DOMACA NALOGA - LABORATORIJSKE VAJE

NALOGA 1

- **Dani sta kompleksni stevili $z_1 = -5 + 2 * 3 i$ in $z_2 = 3 * 8 - 5 i$.**

Kompleksno stevilo je definirano kot : z = a + *i* b,a predstavlja realno, b pa imaginarno komponento.

```
z1 = -5 + 2 * 3 i
```

$-5 + 6 i$

```
z2 = 3 * 8 - 5 i
```

$24 - 5 i$

- **a) Izracunajte kompleksno stevilo $w = z_1 * z_2 + 1 / z_1 - 1 / z_2$:**

```
N[w = z1 * z2 + 1 / z1 - 1 / z2]
```

$-90.1219 + 168.893 i$

- **b) Izracunajte absolutno vrednost in argument stevila w.**

Absolutno vrednost kompleksnega stevila izracunamo kot | z | = Sqrt(a^2 + b^2)

```
Abs[w]
```

$$\sqrt{\frac{1\,343\,512\,385}{36\,661}}$$

```
Arg[w]
```

$$\pi - \text{ArcTan}\left[\frac{6\,191\,798}{3\,303\,959}\right]$$

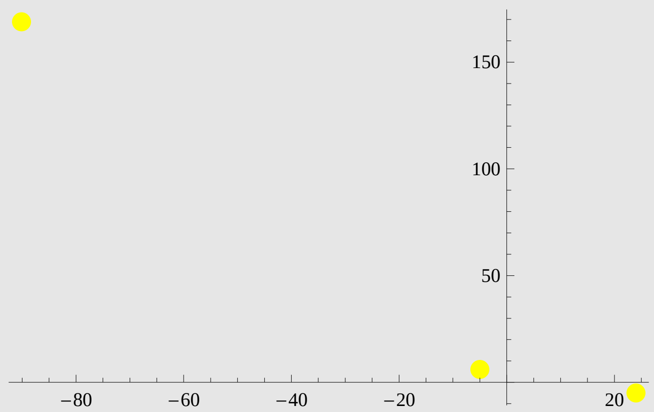
- c) Izracunajte razdaljo med z_1 in z_2.

```
Abs[z1 - z2]
```

$\sqrt{962}$

- d) Narisite vsa tri kompleksna stevila v kompleksni ravnini.

```
ListPlot[{{Re[z1], Im[z1]}, {Re[z2], Im[z2]}, {Re[w], Im[w]}},
 PlotStyle → {PointSize[0.03], RGBColor[1, 1, 0]}]
```



NALOGA 2

- a) Izracunajte limito funkcije f (x) = $\left(\frac{2+x}{3+x}\right)^{13x}$, ko gre x proti 0 in neskoncno.

$$\left(\frac{2+x}{3+x}\right)^{13x} = f1$$

```
Limit[((2 + x) / (3 + x)) ^ (13 x), x -> 0]
```

1

```
Limit[((2 + x) / (3 + x)) ^ (13 x), x → Infinity]
```

$\frac{1}{e^{13}}$

b) Izracunajte levo in desno limito funkcije f (x) = (13/(x^2 - 1)), ko gre x proti - 1 in 1.

```
f2 = 13 / (x^2 - 1)
```

$$\frac{13}{-1+x^2}$$

Leva in desna limita, ko gre x proti - 1 :

$$\text{Limit}\left[\frac{13}{-1+x^2},\ x \to -1,\ \text{Direction} \to 1\right]$$

∞

$$\text{Limit}\left[\frac{13}{-1+x^2},\ x \to -1,\ \text{Direction} \to -1\right]$$

$-\infty$

Leva in desna limita, ko gre x proti 1 :

$$\text{Limit}\left[\frac{13}{-1+x^2},\ x \to 1,\ \text{Direction} \to 1\right]$$

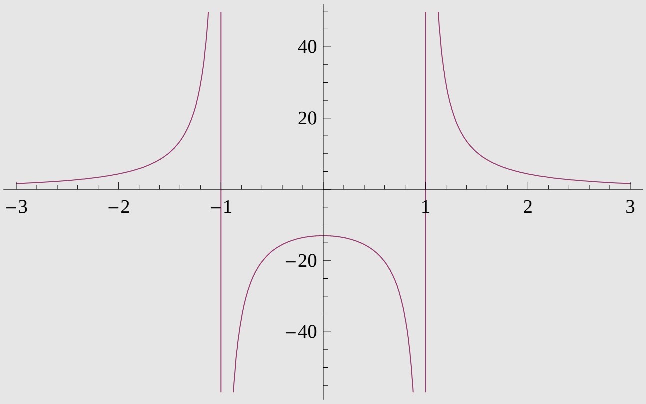
$-\infty$

$$\text{Limit}\left[\frac{13}{-1+x^2},\ x \to 1,\ \text{Direction} \to -1\right]$$

∞

Graf funkcije:

```
Plot[{f1, f2}, {x, -3, 3}]
```



NALOGA 3

```
ClearAll[f]
```

- **Definirajte funkciji f (x) = -x^2 + 13, g (x) = 5/(x^2 + 1).**

```
f = -x^2 + 13
```

