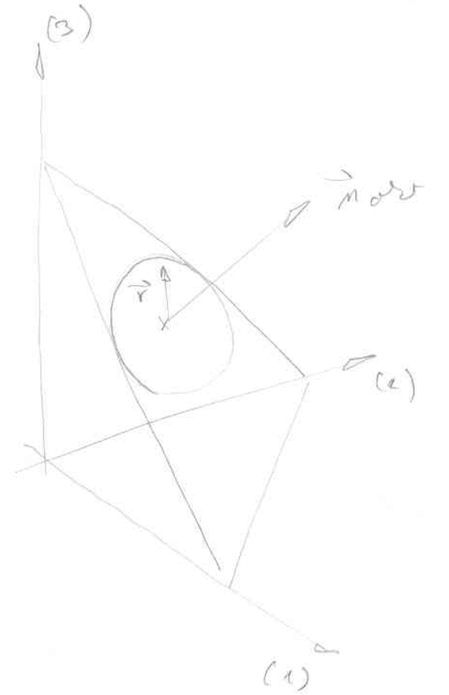
~~Dokaz enakosti tangencialnih napetosti~~
~~Airy-jeva funkcija~~
~~Von missesov pogoj tečenja~~
~~Navier lamillove enačbe~~
~~Princip virtualnih pomikov~~
~~Določitev modula plastičnosti~~
~~Napetostno deformacijske zveze v plastičnem področju~~
~~Vpliv temperaturne obremenitve na kontinuum~~
~~Ravnotežne enačbe mehanike~~
? Dokaz enakosti tangencialnih napetosti
~~Vpliv temperaturne obremenitve na elastično telo~~
~~Rotacijsko simetrično delovanje napetostnega stanja –airy-jeva funkcija~~ ? ?
~~Določitev modula plastičnosti~~
~~Napetostno deformacijske zveze v plastičnem področju~~
Določitev modula elastičnosti
Maxwellovo telo
Zapis napetosti v oktaedrični ravnini
Hookov zakon

KOMPRESIJSKI MODUL 9 5.

KOMPAT ISTNA ENAČBA

VON MISESOV POGOJ TEČENJA



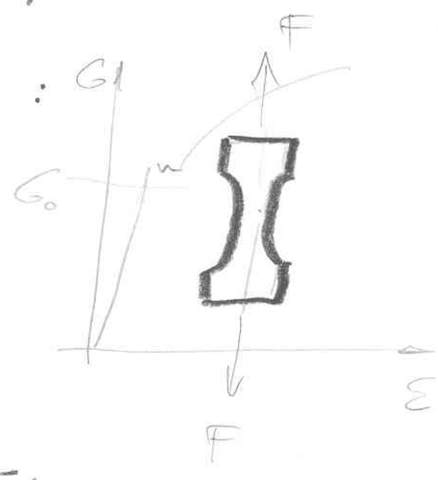
$$\vec{n}_{okt} \times \vec{r} = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 1 & 1 \\ \sigma_1 & \sigma_2 & \sigma_3 \end{vmatrix} \frac{\sqrt{3}}{3}$$

$$= \frac{\sqrt{3}}{3}\left[e_1(\sigma_3-\sigma_2) - e_2(\sigma_3-\sigma_1) + e_3(\sigma_2-\sigma_1)\right]$$

$$|\vec{n}_{okt} \times \vec{r}|^2 = r^2 \sin^2 90 = r^2$$

$$\frac{1}{3}\left[(\sigma_3-\sigma_2)^2 + (\sigma_3-\sigma_1)^2 + (\sigma_2-\sigma_1)^2\right] = r^2$$

Za enosni obremenitveni primer:



