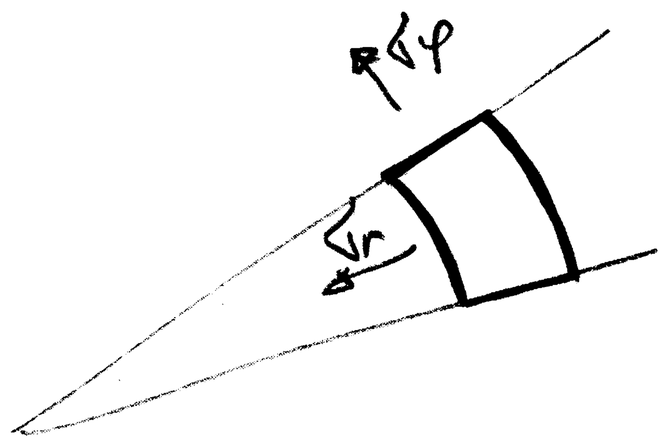
$\nu = 0{,}3$

$\alpha = 1{,}2 \cdot 10^{-5}\ K^{-1}$

$\Delta T = 50\,°C$

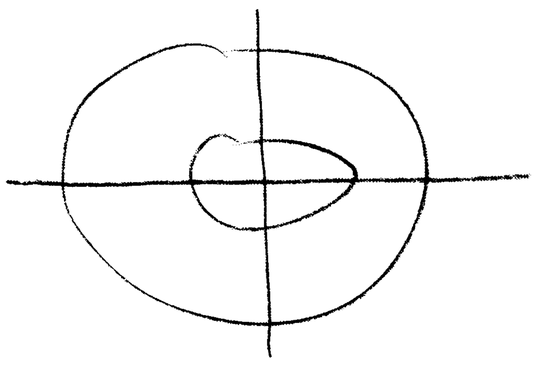
$\sigma_r(a) = ?$

$u_r(b) = ?$



$\sigma_z = \emptyset$; $\varepsilon_{zz} \neq \emptyset$ $\rightarrow$ RNS :

Robni pogoji!



$u_r(a) = \emptyset$

$\sigma_r(b) = \emptyset$; $\omega = \emptyset$

$$u_r(a) = \emptyset = \frac{a}{E}\left[(1-\nu)A - (1+\nu)B a^{-2} - \emptyset\right] + \alpha \Delta T a$$

$$\sigma_r(b) = A + B b^{-2} = \emptyset \rightarrow A = -B b^{-2}$$

$$u_r(a) = \emptyset = \frac{0{,}2}{2 \cdot 10^5}\left[(1-0{,}3)\cancel{B}\, 0{,}1^{-2} - (1+0{,}3)\cancel{B}\, 0{,}2^{-2})B\right] + 1{,}2 \cdot 10^{-5}\, 50\, 0{,}2 = 0$$

$$\frac{m\ \cancel{m^2}}{10^5\ \cancel{N}}\ \frac{1\ \cancel{N}}{\cancel{m^2}} \qquad + \frac{1}{\cancel{K}}\cancel{K}\ m$$

