DOMACA NALOGA - LABORATORIJSKE VAJE

NALOGA 1

Dani sta kompleksni stevili $z_1 = -5 + 2 * 3\,i$ in $z_2 = 3 * 8 - 5\,i$.

$$z_1 = -5 + 2 * 3\,i$$

$$-5 + 6\,i$$

$$z_2 = 3 * 8 - 5\,i$$

$$24 - 5\,i$$

Izracunajte kompleksno stevilo $w = z_1 * z_2 + 1 / z_1 - 1 / z_2$:

$$w = z_1 * z_2 + 1 / z_1 - 1 / z_2$$

$$-\frac{3\,303\,959}{36\,661} + \frac{6\,191\,798\,i}{36\,661}$$

Izracunajte absolutno vrednost in argument stevila w.

$$\text{Abs}[w]$$

$$\sqrt{\frac{1\,343\,512\,385}{36\,661}}$$

$$\text{Arg}[w]$$

$$\pi - \text{ArcTan}\left[\frac{6\,191\,798}{3\,303\,959}\right]$$

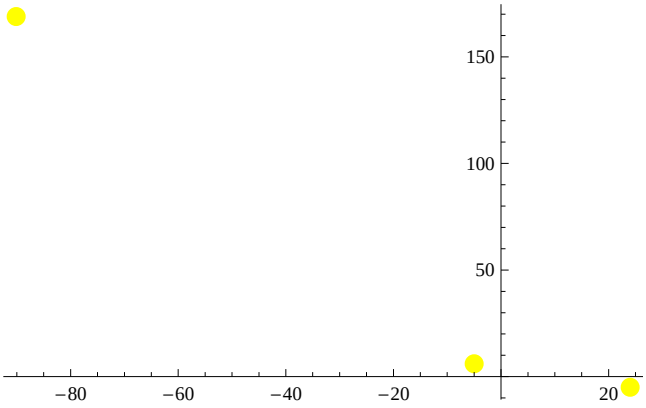
Izracunajte razdaljo med z_1 in z_2.

$$\text{Abs}[z_1 - z_2]$$

$$\sqrt{962}$$

Narisite vsa tri kompleksna stevila v kompleksni ravnini.

```
ListPlot[{{Re[z1], Im[z1]}, {Re[z2], Im[z2]}, {Re[w], Im[w]}},
 PlotStyle → {PointSize[0.03], RGBColor[1, 1, 0]}]
```



NALOGA 2

Izracunajte limito funkcije f (x) = $\left(\frac{2+x}{3+x}\right)^{13\,x}$, ko gre x proti 0 in neskoncno.

$$\left(\frac{2+x}{3+x}\right)^{13\,x} = f1$$

```
Limit[((2 + x) / (3 + x))^(13 x), x -> 0]
```

1

```
Limit[((2 + x) / (3 + x))^(13 x), x → Infinity]
```

$$\frac{1}{e^{13}}$$

Izracunajte levo in desno limito funkcije f (x) = (13/(x^2 - 1)), ko gre x proti - 1 in 1.

```
f2 = 13 / (x^2 - 1)
```

$$\frac{13}{-1+x^2}$$

```
Limit[13/(-1 + x^2), x → -1, Direction → 1]
```

∞

```
Limit[13/(-1 + x^2), x → -1, Direction → -1]
```

−∞

```
Limit[13/(-1 + x^2), x → 1, Direction → 1]
```

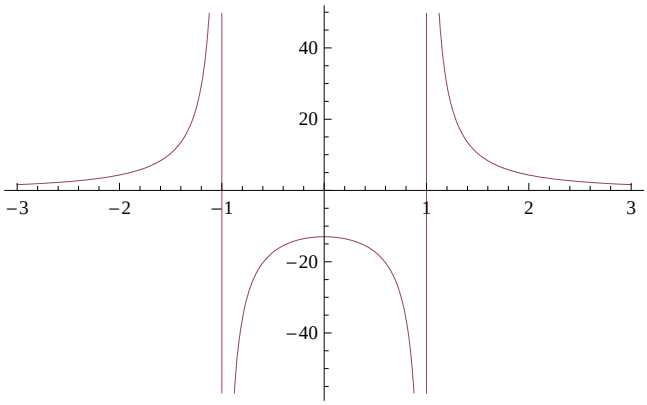
−∞

```
Limit[13/(-1 + x^2), x → 1, Direction → -1]
```

∞

Narisite grafa obeh funkcij.

```
Plot[{f1, f2}, {x, -3, 3}]
```



NALOGA 3

```
ClearAll[f]
```

Definirajte funkciji f (x) = -x^2 + 13, g (x) = 5/(x^2 + 1).

```
f = -x^2 + 13
```