$$\sigma_0 = \sigma_{0_2} = \sigma_{PL} = \sigma_y$$

$$\sigma_1 = \sigma_0 \qquad \sigma_2 = \sigma_3 = 0$$

$$\frac{1}{3}\left[0^2 + \sigma_0^2 + \sigma_0^2\right] = r^2$$

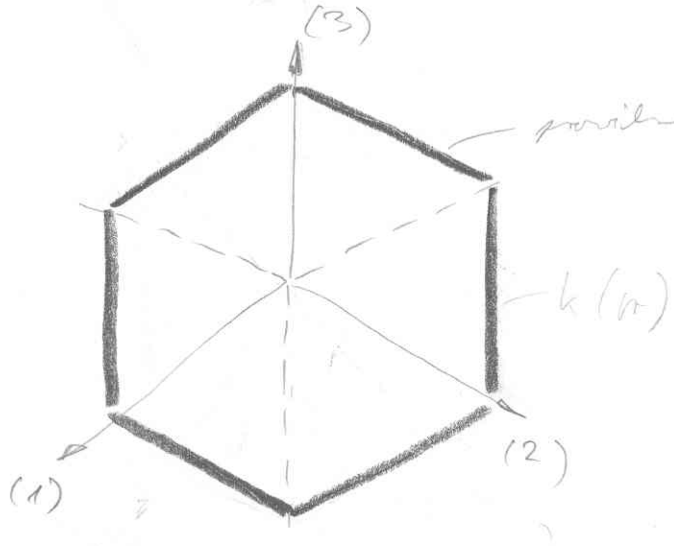
$$\frac{2}{3}\sigma_0^2 = r^2 \quad \Rightarrow \quad \boxed{2\sigma_0^2 = 3r^2}$$

$$\alpha(\sigma_{ij}) = (\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 = 2\sigma_0$$

$$\boxed{\alpha(\sigma_{ij}) \begin{matrix} < \\ = \\ > \end{matrix} 2\sigma_0^2}$$

- elastično območje
- na meji plastičnost
- v elastoplastičnem območju

TRESCA-jev pogoj tečenja



Enačba šestkotnika

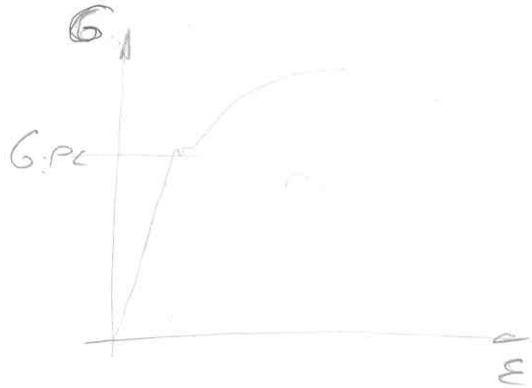
$$\left[(\sigma_1-\sigma_2)^2-k^2\right]\left[(\sigma_2-\sigma_3)^2-k^2\right]\left[(\sigma_3-\sigma_1)^2-k^2\right]=\phi$$

$$(\sigma_1-\sigma_2)^2-k=0$$

$$(\sigma_2-\sigma_3)^2-k=0$$

$$(\sigma_3-\sigma_1)^2-k=0$$

Za enoosno napetostno stanje



$$\sigma_1=\sigma_0$$

$$\sigma_2=\sigma_3=0$$

$$(\sigma_1-\sigma_2)^2-\sigma_0^2=0$$

$$(\sigma_1-\sigma_2)=\pm\sigma_0$$

$$(\sigma_2-\sigma_3)=\pm\sigma_0$$

$$(\sigma_3-\sigma_1)=\pm\sigma_0$$

$$\sigma_1-\sigma_2=\pm 2\tau_3$$

$$\tau_1=\pm\frac{\sigma_1-\sigma_2}{2}$$

$$\tau_2=\pm\frac{\sigma_2-\sigma_3}{2}$$

$$\tau_3=\pm\frac{\sigma_3-\sigma_1}{2}$$

AIRY-jeva funkcija

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}$$

$$\tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} - xY - yX$$

$$\sigma_y = \frac{\partial^2 F}{\partial x^2}$$

$$\frac{\partial^2 \tau_{xy}}{\mu\, \partial x \partial y} = \frac{1}{E}\left[\frac{\partial^2 \sigma_x}{\partial y^2} - \nu \frac{\partial^2 \sigma_y}{\partial y^2} + \frac{\partial^2 \sigma_y}{\partial x^2} - \nu \frac{\partial^2 \sigma_x}{\partial x^2}\right] + \alpha\left[\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial x^2}\right]$$