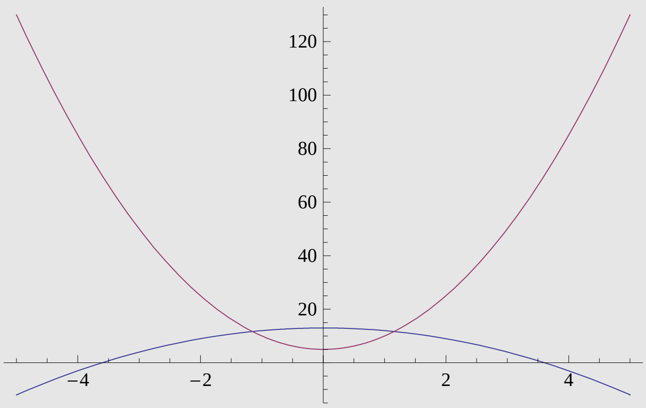
$13 - x^2$

```
g = 5 (x^2 + 1)
```

$5 (1 + x^2)$

- **a) Narisite grafa obeh funkcij in izracunajte njuna presecisca.**

```
Plot[{f, g}, {x, -5, 5}]
```



Presecisce izracunamo tako, da funkciji izenacimo :

```
res = x /. NSolve[13 - x^2 == 5 (1 + x^2), x]
```

```
{-1.1547, 1.1547}
```

- **b) Izracunajte ploscino lika, ki ga funkciji oklepata.**

Ploscino izracunamo po formuli : $S = \int | f(x) - g(x) | dx$, v mejah od a do b.

```
NIntegrate[Abs[13 - x^2 / 5 (1 + x^2)], {x, 0, 3}]
```

```
28.6651
```

- **c) Izracunajte volumen in povrsino vrtenine, ki jo dobite pri vrtenju pozitivnega dela funkcije f okrog osi x.**

Volumen izracunamo po formuli : $V = \pi \int y^2 dx$, v mejah od a do b.

```
N[Pi Integrate[13 - x^2, {x, 0, 2}]]
```

```
73.3038
```

Povrsino izracunamo po formuli : $P = 2\pi \int y \sqrt{(1 + y^2)} dx$, v mejah od a do b.

```
N[2 Pi Integrate[(13 - x^2) Sqrt[1 + (D[13 - x^2, x])^2], {x, 0, 2}]]
```

```
326.33
```

NALOGA 4

- **Definirajte racionalno funkcijo f (x) = (8 x^3 + x^2 + x - 1)/(x^2 - 5).**

```
ClearAll[f]
```

```
f[x_] = (8 x^3 + x^2 + x - 1) / (x^2 - 5)
```

$$\frac{-1+x+x^2+8x^3}{-5+x^2}$$

a) in b) Izracunajte nicle, pole in narisite graf funkcije.

Nicle funkcije so nicle polinoma v stevcu, poli pa nicle polinoma v imenovalcu :

```
NSolve[Numerator[f[x]] == 0, x]
```

{{x → -0.255958 - 0.507497 ⅈ}, {x → -0.255958 + 0.507497 ⅈ}, {x → 0.386915}}

```
NSolve[Denominator[f[x]] == 0, x]
```

{{x → -2.23607}, {x → 2.23607}}

c) Izracunajte asimptoto.

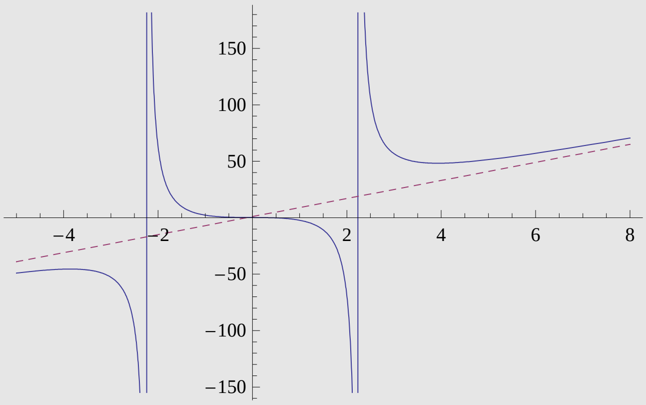
Asimptoto dobimo tako, da med sabo delimo imenovalec in stevec, ce je stopnja stevca vecja kot stopnja imenovalca :

```
a = PolynomialQuotient[Numerator[f[x]], Denominator[f[x]], x]
```

1 + 8 x

Graf :

```
Plot[{f[x], a}, {x, -5, 8},
 PlotStyle → {Dashing[1], Dashing[0.01]}]
```



d) Izracunajte vse lokalne ekstreme in dolocite njihovo naravo (minimum, maksimum).

