DIFERENCIALNE ENAČBE:

(Izberi svoj način dela; namreč, na začetku so naloge razvrščene po snoveh in ti tipa DE ni treba prepoznati, na koncu pa so dodane (nekatere iste) naloge v mešanem vrstnem redu.)

OSNOVNO:

1. Danim družinam krivulj poišči ustrezne diferencialne enačbe:
   (a) $y = kx$, parameter je k; R: $y'x - y = 0$
   (b) $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$, parameter je a; R: $y^2 - yy'x - 4 = 0$
   (c) $ty - \cos tx = 0$, parameter: t; $y' + \sin \frac{x\sqrt{1-y'^2}}{y} = 0$
   (d) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, parametra: a in b; R: $yy' - y'^2x - xyy'' = 0$
2. Preveri, če je dana funkcija rešitev dane diferencialne enačbe:
   (a) $y = x^3 (\ln x - 1) + 2x^2$, $xdy - 2ydx = x^3 \ln x dx$
   (b) $y = C_1 \sin x + C_2 \cos x$, $y'' + y = 0$

DE Z LOČLJIVIMA SPREMENLJIVKAMA:

1. Poišči tisto rešitev DE $y' - 2x = 0$, ki ustreza pogoju $y = 1$ pri $x = 0$. R: $y = x^2 + 1$.
2. $xyy\prime = 1 - x^2$, R: $x^2 + y^2 = \ln Dx^2$.
3. $tgydx - ctgxdy = 0$, R: $\sin y \cos x = D$.
4. $y\prime = \frac{1+y^2}{1+x^2}$, R: $\arctan y = \arctan x + C$.
5. $(2x + 1)\, dy + y^2 dx = 0$. Poišči tisto partikularno rešitev, ki gre skozi točko $T\,(1, 4)$. R: $3e^{\frac{1}{y}} = 3\sqrt{2x + 1}$.
6. $(xy - x)\, dx + (xy + x - y - 1)\, dy = 0$. R: $(x - 1)\,(y - 1)^2\, e^{x+y} = C$.
7. $xy' - y = y^3$, R: $x = \frac{Dy}{\sqrt{y^2+1}}$.
8. $y' = \sqrt{3x + 4y + 1}$, Namig: nova spremenljivka $z = 3x + 4y + 1$, R: $\frac{1}{2}\sqrt{3x + 4y + 1} + \frac{3}{8} - \frac{3}{8} \ln \left(4\sqrt{3x + 4y + 1} + 3\right) = x + C$.
9. $y' - y = 2x - 3$, Namig: nova sprem.: $z = 2x + y - 3$, R: $\ln (2x + y - 1) = x + C$.

HOMOGENA DE:

1. $y' = \frac{y}{x} - 1$, R: $y = x \ln \frac{C}{x}$.

2. $y' = e^{\frac{y}{x}} + \frac{y}{x}$, R: $\ln Dx + e^{-\frac{y}{x}} = 0$.

3. $xy' - y = \sqrt{x^2 + y^2}$, R: $Cx^2 = y + \sqrt{x^2 + y^2}$.

4. $y' = \frac{(x+y)^2}{2x^2}$, R: $y = x \tan\left(\frac{1}{2} \ln x + D\right)$.

5. $\left(x^2 + 3y^2\right) dx - 2xydy = 0$, R:$\left(1 + \frac{y^2}{x^2}\right) = xD$.

NEPOPOLNE D.ENAČBE:

1. $e^{y'} + y' = 1$, R: $e^{\frac{y-C}{x}} + \frac{y-C}{x} = 1$.

2. $x = y'^3 - y' - 1$, R: $x = t^3 - t - 1$, $y = \frac{3}{4}t^4 - \frac{t^2}{2} + C$.

3. $x + y' + y'^5 = 0$, R: $x = -t - t^5$, $y = -\frac{t^2}{2} - \frac{5}{6}t^6 + C$.

