

OPOMBA: Primeri nalog so iz knjige avtorjev dr. Boštjana Brešarja in dr. Franceta Brešarja **ANALIZA 2**. Naloge so iz prvega poglavja o nedoločenem integralu, izbrani primeri so oštevilčeni tako kot v knjigi, kjer lahko najdete njihove rešitve skupaj z napotki za reševanje.

Tabela nedoločenih integralov:

- (1) $\int x^r dx = \frac{x^{r+1}}{r+1}$, $r \in \mathbb{R} \wedge r \neq -1$
- (2) $\int x^{-1} dx = \int \frac{dx}{x} = \ln|x| + C$
- (3) $\int e^x dx = e^x + C$
- (4) $\int a^x dx = \frac{a^x}{\ln a} + C$
- (5) $\int \cos x dx = \sin x + C$
- (6) $\int \sin x dx = -\cos x + C$
- (7) $\int \frac{dx}{\cos^2 x} = \tan x + C$
- (7') $\int (1 + \tan^2 x) dx = \tan x + C$
- (8) $\int \frac{dx}{\sin^2 x} = -\cot x + C$
- (8') $\int (1 + \cot^2 x) dx = -\cot x + C$
- (9) $\int \cosh x dx = \sinh x + C$
- (10) $\int \sinh x dx = \cosh x + C$
- (11) $\int \frac{dx}{\cosh^2 x} = \tanh x + C$
- (12) $\int \frac{dx}{\sinh^2 x} = -\coth x + C$
- (13) $\int \frac{dx}{1+x^2} = \arctan x + C = -\operatorname{arccot} x + C$
- (14) $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C = -\arccos x + C$
- (15) $\int \frac{dx}{\sqrt{x^2+a}} = \ln|x + \sqrt{x^2+a}| + C$

Naloge:

V nalogah 1. - 14. izračunajte dane preproste integrale.

1. $\int x^7 dx$, 2. $\int \sqrt{x} dx$, 3. $\int dx/\sqrt[3]{x}$, 4. $\int \sqrt{x}\sqrt{x}\sqrt{x} dx$,
5. $\int (x^2 - 3x + 1) dx$, 6. $\int x^2(2-x)^3 dx$, 7. (a) $\int \frac{\sqrt{x} - \sqrt[3]{x+2x}}{\sqrt{x}} dx$,
 (b) $\int \frac{\sqrt{2x+3} - \sqrt[3]{2x+3} + 2(2x+3)}{\sqrt[4]{2x+3}} dx$, 8. $\int \sqrt{x^4 - 2x + x^{-2}} dx$,
9. $\int \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1-x^4}} dx$, 10. $\int \frac{x^4 + x^3 + x^2 + x + 1}{x^2 + 1} dx$, 11. (a) $\int 3^x dx$,
 (b) $\int 3^x \cdot 2^{-x} dx$, (c) $\int \frac{3^x + 2^x}{5^x} dx$, 12. $\int \tan^2 x dx$, 13. $\int \sqrt{1 + \sin 2x} dx$,
14. $\int \frac{\sin^4 x + \cos^4 x}{\sin^2 x \cos^2 x} dx$, 15. $\int \frac{dx}{\sqrt{x} - \sqrt{x-1}} dx$.

V nalogah 15. - 28. si pomagajte z vpeljavo nove spremenljivke (z metodo substitucije)!

- 16.** $\int (\sqrt[3]{1-x} + \sin(1-x) + e^{1-x} + 1-x) dx$, **17.** $\int \frac{2+x}{\sqrt[3]{2-3x}} dx$,
18. $\int \frac{x+1}{\sqrt{2x+1}} dx$, **20.** $\int \frac{dx}{x^2+2}$, **21.** $\int \frac{dx}{\sqrt{4x^2-1}}$, **22.** $\int \frac{dx}{3x^2+2}$, **23.** $\int \frac{xdx}{x^4+4}$, **24.**
 $\int \frac{x^2 dx}{x^6+2x^3+1}$,
25. $\int \sin x \cos x dx$, $\int e^{x^2-x} (2x-1) dx$, $\int \frac{-2x dx}{\sqrt{1-x^2}}$, $\int \frac{\arcsin x}{\sqrt{1-x^2}} dx$, $\int \frac{(3x^2+4x) dx}{x^3+2x^2-1}$;
26. $\int \frac{xdx}{3-4x^2}$, $\int \frac{xdx}{1+(3-4x^2)^2}$, $\int x \sin(3-4x^2) dx$, $\int x e^{3-4x^2} dx$,
27. (a) $\int \frac{dx}{x\sqrt{1-x^2}}$, (b) $\int \frac{dx}{(x+1)\sqrt{x^2+2x-2}}$, **28.** $\int \frac{\arctan \sqrt{x}}{1+x} \frac{dx}{\sqrt{x}}$.