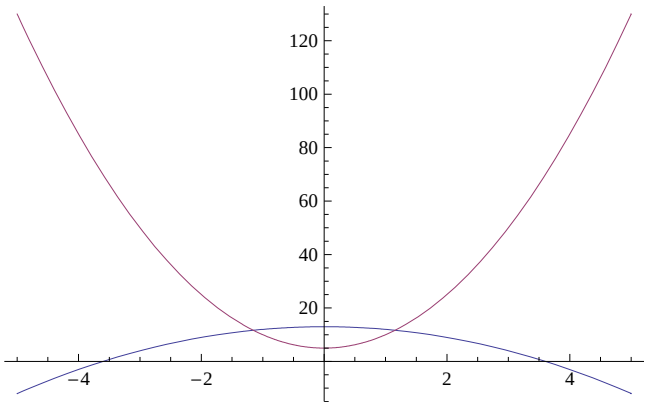
13 - x^2

```
g = 5 (x^2 + 1)
```

$5\left(1 + x^2\right)$

Narisite grafa obeh funkcij in izracunajte njuna presecisca.

```
Plot[{f, g}, {x, -5, 5}]
```




```
res = x /. NSolve[13 - x^2 == 5 (1 + x^2), x]
```

{-1.1547, 1.1547}

Izracunajte ploscino lika, ki ga funkciji oklepata.

```
NIntegrate[Abs[13 - x^2 / 5 (1 + x^2)], {x, 0, 3}]
```

```
28.6651
```

Izracunajte volumen in povrsino vrtenine, ki jo dobite pri vrtenju pozitivnega dela funkcije f okrog osi x.

```
N[Pi Integrate[13 - x^2, {x, 0, 2}]]
```

```
73.3038
```

```
N[2 Pi Integrate[(13 - x^2) Sqrt[1 + (D[13 - x^2, x])^2], {x, 0, 2}]]
```

```
326.33
```

NALOGA 4

Definirajte racionalno funkcijo f (x) = (8 x^3 + x^2 + x - 1)/(x^2 - 5).

```
ClearAll[f]
```

```
f = (8 x^3 + x^2 + x - 1) / (x^2 - 5)
```

$$\frac{-1 + x + x^2 + 8\,x^3}{-5 + x^2}$$

```
P[x_] = 8 x^3 + x^2 + x - 1
```

$$-1 + x + x^2 + 8\,x^3$$

```
Q[x_] = x^2 - 5
```

$$-5 + x^2$$

Izracunajte nicle, pole in narisite graf funkcije.

```
NSolve[Numerator[f[x]] == 0, x]
```

```
{{x → -0.255958 - 0.507497 i}, {x → -0.255958 + 0.507497 i}, {x → 0.386915}}
```

```
NSolve[Denominator[f[x]] == 0, x]
```

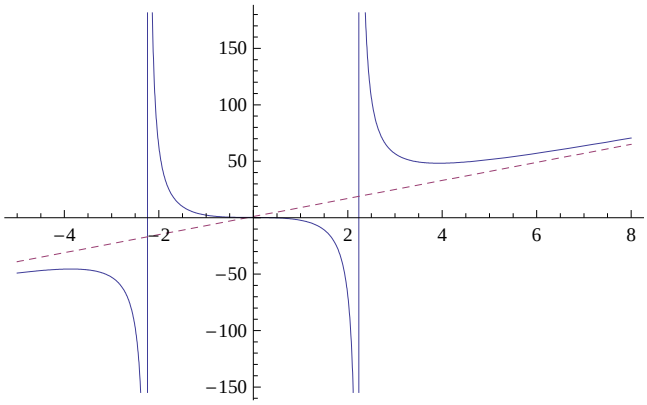
```
{{x → -2.23607}, {x → 2.23607}}
```

Izracunajte asimptoto.

```
a = PolynomialQuotient[Numerator[f], Denominator[f], x]
```

```
1 + 8 x
```

```
Plot[{f, a}, {x, -5, 8}, PlotStyle → {Dashing[1], Dashing[0.01]}]
```



Izracunajte vse lokalne ekstreme in dolocite njihovo naravo (minimum, maksimum).

```
Df = Simplify[D[f, x]]
```

$$\frac{-5 - 8\,x - 121\,x^2 + 8\,x^4}{\left(-5 + x^2\right)^2}$$

```
Df2 = Simplify[D[f, {x, 2}]]
```

$$\frac{2\left(20 + 615\,x + 12\,x^2 + 41\,x^3\right)}{\left(-5 + x^2\right)^3}$$

```
m = NSolve[Df == 0, x]
```

```
{{x → 3.92685}, {x → -3.86108}, {x → -0.032888 + 0.20035 i}, {x → -0.032888 - 0.20035 i}}
```