4. $y'^2 - 4x^2 = 0$, R: $\left(y - x^2 - C\right)\left(y + x^2 - C\right) = 0$.

5. $x\sqrt{1 + y'^2} = y'$, R: $x^2 + (y - C)^2 = 1$, tako obliko rešitve dobiš po eliminaciji parametra.

6. $2y' + 3y + y'^4 = 0$, R: $x = -\frac{2 \ln t}{3} - \frac{4t^3}{9} + C$, $y = -\frac{2t}{3} - \frac{t^4}{3}$.

EKSAKTNA DE:

1. $\left(3x^2y^2 + y\right) dy + \left(2x^3y + x + 1\right) dy = 0$, R: $x^3y^2 + xy + y = E$.

2. $(x + y + 1)\, dx + \left(x - y^2 + 3\right) dy = 0$. R: $\frac{x^2}{2} + xy + x - \frac{y^y}{3} + 3y = C$.

3. $\frac{2x}{y^3} dx + \frac{y^2 - 3x^2}{y^4} dy = 0$, R: $\frac{x^2}{y^3} - \frac{1}{y} = C$.

LINEARNA DIFERENCIALNA ENAČBA:

1. Katera linearna homogena DE ima rešitev $y = e^{2x} + 2$? R: $\left(1 + 2e^{-2x}\right) y' - 2y = 0$.

2. $xy' + y = \ln x + 1$, R: $y = \ln x + \frac{C}{x}$.

3. $y' - \frac{1+2x}{x+x^2} y = \frac{1+2x}{x+x^2}$, R: $y = C\left(x^2 + x\right) - 1$.

4. $y' + 2xy = e^{-x^2}$, R: $y = (x + C)\, e^{-x^2}$.

5. Reši DE $y' + ay = e^{mx}$, kjer sta a in m konstanti. R: $y = \frac{e^{mx}}{a+m} + Ce^{-ax}$, če $a \neq -m$, če pa je $a = -m$, potem je rešitev $y = (x + C)\, e^{-ax}$.

6. $(1+e^x)(y'+y)=1$, R: $y=e^{-x}\left(\ln\left(e^x+1\right)+C\right)$.

BERNOULLIJEVA DE:

1. $xy'+y=-xy^2$, R: $y=\frac{1}{Cx+x\ln x}$.
2. $y'-\frac{1}{x}y=\frac{1}{2y}$, R: $y^2=Cx^2-x$.
3. $y'+xy=x\sqrt{y}$, R: $y=\left(1+Ce^{-\frac{x^2}{4}}\right)^2$.
4. $y'ctgy+4x^3\sin y=2x$, Namig: uvedi novo sprem. $z=\sin y$, da DE pretvoriš na Bernoullijevo, R: $\frac{1}{\sin y}=2\left(x^2-1\right)+Ce^{-x^2}$.
5. $x^{\frac{5}{2}}y'=\frac{5}{2}y^{\frac{5}{2}}+yx^{\frac{3}{2}}$, $y(4)=0$, R: $y=x\left(\frac{15}{4}\right)^{-\frac{2}{3}}\left(\ln 4-\ln x\right)^{-\frac{2}{3}}$.

RICCATIJEVA DE:

1. $y'=y^2-2ye^x+e^{2x}+e^x$, $y(0)=0$, R: $y=\frac{-1}{1+x}+e^x$.
2. $y'=y^2-\frac{2}{x^2}$, R: $y=\frac{x^2}{-x+C}+\frac{1}{x}$.
3. $3y'+y^2+\frac{2}{x^2}=0$, R: $y=\frac{1}{x}+\frac{1}{x+Cx^{\frac{2}{3}}}$.

CLAIRAUTOVA DE:

1. $y=xy'+y'^2$, R: splošna rešitev: $y=Cx+C^2$, singularna rešitev: $y=\frac{-x^2}{4}$.
2. $y'=xy''+y''^2$, R: $y=\frac{Cx^2}{2}+C^2x+C_1$, $y=-\frac{x^3}{12}+C$.