$$\cancel{10^{-4} - 10{,}25\, 10^{-5} B = 12 \cdot 10^{-5}}$$

$$\cancel{B = -\frac{12}{10{,}25} = 1{,}17}$$

$[N]$ original

$$-1{,}025 \cdot 10^{-10} B = 12 \cdot 10^{-5}$$

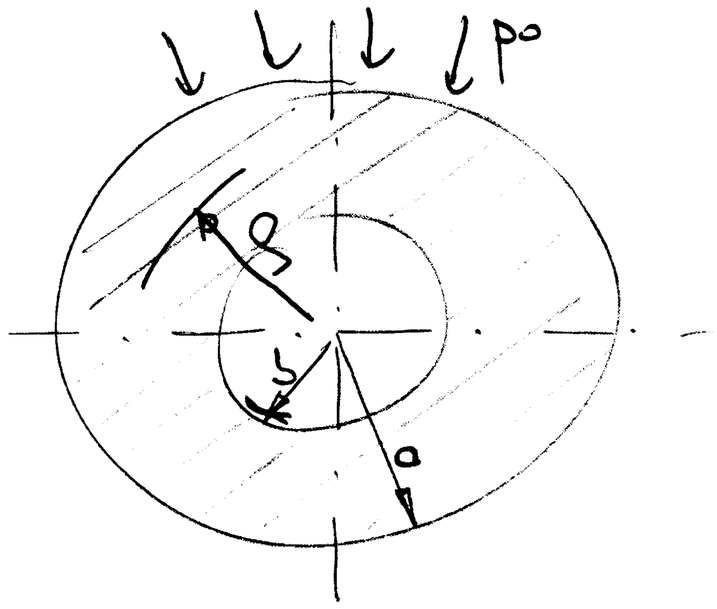
$$B = 1170731{,}7\ N$$

$$B = 1{,}17073 \cdot \cancel{10}\ MN$$

$A = -117{,}073\ MPa$ $\quad [N/m^2]$ original

$$\sigma_r(a) = A + B a^{-2} = 87{,}80\ MPa$$

$$u_r(b) = \frac{b}{E}\left[(1-\nu)A - (1+\nu)B b^{-2}\right] + \alpha \Delta T b = -0{,}057073\ mm$$



$\sigma_0 = 220$ MPa : verjetno kdaj, kje plastifikacija

$a = 100$ mm

$b = 60$ mm

p_0

$\sigma_r(a) = -p_0$

$\sigma_r(b) = \emptyset$

RNS

$$-p_0 = A + Ba^{-2}$$

$$\emptyset = A + Bb^{-2} \rightarrow A = -Bb^{-2} = -\frac{p_0 b^{-2}}{(b^{-2} - a^{-2})} = A$$

$$+p_0 = -Bb^{-2} + Ba^{-2} = +B(b^{-2} - a^{-2}) \rightarrow B = \frac{p_0}{(b^{-2} - a^{-2})}$$