$$\mu = \frac{E}{2(1+\nu)}$$

$$-\frac{2(1+\nu)}{E}\frac{\partial^4 F}{\partial x^2 \partial y^2} = \frac{1}{E}\left[\frac{\partial^4 F}{\partial y^4} - \nu \cancel{\frac{\partial^4 F}{\partial x^2 \partial y^2}} + \frac{\partial^4 F}{\partial x^4} - \nu \cancel{\frac{\partial^4 F}{\partial x^2 \partial y^2}}\right] + \alpha \Delta T = 0$$

$$\underbrace{\left(\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4}\right)}_{\Delta\Delta F} + \alpha E \Delta T = 0$$

$$\boxed{\Delta\Delta F + \alpha E \Delta T = 0}$$

$$\Delta\left[\Delta F + \alpha E T\right] = 0$$

za termostatični primer $\Delta T = 0 \Rightarrow \Delta\Delta F = 0$

Navier [illegible] [illegible]

$$\sigma_{ij,j} + x_i = \phi$$

$$\sigma_{ij,j}\left[\varepsilon_{ij}(u_i)\right] + x_i = 0$$

$$\sigma_{ij} = 2\mu\,\varepsilon_{ij} + \lambda\,\varepsilon_{kk}\,\delta_{ij} - (2\mu + 3\lambda)\alpha T\,\delta_{ij}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$$

$$\sigma_{ij,j} = 2\mu\,\varepsilon_{ij,j} + \lambda\,\overbrace{\varepsilon_{kk,j}\,\delta_{ij}}^{\varepsilon_{v,i}} - (2\mu + 3\lambda)\,\alpha\,\overbrace{T_{,j}\,\delta_{ij}}^{T_{,i}}$$

$$\varepsilon_{ij,j} = \frac{1}{2}\left(u_{i,jj} + u_{j,ij}\right)$$

$$u_{k,k} = \varepsilon_v$$

$$u_{i,jj} = u_{i,kk} = \Delta u_i \qquad u_{i,ij} = u_{j,ij} = \frac{\partial}{\partial x_i}\left(u_{k,k}\right)$$

$$\varepsilon_v = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = u_{k,k} = \varepsilon_{kk}$$

$$u_{j,ij} = \varepsilon_{kk,i} = \varepsilon_{v,i}$$

$$\varepsilon_{ij,j} = \frac{1}{2}\left(\Delta u_i + \varepsilon_{v,i}\right)$$

$$\mu(\Delta u_i + \varepsilon_{v,i}) + \lambda\,(\varepsilon_{v,i} - (2\mu + 3\lambda)\alpha\,T_{,i} + X_i = 0$$

$$\boxed{\mu\left(\Delta u_i + \frac{\mu + \lambda}{\mu}\,\varepsilon_{v,i}\right) - (2\mu + 3\lambda)\,\alpha\,T_{,i} + X_i = 0}$$

$$\begin{array}{l|l}
x_i & \mu\left(\Delta u + \dfrac{\mu+\lambda}{\mu}\dfrac{\partial \varepsilon_v}{\partial x}\right) - (2\mu + 3\lambda)\,\alpha\,\dfrac{\partial T}{\partial x} + X = 0 \\
y_i & \mu\left(\Delta v + \dfrac{\mu+\lambda}{\mu}\dfrac{\partial \varepsilon_v}{\partial y}\right) - (2\mu + 3\lambda)\,\alpha\,\dfrac{\partial T}{\partial y} + Y = 0 \\
z_i & \mu\left(\Delta w + \dfrac{\mu+\lambda}{\mu}\dfrac{\partial \varepsilon_v}{\partial z}\right) - (2\mu + 3\lambda)\,\alpha\,\dfrac{\partial T}{\partial z} + Z = 0
\end{array}$$

Določitev modula plastičnosti

$$d\varepsilon_{ij}^{pl} = d\lambda\, s_{ij}$$

$$d\varepsilon_{ij}^{pl}\cdot d\varepsilon_{ij}^{pl} = d\lambda^2\, s_{ij}\cdot s_{ij}$$

deviator deformacij

$$e_{ij} = \varepsilon_{ij} + \frac{\varepsilon_{kk}}{3}\delta_{ij}$$

$$e_{ij}^{pl} = \varepsilon_{ij}^{pl} + \frac{\varepsilon_v^{pl}}{3}\delta_{ij} \qquad \varepsilon_v^{pl} = 0$$