Prvi odvod :

```
Df = Simplify[D[f[x], x]]
```

$$\frac{-5-8x-121x^2+8x^4}{\left(-5+x^2\right)^2}$$

Drugi odvod :

```
Df2 = Simplify[D[f[x], {x, 2}]]
```

$$\frac{2\left(20+615x+12x^2+41x^3\right)}{\left(-5+x^2\right)^3}$$

V ekstremih je vrednost prvega odvoda nic.

```
m = NSolve[Df == 0, x]
```

{{x → 3.92685}, {x → -3.86108}, {x → -0.032888 + 0.20035 i}, {x → -0.032888 - 0.20035 i}}

Ekstremi :

```
b = {x, f[x]} /. m[[1]]
```

{3.92685, 48.2496}

```
c = {x, f[x]} /. m[[2]]
```

{-3.86108, -45.4624}

Minimum :
Drugi odvod mora biti pozitiven.

```
Df2 /. m[[1]]
```

9.01998

Maksimum :
Drugi odvod mora biti negativen.

```
Df2 /. m[[2]]
```

```
-9.32656
```

e) Dolocite intervale narascanja in padanja.

Funkcija narasca :

```
N[Reduce[D[f[x], x] > 0, x]]
```

```
x < -3.86108 || x > 3.92685
```

Funkcija pada :

```
N[Reduce[D[f[x], x] < 0, x]]
```

```
-3.86108 < x < -2.23607 || -2.23607 < x < 2.23607 || 2.23607 < x < 3.92685
```

f) Dolocite intervale konveksnosti in konkavnosti.

f'' (x) > 0 - pogoj za konveksnost.

```
N[Reduce[D[f[x], {x, 2}] > 0, x]]
```

```
-2.23607 < x < -0.0325387 || x > 2.23607
```

f'' (x) < 0 - pogoj za konkavnost.

```
N[Reduce[D[f[x], {x, 2}] < 0, x]]
```

```
x < -2.23607 || -0.0325387 < x < 2.23607
```

NALOGA 5

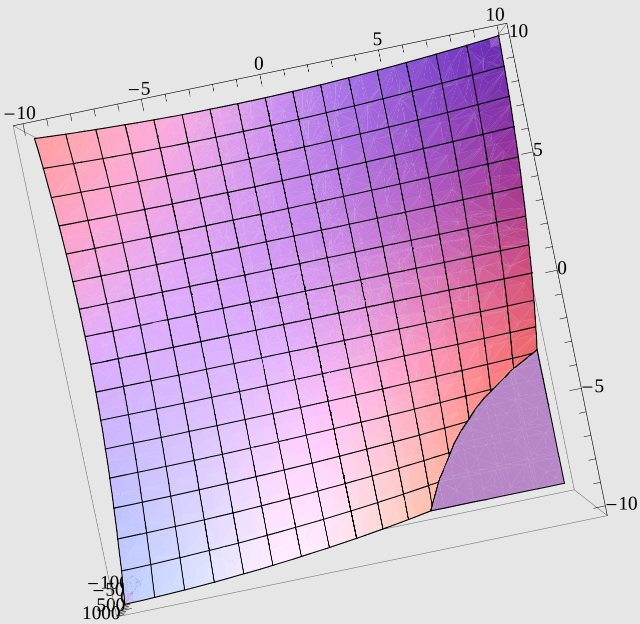
Definirajte funkcijo dveh spremenljivk f (x, y) = 13 xy + x^2 y - y^2 x.

```
Clear[f]
```

```
f[x_, y_] := 13 x * y + x^2 y - y^2 x
```

- **a) Nari itegraf funkcije.**

```
Plot3D[f[x, y], {x, -10, 10}, {y, -10, 10}]
```



- **b) Izracunajte parcialne odvode prvega in drugega reda.**

Prvi odvod po x :

```
fx = D[f[x, y], {x, 1}]
```

$13\, y + 2\, x\, y - y^2$

Prvi odvod po y :

```
fy = D[f[x, y], {y, 1}]
```

$13\, x + x^2 - 2\, x\, y$

Drugi odvod po x :

```
fxx = D[f[x, y], {x, 2}]
```

$2\, y$

Drugi odvod po y :

```
fyy = D[f[x, y], {y, 2}]
```

$-2x$

c) Izracuanjte vse stacionarne tocke.

```
res = Solve[{fx == 0, fy == 0}, {x, y}]
```

$\left\{\{x \to -13, y \to 0\}, \left\{x \to -\frac{13}{3}, y \to \frac{13}{3}\right\}, \{x \to 0, y \to 0\}, \{x \to 0, y \to 13\}\right\}$

```
t = {x, y} /. Solve[{fx == 0, fy == 0}, {x, y}]
```

$\left\{\{-13, 0\}, \left\{-\frac{13}{3}, \frac{13}{3}\right\}, \{0, 0\}, \{0, 13\}\right\}$

d) Dolocite vse lokalne minimume in maksimume.

```
Hess = D[f[x, y], {{x, y}, 2}]
```

$\{\{2y, 13 + 2x - 2y\}, \{13 + 2x - 2y, -2x\}\}$

```
Det[Hess] /. res
```

$\left\{-169, \frac{169}{3}, -169, -169\right\}$