Izračunaj integral funkcije $f(x,y) = x^2y$ po pravokotniku $D = \{(x,y); 0 \leq x \leq 3, 1 \leq y \leq 2\}$. Kolikšen je volumen telesa, ki leži pod grafom funkcije f in nad pravokotnikom D? Izračunaj $\int\int_R y\sin(xy)dxdy$, kjer je $D = \{(x,y); 1 \leq x \leq 2, 0 \leq y \leq \pi\}$.

$\int\int_R y\sin(xy)dxdy = \int_1^2 (\int_0^\pi y\sin(xy)\,dy)\,dx$. Notranji integral izračunamo z metodo per partes, kjer je

$u = y$

$du = dy$ in

$dv = \sin(xy)dy$

$v = -\frac{\cos(xy)}{x}$

$\int_0^\pi y\sin(xy)dy = -\frac{y\cos(xy)}{x}\Big|_{y=0}^{y=\pi} + 1/x\int_0^\pi \cos(xy)\,dy = \frac{-\pi\cos(\pi x)}{x} + \frac{1}{x^2}(sin(xy))\Big|_{y=0}^{y=\pi} = \frac{-\pi\cos(\pi x)}{x} + \frac{\sin(\pi x)}{x^2}.$

Prvi člen integriramo per partes $u = -1/x$ in $dv = \pi\cos(\pi x)dx$, $du = dx/x^2$ in $v = \sin\pi x$,

$\int(\frac{-\pi\cos(\pi x)}{x})\,dx = -\frac{\sin(\pi x)}{x} - \int\frac{\sin(\pi x)}{x^2}\,dx.$

Tako $\int\left(\frac{-\pi\cos(\pi x)}{x} + \frac{\sin(\pi x)}{x^2}\right)dx = -\frac{\sin(\pi x)}{x}$. Torej $\int_1^2(\int_0^\pi y\sin(xy)\,dy)\,dx = -\frac{\sin(\pi x)}{x}\Big|_1^2 = -\frac{\sin(\pi 2)}{2} + \sin\pi =$