$$e_{ij}^{pl} = 0 \Rightarrow e_{ij}^{pl} = \varepsilon_{ij}^{pl}$$

$$\Rightarrow de_{ij}^{pl} = d\varepsilon_{ij}^{pl}$$

$$de_{ij}^{pl}\, de_{ij}^{pl} = d\lambda^2\, s_{ij}\cdot s_{ij}$$

$$s_{ij}s_{ij} = 3\tau_{okt}^2$$

$$e_{ij}e_{ij} = 3\gamma_{okt}^2$$

$$3(d\gamma_{okt}^{pl})^2 = d\lambda^2\, 3\tau_{okt}^2$$

$$d\lambda = \frac{d\gamma_{okt}^{pl}}{\tau_{okt}^2}$$

$$\tau_{okt}^2 = \frac{1}{9}\underbrace{\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]}_{D(\sigma_{ij})}$$

$\sigma_1 \neq 0$ $\qquad$ $\sigma_2 = \sigma_3 = 0$ – enoosno napetostno stanje

$$D\sigma_{ij} = \frac{1}{9}\left[\sigma_1^2 + \sigma_1^2\right] = \frac{2}{9}\sigma_1^2$$

$$\tau_{okt}[\sigma_{ij}] = \frac{\sqrt{2}}{3}\sigma_1$$

Napetostno deformacijske veze u plastičnom području

$$\sigma_{ij} = -p\,\delta_{ij} + 2\mu\,\dot{\varepsilon}_{ij} - \frac{2}{3}\mu\,\dot{\varepsilon}_v\,\delta_{ij}$$

$$\sigma_{ij} = s_{ij} + \frac{\sigma_{kk}}{3}\delta_{ij}$$

$$-p = \frac{\sigma_{kk}}{3}$$

$$\sigma_{ij} = s_{ij} + (-p\,\delta_{ij}) = -p\,\delta_{ij} + 2\mu\,\dot{\varepsilon}_{ij}^{PL} - \frac{2}{3}\cancel{\mu}\,\dot{\varepsilon}_v^{PL}\,\delta_{ij}$$

$$s_{ij} = 2\mu\,\dot{\varepsilon}_{ij}^{PL} = 2\mu\,\frac{d\varepsilon_{ij}^{PL}}{dt}$$

$$d\varepsilon_{ij}^{PL} = \frac{dt}{2\mu}\,s_{ij}$$ – Saint-Venant-ova značka

$d\lambda$ – modul plastičnosti

$$\begin{pmatrix} d\varepsilon_{11}^{PL}, & d\varepsilon_{12}^{PL} = \frac{1}{2}d\gamma_{12}^{PL}, & d\varepsilon_{13}^{PL} = \frac{1}{2}d\gamma_{13}^{PL} \\ \frac{1}{2}d\gamma_{21}^{PL}, & d\varepsilon_{22}^{PL}, & d\varepsilon_{23}^{PL} = \frac{1}{2}d\gamma_{23}^{PL} \\ \frac{1}{2}d\gamma_{31}^{PL} & \frac{1}{2}d\ d\gamma_{32}^{PL} & , d\varepsilon_{33}^{PL} \end{pmatrix} =$$

$$= d\lambda \begin{pmatrix} \frac{2\sigma_{11}-\sigma_{22}-\sigma_{33}}{3} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \frac{2\sigma_{22}-\sigma_{11}-\sigma_{33}}{3} & \sigma_{32} \\ \sigma_{31} & \sigma_{32} & \frac{2\sigma_{33}-\sigma_{11}-\sigma_{22}}{3} \end{pmatrix}$$

$$d\varepsilon_{11}^{PL} = \frac{2}{3}d\lambda\left[\sigma_{11} - \frac{1}{2}(\sigma_{22}-\sigma_{33})\right]$$

$$\frac{1}{2}d\gamma_{12}^{PL} = d\lambda\,\sigma_{12} \qquad d\gamma_{12}^{PL} = 2\,d\lambda\,\sigma_{12}$$

$$\varepsilon_{11} = \frac{1}{E}\left[\sigma_{11} - \nu(\sigma_{22}+\sigma_{33})\right] \qquad E_{PL} = \frac{3}{2\,d\lambda} \qquad \nu^{PL} = \frac{1}{2}$$

$$\frac{1}{E_{PL}} = \frac{2\,d\lambda}{3}$$

Princip virtualnih pomikov

Def : Virtualno delo ravnotežnega deformacijskega sistema je enako nič.