V nalogah 29. - 38. integrirajte dane racionalne funkcije!

- 29.** $f(x) = (x+1)^3 / (x+4)$, **30.** $f(x) = x^3 / (x^2 - 3x + 2)$,
31. $f(x) = x^4 / (x^2 + 1)$, **32.** $f(x) = \frac{1}{x(x+1)(x+3)}$, **33.** (a) $f(x) = 1 / (x^2 + 2x - 3)$, (b) $g(x) = (x+1) / (x^2 + 2x - 3)$, **34.** $f(x) = 1 / (x^3 + 1)$,
35. $f(x) = (x-1) / (x^3 - 3x^2 + 4)$, **36.** $f(x) = x^3 / (4x^2 - 1)^2$,
37. $f(x) = \frac{1}{x^2(1+x^2)^2}$, **38.** $f(x) = (x^2 - 2x + 2)^{-3}$, **39.** $f(x) = \frac{x^2+x+3}{x(x^3+4x^2+x-6)}$,
40. $f(x) = \frac{x+1}{(x-1)(x+3)(x^2+2x+2)}$.

V nalogah 41. - 53. integrirajte iracionalne funkcije!

- 41.** $\int (\sqrt{x\sqrt{x^3}} + (2x+1)\sqrt{x^2+x+1}) dx$, **42.** $\int x\sqrt[3]{2x+3} dx$, **43.**
 $\int \frac{dx}{1+\sqrt{x+1}}$, **44.** $\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}$, **45.** $\int \sqrt{\frac{x-1}{x+1}} dx$, **46.** $\int \frac{dx}{\sqrt{4x-x^2}}$, **47.** $\int \frac{dx}{\sqrt{1+6x-9x^2}}$,
48. $\int \frac{dx}{\sqrt{x^2+2x+9}}$, **49.** $\int \frac{dx}{\sqrt{7x^2-x-1}}$, **50.** $\int \frac{x^3}{\sqrt{x^2+4x-1}} dx$, **51.** $\int \frac{2x^3}{\sqrt{1-x^2-x^4}} dx$,
52. $\int \frac{dx}{x\sqrt{3x^2+6x-1}}$, **53.** $\int \frac{dx}{(x+1)\sqrt{7x^2+5x+2}}$.

V nalogah 55. - 70. integrirajte dane kotne funkcije!

- 55.** $\int \frac{1-\sin x}{\cos^2 x} dx$, **56.** (a) $\int \frac{dx}{5-4\cos x}$, (b) $\int \frac{dx}{5+4\sin x}$, **57.** $\int \frac{dx}{5+4\sin x+3\cos x}$,
58. $\int \frac{dx}{(1+\cos^2 x)\sin x}$, **59.** (a) $\int \sin^4 x \cos^5 x dx$, (b) $\int \sin^5 x \cos^4 x dx$,
60. $\int \cos^3 x \sin^3 x dx$, **61.** (a) $\int \cos^2 x dx$, (b) $\int \sin^2 x dx$, **62.** $\int \cos^4 x dx$,
63. $\int \sin^2 x \cos^4 x dx$, **64.** $\int \sin^4 x \cos^4 x dx$, **65.** $\int \tan^3 x dx$, **66.** $\int \cot^3 x dx$,
67. $\int \frac{dx}{\sin^3 x \cos^5 x}$, **68.** $\int \frac{dx}{\sin^4 x \cos^4 x}$, **69.** $\int \sin x \sin 2x \sin 3x dx$,
70. $\int \sin^2 2x \cos^3 3x dx$.

V nalogah 71. - 76. izračunajte integrale eksponentne funkcije in hiperboličnih funkcij!

$$\mathbf{71.} \int \frac{e^x - 1}{e^x + 1} dx, \mathbf{72.} \int \frac{3e^{2x} + 2e^x + 1}{e^{2x} + e^x + 1} dx, \mathbf{73.} \int \sqrt{\frac{e^x - 1}{e^x + 1}} dx,$$

V nalogah 77. - 79. integrirajte po delih (per partes).

$$\mathbf{77.} \int (x^2 + 2x) e^x dx, \mathbf{78.} \int x^2 \cos x dx, \mathbf{80.} \int e^{\sqrt{x}} dx,$$

V nalogah 81. - 85. integrirajte po delih tako, da logaritemsko oz. ciklotometrično funkcijo odvajate!

$$\mathbf{81.} \int \ln x dx, \mathbf{82.} \int x \ln(x^2 - a^2) dx, \mathbf{83.} (a) \int \arcsin x dx, \\ (b) \int \arctan x dx, \mathbf{84.} \int \ln^2 x dx, \mathbf{85.} \int (3x^2 + 1) \arctan x dx.$$