LAGRANGEOVA DE:

1. $y'=xy'^2+y'^2$, R: $x=-1+\frac{C}{(t-1)^2}$, $y=\frac{Ct^2}{(t-1)^2}$ in $y=0$ ter $y=x+1$.
2. $2y=\frac{xy'^2}{y'+2}$, R: $Dy=(x-D)^2$ in $y=0$ ter $y=-4x$.
3. $y=xy'^2-2y'^3$, R: $y=xt^2-2t^3$ in $y=0$ ter $y=2-x$.

ORTOGONALNE PRESEČNICE:

1. Poišči ortogonalne trajektorije družine parabol $y=Cx^2$. R: $\frac{x^2}{4}+\frac{y^2}{2}=D$.
2. Poišči ortogonalne trajektorije družine $y^2=Cx$. R: $y^2+2x^2=D^2$.

LINEARNE DE VIŠJEGA REDA:

1. $y'' = xe^x$, $y(0) = 1$, $y'(0) = 0$, R: $y = (x-2)e^x + x + 3$.
2. $y''^2 - 3y'' + 2 = 0$, R: $\left(y - \frac{x^2}{2} - C_1x - C_2\right)\left(y - x^2 - C_1x - C_2\right) = 0$.

HOMOGENA LINEARNA DE VIŠJEGA REDA S KONSTANTNIMI KOEFICIENTI:

1. Reši DE $y'' - 4y' + 3y = 0$. Poišči tisto partikularno rešitev, ki ustreza začetnemu pogoju $y = 2$, $y' = 4$ pri $x = 0$. R: $y = C_1e^x + C_2e^{3x}$, $y_p = e^x + e^{3x}$.
2. $y'' - 4y' + 4y = 0$, R: $y = (C_1 + C_2x)e^{2x}$.
3. $y'' - 6y' + 13y = 0$, R: $y = e^{3x}(C_1\cos 2x + C_2\sin 2x)$.
4. $y^{(7)} - 2y^{(5)} + y''' = 0$, R: $y = A + Bx + Cx^2 + De^x + Fxe^x + Ge^{-x} + Hxe^{-x}$.
5. $y^{(4)} - y = 0$, R: $y = C_1e^x + C_2e^{-x} + C_3\cos x + C_4\sin x$.
6. $y''' - 8y = 0$, R: $y = Ae^{2x} + Be^{-x}\cos\left(\sqrt{3}x\right) + Ce^{-x}\sin\left(\sqrt{3}x\right)$.
7. $y'' - 4y' + 5y = 0$, R: $y = Ae^{2x}\cos x + Be^{2x}\sin x$.
8. $y'' + y = 0$, R: $y = A\cos x + B\sin x$.

NEHOMOGENA LDE VIŠJEGA REDA S KONSTANTNIMI KOEFICIENTI:

1. $y'' + y = \sin x$, R: $y = A\cos x + B\sin x + \frac{\sin 2x\cos x}{4} - \frac{x\cos x}{2} + \frac{\sin^3 x}{2}$.
2. $y'' + 3y' + 2y = \frac{1}{e^x+1}$, R: $y = Ae^{-2x} + Be^{-x} + e^{-2x}(e^x+1)(\ln(e^x+1) - 1)$.
3. Poišči rešitev homogenega dela enačbe ter nastavek za partikularno rešitev:
   (a) $y'' + 4y = 8x^2$, R: $y_H = C_1\cos 2x + C_2\sin 2x$, $y_P = Ax^2 + Bx + C$.
   (b) $y'' - 2y' + y = xe^x$, R: $y_H = C_1e^x + C_2xe^x$, $y_P = Ax^3e^x + Bx^2e^x$.
   (c) $y'' + 2y' + 5y = 16e^x + \sin 2x$, R: $y_H = e^{-x}(C_1\cos 2x + C_2\sin 2x)$.
   (d) $y'' + y' = 5x$, R: $y_H = A + Be^{-x}$, $y_P = Ax^2 + Bx$.
   (e) $y'' + y' - 6y = xe^{2x}$, $y_H = Ae^{-3x} + Be^{2x}$, $y_P = \left(Ax^2 + Bx\right)e^{2x}$.
   (f) $y'' + y' - 6y = e^{2x}\cos x$, $y_H = Ae^{-3x} + Be^{2x}$, $y_P = e^{2x}(A\cos x + B\sin x)$.
   (g) $y'' - 2y' + 5y = xe^x\cos 2x$, $y_H = Ae^x\cos 2x + Be^x\sin 2x$, $y_P = xe^x((Ax+B)\cos 2x + (Cx+D)\sin 2x)$.
   (h) $y^{(4)} - 16y = xe^{2x}\sin 2x$, $y_H = Ae^{2x} + Be^{-2x} + C\cos 2x + D\sin 2x$, $y_P = e^{2x}((Ax+B)\cos 2x + (Cx+D)\sin 2x)$.
4. $y'' + 4y' + 5y = 5x^2 - 32x + 5$, R: $y = e^{-2x}(A\cos x + B\sin x) + x^2 - 8x + 7$.

5. $y'' + 3y' = 9x$, R: $y = A + Be^{-3x} + \frac{3}{2}x^2 - x$.

6. $y^{(4)} - y = 4e^x$, R: $y = Ae^x + Be^{-x} + C\cos x + D\sin x + xe^x$.

7. $y'' - 3y' + 2y = 3x + 5\sin 2x$, R: $y = C_1e^x + C_2e^{2x} + \frac{3}{2}x + \frac{3}{4} + -\frac{1}{4}\sin 2x + \frac{3}{4}\cos 2x$.

SISTEMI DIFERENCIALNIH ENAČB:

1. Reši sistem DE, podan z matriko $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$. R: $x = e^{2t}\left(C_3 + C_2t + C_1\left(\frac{t^2}{2} + t\right)\right)$, $y = e^{2t}\left(C_2 + C_1t\right)$, $z = C_1e^{2t}$.

2. Z uporabo matrik reši sistem $\dot{x} = 2x - 3y + 2$, $\dot{y} = x - 2y + 4e^t$. R: $x = -6te^t + 3e^t - 4 + 3C_1e^t + C_2e^{-t}$, $y = -2te^t + 3e^t - 2 + C_2e^t + C_1e^t$.

3. $\dot{x} = 3x + 5y$, $\dot{y} = x - y$, $x(0) = 1$, $y(0) = 5$, R: $x = 5e^{4t} - 4e^{-2t}$, $y = e^{4t} + 4e^{-t}$.

4. Reši sistem DE, podan z matriko $A = \begin{bmatrix} 1 & -1 & 1 \\ -5 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. R: $x = C_1e^t + C_2e^{-t} + C_3e^{4t}$, $y = -5C_1e^t + 3C_2e^{-t} - 2C_3e^{4t}$, $z = -5C_1e^t + C_2e^{-t} + C_3e^{4t}$.

5. * $\dot{x} = x - 2y$, $\dot{y} = 4x + 5y$, R: $x = -e^{3t}\left(A\cos 2t + B\sin 2t\right)$, $y = e^{3t}\left((A + B)\cos 2t + (A - B)\sin 2t\right)$.

6. * Reši sistem DE, podan z matriko $A = \begin{bmatrix} 2 & 1 & -2 \\ -1 & 0 & 0 \\ 1 & 1 & -1 \end{bmatrix}$. R: $x = Ce^t + A\cos t + B\sin t$, $y = -Ce^t + B\cos t - A\sin t$, $z = A\cos t + B\sin t$.