$$W^z = W^u \quad / \delta$$

$$\delta W^z = \delta W^u$$

δu_i – virtualni pomik i-te točke

$$\sigma_{ij,j} + \rho g_i = \rho a_i$$

$$\sigma_{ij,j} + \rho (g_i - a_i) = 0$$ – ravnotežna enačba za vsako notranjo točko

$$\sigma_{ij,j} \delta u_i + \rho (g_i - a_i) \delta u_i = 0$$

$$\boxed{\int_V \left[\sigma_{ij,j} \delta u_i + \rho (g_i - a_i) \delta u_i \right] dV = 0}$$ ravnotežna enačba

$$\sigma_{ij,j} \delta u_i = (\sigma_{ij} \delta u_i)_{,j} - \sigma_{ij} \delta u_{i,j}$$

$$\delta u_{i,j} = \delta \overline{\varepsilon_{ij}} = \delta \varepsilon_{ij} + \delta \omega_{ij}$$ – translacija – rotacija

$$\sigma_{ij} \delta u_{i,j} = \sigma_{ij} (\delta \varepsilon_{ij} + \delta \omega_{ij})$$

$$= \sigma_{ij} \delta \varepsilon_{ij} + \underbrace{\sigma_{ij} \delta \omega_{ij}}_{0}$$

$$\boxed{\sigma_{ij} \delta u_{i,j} = \sigma_{ij} \delta \varepsilon_{ij}}$$

$$\sigma_{ij,j} = (\sigma_{ij} \delta u_i)_{,j} - \sigma_{ij} \delta \varepsilon_{ij}$$

Vpliv temperature obremenitve na kontinuum

ENAČBA ZA VEZANO TERMODINAMSKO STANJE

$$\varkappa \Delta T + \frac{Q}{\rho c_\varepsilon} = \frac{d}{dt}\left(T + \frac{m}{\rho c_\varepsilon} T_0 \varepsilon_v\right)$$

α - razteznostni koeficient

T - razlika temperature $\quad T=(T_2-T_1)$

$T>0$ segrevanje

$T<0$ ohlajanje

$\varkappa$ - koeficient konduktivnosti

Q - notranja generacija toplote

ρ - gostota

c_ε - specifična toplota na enoto volumna pri konstantni specifični deformaciji

T_0 - temperatura pri kateri ni notranjih deformacij

m - masa

ε_v - plastifikacija materiala

UTICAJ TEMPARATURE OPTEREĆENJE NA ELASTIČNO TELO

H

$$\varepsilon_{ij} = \frac{1}{2\mu}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij} = \varepsilon_{ij}^{el}$$

$$\varepsilon_{ij}^{T} = \alpha T \delta_{ij} \Rightarrow \varepsilon_{ij}^{tot} = \varepsilon_{ij}^{el} + \varepsilon_{ij}^{T} = \varepsilon_{ij}$$

$$\varepsilon_{ij} = \frac{1}{2}\mu\,\sigma_{ij} - \left(\frac{\nu}{E}\sigma_{kk} - \alpha T\right)\delta_{ij}$$

$$\sigma_{ij} = 2\mu\,\varepsilon_{ij} + 2\mu\left(\frac{\nu}{E}\sigma_{kk} - \alpha T\right)\delta_{ij}$$

$$\underbrace{\sigma_{ij}\delta_{ij}}_{\sigma_{kk}} = 2\mu\,\underbrace{\varepsilon_{ij}\delta_{ij}}_{\varepsilon_{kk}} + 2\mu\left(\frac{\nu}{E}\sigma_{kk} - \alpha T\right)\underbrace{\delta_{ij}\delta_{ij}}_{3}$$

$$\sigma_{kk}\left(1 - 2\mu\frac{\nu 3}{E}\right) = 2\mu\,\varepsilon_{kk} - 6\mu\alpha T$$