$A = -1{,}5625\, p_0$ [Pa]
$-1{,}5625$

$B = 0{,}005625\, p_0$ [N]
$0{,}5625$

$$K = \frac{|A|}{|B|} b^2 = 1$$

$$A \cdot B = -0{,}00878\, p_0^2 < 0$$

$$\sigma_\varphi = \pm \sigma_0$$

$$A - Br^{-2} = \pm \sigma_0$$

$$+Br^{-2} = \pm \sigma_0 + A$$

$$\frac{1}{r^2} = \frac{\pm \sigma_0 + A}{B}$$

$$r = \sqrt{\frac{B}{\pm \sigma_0 + A}}$$

$r_1 = 93$ mm

$r_2 =$

① $\sigma_r - \sigma_\varphi = \pm \sigma_0$

$$\cancel{A} + Br^{-2} - \cancel{A} + Br^{-2} = \pm \sigma_0$$

$$r^{-2} = \frac{\pm \sigma_0}{2B}$$

$$r = \sqrt{\frac{2B}{\sigma_0}}$$

$r = 71$ mm

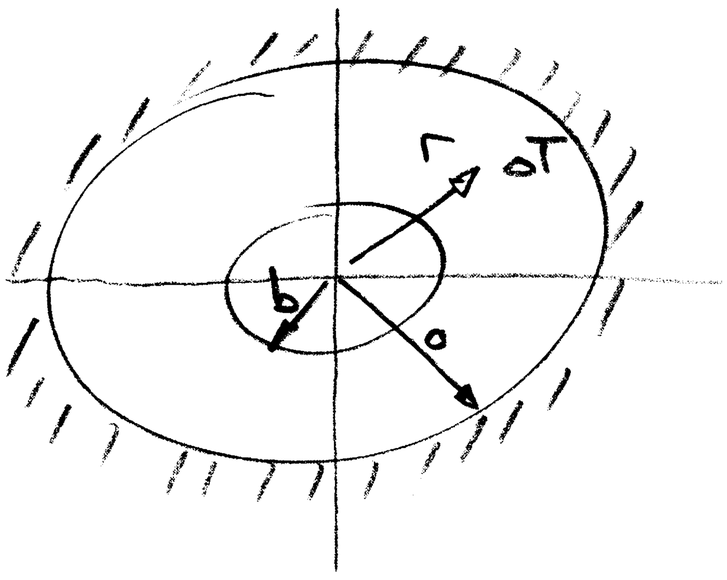
③ $\sigma_r = \pm \sigma_0$

$$A + Br^{-2} = \pm \sigma_0$$

$$r^{-2} = \frac{\pm \sigma_0 - A}{B}$$

$$r = \sqrt{\frac{B}{\sigma_0 - A}}$$

$r = 38$ mm



$a = 200\ mm$

$b = 100\ mm$

$\nu = 1/3$

$\alpha = 1{,}2 \cdot 10^{-5}\ K^{-1}$

$\sigma_0 = 240\ MPa$

$E = 200\ GPa = 200.000\ MPa$

$= 2 \cdot 10^5 \cdot 10^6\ Pa$

$= 2 \cdot 10^{11}\ Pa$

$\Delta T = ?$

Robni pogoji!

RNS:

$u_r^d(a) = \emptyset$

$\sigma_r^d(b) = \emptyset$

$$\emptyset = A + B b^{-2} \rightarrow A = -B b^{-2}$$

$$\emptyset = \frac{a}{E}\left[-(1-\nu) B b^{-2} - (1+\nu) B a^{-2}\right] + \alpha \Delta T a$$

$$-\frac{\alpha \Delta T \cancel{a} E}{\cancel{a}} = B\left[-(1-\nu) b^{-2} - (1+\nu) a^{-2}\right]$$

$$-2{,}4 \cdot 10^{-5}\ K^{-1}\ Pa\ \Delta T = -102{,}5\ B\ [m^{-2}]$$

$$B = 23414{,}6\ \frac{K^{-1} N}{\cancel{m^2}\ \cancel{m^{-2}}} \Delta T = \underline{23414{,}6\ K^{-1} N\ \Delta T} = B_1 \Delta T$$

$$A = \underline{-2{,}341460 \cdot 10^{+6}\ \Delta T\ K^{-1} N\ m^{-2}}$$

1) $\sigma_r - \sigma_\varphi = \pm \sigma_0$

$$A + B r^{-2} - (A - B r^{-2}) = \pm \sigma_0$$

$$\cancel{A} + B r^{-2} - \cancel{A} + B r^{-2} = \pm \sigma_0$$

$$2 B r^{-2} = \pm \sigma_0$$

$$2 B_1 \Delta T r^{-2} = \pm \sigma_0$$

$r = b$ ... tu se zgodi plastifikacija

predznak izberi tako da bo glede na konstante ΔT pozitiven

$$\Delta T = \frac{\sigma_0}{2 B_1 b^{-2}} \qquad \left[\frac{10^6\ \cancel{Pa}\ \cancel{N}}{K^{-1}\ \cancel{N}\ \cancel{m^{-2}}\ \cancel{m^{-2}}}\right]$$

$$\Delta T = \underline{\underline{51{,}25\ °C}}$$

$\Delta T_0 = 205\ °C$

2) $\sigma_\varphi = \pm \sigma_0$

$\Delta T A_1 - B_1 r^{-2} \Delta T = \pm \sigma_0$; $r = b$

$\Delta T = \dfrac{\pm \sigma_0}{A - B_1 b^{-2}}$ $\qquad \dfrac{10^6 \cancel{N}}{\cancel{m^2}\,[K^{-1} \cancel{N} \cancel{m^{-2}} - K^{-1} \cancel{N} \cancel{m^{-2}}]}$