Ekstremi :

```
b = {x, f} /. m[[1]]
```

```
{3.92685, 48.2496}
```

```
c = {x, f} /. m[[2]]
```

```
{-3.86108, -45.4624}
```

Minimum :

```
Df2 /. m[[1]]
```

```
9.01998
```

Maksimum :

```
Df2 /. m[[2]]
```

```
-9.32656
```

Dolocite intervale narascanja in padanja.

```
Reduce[D[f, x] > 0, x]
```

```
x < Root[-5 - 8 #1 - 121 #1^2 + 8 #1^4 &, 1] || x > Root[-5 - 8 #1 - 121 #1^2 + 8 #1^4 &, 2]
```

```
Reduce[D[f, x] < 0, x]
```

$$\text{Root}\left[-5-8\,\#1-121\,\#1^2+8\,\#1^4\,\&,\,1\right] < x < -\sqrt{5} \;||\; -\sqrt{5} < x < \sqrt{5} \;||\; \sqrt{5} < x < \text{Root}\left[-5-8\,\#1-121\,\#1^2+8\,\#1^4\,\&,\,2\right]$$

Dolocite intervale konveksnosti in konkavnosti.

NALOGA 5

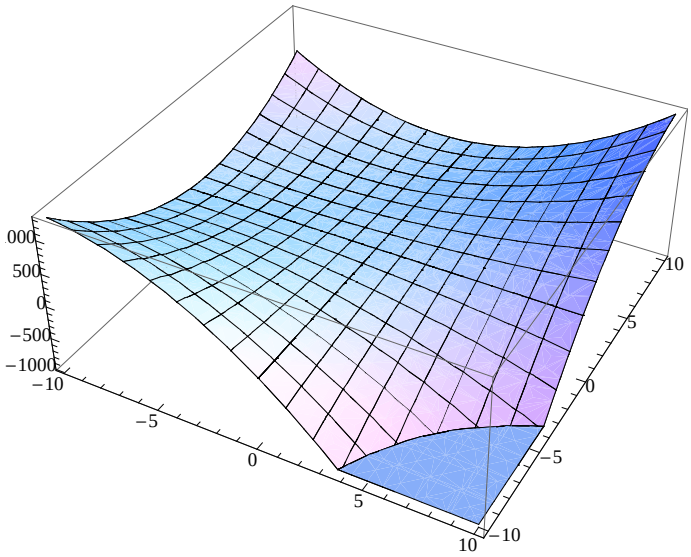
Definirajte funkcijo dveh spremenljivk f (x, y) = 13 xy + x^2 y - y^2 x.

```
Clear[f]
```

```
f[x_, y_] := 13 x * y + x^2 y - y^2 x
```

Nari itegraf funkcije.

```
Plot3D[f[x, y], {x, -10, 10}, {y, -10, 10}]
```



Izracunajte parcialne odvode prvega in drugega reda.

```
fx = D[f[x, y], {x, 1}]
```

$$13\,y + 2\,x\,y - y^2$$

```
fy = D[f[x, y], {y, 1}]
```

$$13\,x + x^2 - 2\,x\,y$$

```
fxx = D[f[x, y], {x, 2}]
```

$$2\,y$$

```
fyy = D[f[x, y], {y, 2}]
```

```
-2 x
```

Izracuanjte vse stacionarne tocke.

```
res = Solve[{fx == 0, fy == 0}, {x, y}]
```

$$\left\{\{x \to -13,\ y \to 0\},\ \left\{x \to -\frac{13}{3},\ y \to \frac{13}{3}\right\},\ \{x \to 0,\ y \to 0\},\ \{x \to 0,\ y \to 13\}\right\}$$

```
t = {x, y} /. Solve[{fx == 0, fy == 0}, {x, y}]
```

$$\left\{\{-13,\ 0\},\ \left\{-\frac{13}{3},\ \frac{13}{3}\right\},\ \{0,\ 0\},\ \{0,\ 13\}\right\}$$

Dolocite vse lokalne minimume in maksimume.

```
Hess = D[f[x, y], {{x, y}, 2}]
```

$$\{\{2\,y,\ 13 + 2\,x - 2\,y\},\ \{13 + 2\,x - 2\,y,\ -2\,x\}\}$$

```
Det[Hess] /. res
```

$$\left\{-169,\ \frac{169}{3},\ -169,\ -169\right\}$$