NEKATERE NALOGE - MEŠANO:
Prepoznaj in reši naslednje diferencialne enačbe:

1. $\tan y dx - c\tan x dy = 0$, $\sin y\cos x = D$

2. $xy' - y = y^3$, $Cy = x\sqrt{y^2 + 1}$

3. $y' = e^{\frac{y}{x}} + \frac{y}{x}$, $\ln Dx + e^{-\frac{y}{x}} = 0$

4. $x + y' + y'^5 = 0$, $x = -t - t^5; y = -\frac{t^2}{2} - \frac{5t^6}{6} + C$

5. $y' - \frac{1+2x}{x+x^2}y = \frac{1+2x}{x+x^2}$, $y = C(x^2 + x) - 1$

6. $y' - 2xy = -4x^3y^2$, $\frac{1}{y} = 2(x^2 - 1) + Ce^{-x^2}$

7. $y'(x-x^2) = 2yx - 6x^2$, $y(x-1)^2 = 2x^3 - 3x^2 + C$
8. $3y'\cos x + y\sin x - \frac{1}{y^2} = 0$, $y^3 = \sin x + C\cos x$
9. $y' = \frac{-2y}{x} + \frac{y^2}{x}$, $\frac{x}{C} = \sqrt{\frac{y-2}{2}}$
10. $\frac{dx}{dy}(y-y^2) = 2xy + 2y$, $(x+1)(y-1)^2 = C$
11. $y' = \frac{1+y^2}{1+x^2}$, $\arctan y = \arctan x + C$
12. $xy' - y = \sqrt{x^2+y^2}$, $Cx^2 = y + \sqrt{x^2+y^2}$
13. $y'^2 - 4x^2 = 0$, $y = \pm x^2 + C$
14. $y' + 2xy = e^{-x^2}$, $y = (x+C)e^{-x^2}$
15. $y' = \frac{(x+y)^2}{2x^2}$, $y = x\tan(\frac{1}{2}\ln x + D)$
16. $(2x+1)dy + y^2dx = 0$, $e^{\frac{1}{y}} = D\sqrt{2x+1}$
17. $2y' + 3y + y'^4 = 0$, $x = -\frac{2}{3}\ln t - \frac{4}{9}t^3 + C$; $y = -\frac{2}{3}t - \frac{1}{3}t^4$
18. $xy' + y = -xy^2$, $y(Cx + x\ln x) = 1$
19. $e^{y'} + y' = 1$, $e^{\frac{y-C}{x}} + \frac{y-C}{x} = 1$
20. $2y\frac{1}{y'} = \frac{y^2}{y'(y+2)} + x\frac{y^2+4y}{(y+2)^2}$, $x = C(y+2)$
21. $y' + y = xy^{\frac{2}{3}}$, $y = (x - 3 + Ce^{-\frac{x}{3}})^3$
22. $xyy' = 1 - x^2$, $\frac{y}{2} = \ln x - \frac{x^2}{2} + C$
23. $xy' + y = \ln x + 1$, $y = \ln x + \frac{C}{x}$
24. $(xy - x)dx + (xy + x - y - 1)dy = 0$, $(x-1)(y-1)^2e^{x+y} = A$
25. $x = y'^3 - y' - 1$, $x = t^3 - t - 1$; $y = \frac{3}{4}t^4 - \frac{t^2}{2} + C$
26. $y' - \frac{1}{xy} = \frac{1}{2y}$, $y^2 = Cx^2 - x$
27. $y' + x = x^2 - 2$, $e^{3x} = \frac{y-2}{D(y+1)}$
28. $x^{\frac{5}{2}}y' = \frac{5}{2}y^{\frac{5}{2}} + yx^{\frac{3}{2}}$, $y = x(C - \frac{15}{4}\ln x)^{-\frac{2}{3}}$
29. $y' = \frac{2}{x}y + y^2$, $x^2 = y(C - x)$
30. $y' = \frac{y}{x} - 1$, $y = x\ln\frac{C}{x}$
31. $y' + xy = x\sqrt{y}$, $y = (1 + Ce^{-\frac{1}{4}x^2})^2$
32. $y' + \frac{1}{3y^2} = -\frac{2y}{3x}$, $1 = y(x + x^{\frac{2}{3}}C)$