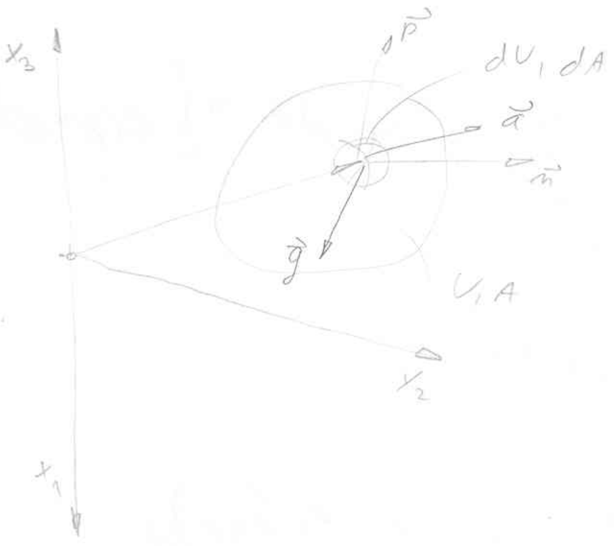
$$1 - \frac{2E}{2(1+\nu)}\frac{3\nu}{E} = \frac{1+\nu-3\nu}{(1+\nu)} = \frac{1-2\nu}{1+\nu}$$

$$\sigma_{kk}\frac{1-2\nu}{(1+\nu)} = \frac{2E}{2(1+\nu)}\varepsilon_{kk} - \frac{3\cancel{6}E}{\cancel{2}(1+\nu)}\alpha T$$

$$\boxed{\sigma_{kk} = \frac{E}{1-2\nu}\varepsilon_{kk} - \frac{3E}{1-2\nu}\alpha T}$$

$$2\mu\frac{\nu}{E}\sigma_{kk} = \underbrace{\frac{2E}{2(1+\nu)}\frac{E}{(1-2\nu)}}_{\lambda}\varepsilon_{kk} - 3\lambda\alpha T$$

$$\boxed{\begin{gathered}\sigma_{ij} = 2\mu\,\varepsilon_{ij} + \lambda\,\varepsilon_{kk}\delta_{ij} - (2\mu + 3\lambda)\alpha T\delta_{ij} \\ \sigma_{kk} = \frac{E}{1+2\nu}\varepsilon_{kk} - \frac{3E}{1-2\nu}\alpha T\end{gathered}}$$



$$\vec{n} = \frac{\vec{\nabla} \varphi}{|\vec{\nabla} \varphi|}$$

$$\vec{\varphi}(x_1, x_2, x_3) = 0 \qquad \vec{r} = (x, y, z)$$

$$p_i = \sigma_{ij} n_j$$

RAVNOTEŽJE SIL

1.) $\sum \vec{F_i} = \vec{\phi}$

2.) $\sum \vec{M_i} = \vec{\phi}$

2.) $\sum \vec{r_i} \times \vec{F_i} = \phi$

DVOJICA SIL

$M = a \cdot F$

VSOTA VSEH SIL $= \phi$

$$\int_A p_i \, dA = \int_A \sigma_{ij} n_j \, dA \qquad p_i = \sigma_{ij} n_j \quad = \int_V \sigma_{ij,j} \, dV$$

$$\int_V \left[\sigma_{ij,j} + \rho(g_i - a_i)\right] dV = \phi$$ RAVNOTEŽNA ENAČBA SIL NA CELOTNEM TELESU

Kompresijski modul

$$\sigma_{ij}^{k} = \frac{\sigma_{kk}}{3}\delta_{ij} = -p\,\delta_{ij}$$

$$\sigma_1 = \sigma_2 = \sigma_3 = -p$$

$$\tau = 0$$

$$\varepsilon_{kk} = \frac{1-2\nu}{E}\sigma_{kk} = -p\,\frac{1}{\dfrac{E}{3(1-2\nu)}}$$

KOMPRESIJSKI MODUL

$$\varepsilon_{kk} = \varepsilon_v = \frac{-p}{K}$$

$K \Rightarrow \infty \Rightarrow \varepsilon_v \to \phi$ – to pomeni daje telo idealno togo

$$\lim_{\nu \to \frac{1}{2}} \frac{E}{3(1-2\nu)} = \lim K \to \infty$$

$$\lim \varepsilon_v \to 0$$

$\nu = \frac{1}{2}$ $\qquad \varepsilon_v^{PL} \approx 0$

$$\varepsilon_v^{CEL} = \varepsilon_v^{EL} + \varepsilon_v^{PL}$$