$\Delta T = \underline{51{,}25\ °C}$

Če razlika σ-ev doseže σ_0 dosežemo plastično tečenje

3) $\sigma_r = \pm \sigma_0$

$\Delta T A_1 + B_1 r^{-2} \Delta T = \pm \sigma_0$

$\Delta T = \dfrac{\sigma_0}{A_1 - B_1 r^{-2}} \longrightarrow K^{-1} N m^{-2} - K^{-1} N m^{-2}$

1) $r = b \rightarrow \Delta T = \cancel{52\ °C}$ = nesmisel, kar na robu ni napetosti

1) $r = a \rightarrow \Delta T = \dfrac{-240 \cdot 10^6}{-2926825} = \underline{82\ °C}$

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1) $\sigma_r - \sigma_\varphi = \pm \sigma_0$

$\sigma_r^{pl} = \pm \sigma_0 \ln(c/r)$

$\sigma_\varphi^{pl} = \pm \sigma_0 (\ln(c/r) + 1)$

$u_r^{pl} = \pm \dfrac{(1-\nu)}{E} (\pm \sigma_0 r + C) + \dfrac{D}{r} + \alpha \Delta T r$

Robni pogoji:

$\sigma_r^{pl}(b) = \emptyset$ $\qquad \rightarrow \left(\begin{array}{l} \emptyset = \cancel{\sigma_0} \ln(c/b) = \cancel{\sigma_0} \ln c - \cancel{\sigma_0} \ln b \\ e^0 = \frac{c}{b} \rightarrow c = b e^0 = b = \\ \ln c = \ln b \rightarrow c = b \end{array} \right)$? ✓

$u_r^{pl}(0) = \emptyset$

$\rightarrow$ 3N, 2E

$\rightarrow$ Novi robni pogoji:

$\sigma_r^{el}(\rho) = \sigma_r^{pl}(\rho) \rightarrow A + B\rho^{-2} = \sigma_0 \ln c/\rho$; $\boxed{A = \sigma_0 \ln c/\rho - \frac{\sigma_0}{2}}$

$\sigma_r^{el}(\rho) - \sigma_\varphi^{el}(\rho) = \pm \sigma_0 \rightarrow \cancel{A} + B\rho^{-2} - \cancel{A} + B\rho^{-2} = \sigma_0$

$$\boxed{B = \frac{\sigma_0}{2\rho^{-2}}}$$

$u_r^{d}(a) = \emptyset \rightarrow$

$$\emptyset = \frac{a}{E}\left[(1-\nu)A - (1+\nu)B a^{-2}\right] + \alpha \Delta T a$$

$A = -286{,}35 \cdot 10^{6}$ [Pa]

$B = 48 \cdot 10^{15}$ [N]

$c = 0{,}1$ [m]

$\rho = a = 0{,}2$ [m]

$$\Delta T = \frac{-0{,}0004}{\alpha a} = \boxed{148{,}51\,°C}$$

2) $\sigma_\varphi = \pm \sigma_0$

$\sigma_r^{l}(b) = \emptyset \rightarrow \emptyset = \sigma_0 + c/b \rightarrow c = -\sigma_0 b \Rightarrow c = 24 \cdot 10^{6}$ [N]

$u_r^{d}(a) = \emptyset \rightarrow \frac{a}{E}\left[(1-\nu)A - (1+\nu)B a^{-2}\right] + \alpha \Delta T a = \emptyset$

$\sigma_r^{d}(\rho) = \sigma_r^{l}(\rho) \rightarrow A + B\rho^{-2} = \sigma_0 + c/\rho$

$\sigma_\varphi^{d}(\rho) = \pm\sigma_0 \rightarrow A - B\rho^{-2} = \sigma_0$

$\Rightarrow 2B\rho^{-2} = c/\rho$

$$B = \frac{c\,\rho^{2}}{2} = \boxed{\frac{\rho c}{2} = B}, \quad A = -\frac{\sigma_0}{B\rho^2} = \frac{\sigma_0\, 2}{\rho c \rho^2} = \frac{2\sigma_0}{c\rho^3}$$

$$\boxed{\Delta T = 83\,°C}$$

$\rho = 0$