

PARAMETRIČNI PREIZKUSI ZNAČILNOSTI

MAJHNI VZORCI velikosti n

Test	Predpostavke	Testna statistika	Porazdelitev	Namen testa
$H_0 (\mu = \mu_0)$	$X \sim N(\mu, \sigma), \sigma$ znana	$Z = \frac{X - \mu_0}{\sigma} \sqrt{n}$	$N(0, 1)$	populacijsko povprečje
$H_0 (\mu = \mu_0)$	$X \sim N(\mu, \sigma), \sigma$ neznana	$T = \frac{X - \mu_0}{S} \sqrt{n}$	$S(n - 1)$	populacijsko povprečje
$H_0 (\sigma = \sigma_0)$	$X \sim N(\mu, \sigma)$	$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2}$	$\chi^2(n - 1)$	populacijski standardni odklon
$H_0 (\mu = \nu)$	$X \sim N(\mu, \sigma), Y \sim N(\nu, \tau), \sigma, \tau$ znana	$T = \frac{X - Y}{S}$	$N(0, 1)$	enakost populacijskih povprečij
	m velikost vzorca X, n vel. vzorca Y	$S = \sqrt{\frac{\sigma^2}{m} + \frac{\tau^2}{n}}$		
$H_0 (\mu = \nu)$	$X \sim N(\mu, \sigma), Y \sim N(\nu, \sigma), \sigma$ neznana	$T = \frac{X - Y}{S} \sqrt{\frac{nm}{n+m}}$	$S(m + n - 1)$	enakost populacijskih povprečij
	m velikost vzorca X, n vel. vzorca Y	$S = \sqrt{\frac{(m-1)S_X^2 + (n-1)S_Y^2}{m+n-2}}$		
$H_0 (\sigma = \tau)$	$X \sim N(\mu, \sigma), Y \sim N(\nu, \tau)$	$F = \frac{S_X^2}{S_Y^2}$	$F(m - 1, n - 1)$	analiza varianc

VELIKI VZORCI velikosti n

Test	Predpostavke	Testna statistika	Porazdelitev	Namen testa
$H_0 (E(X) = \mu_0)$	X kakorkoli, $\sigma = \sigma(X)$ znana	$Z = \frac{X - \mu_0}{\sigma} \sqrt{n}$	$\simeq N(0, 1)$	populacijsko povprečje
$H_0 (E(X) = \mu_0)$	X kakorkoli, $\sigma = \sigma(X)$ neznana	$Z = \frac{X - \mu_0}{S} \sqrt{n}$	$\simeq N(0, 1)$	populacijsko povprečje
$H_0 (\sigma = \sigma_0)$	$X \sim N(\mu, \sigma)$	$Z = \frac{S}{\sigma_0} \sqrt{2(n-1) - \sqrt{2n-3}}$	$\simeq N(0, 1)$	populacijski standardni odklon
$H_0 (p = p_0)$	\bar{p} vzorčni delež, n poskusov	$Z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \sqrt{n}$	$\simeq N(0, 1)$	populacijski delež
$H_0 (p = q)$	\bar{p} vzorčni delež, m poskusov	$Z = \frac{\bar{p} - q}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n+m}}} \sqrt{\frac{nm}{n+m}}$	$\simeq N(0, 1)$	enakost populacijskih deležev
	\bar{q} vzorčni delež, n poskusov	$\hat{p} = \frac{m\bar{p} + n\bar{q}}{m+n}$		
$H_0 (\mu = \nu)$	$X \sim N(\mu, \sigma), Y \sim N(\nu, \sigma), \sigma$ neznana	$Z = \frac{X - Y}{S}$	$\simeq N(0, 1)$	enakost populacijskih povprečij
	m velikost vzorca X, n vel. vzorca Y	$S = \sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}$		
$H_0 (\sigma = \tau)$	$X \sim N(\mu, \sigma), Y \sim N(\nu, \tau)$	$F = \frac{S_X^2}{S_Y^2}$	$F(m - 1, n - 1)$	analiza varianc

NEPARAMETRIČNI PREIZKUSI ZNAČILNOSTI

1. Test χ^2 (tip porazdelitve)

Zalogo vrednosti X razdelimo v r razredov S_k , ki jim pripadajo hipotetične verjetnosti $p_k = P(X \in S_k/H_0)$, vzorec velikosti $n = \sum n_k$, kjer so n_k frekvence razredov. Testna statistika

$$\chi^2 = \sum \frac{(n_k - np_k)^2}{np_k}$$

je porazdeljena po $\chi^2(s)$, kjer je $s = r - 1$, kadar ni potrebno ocenjevanje parametrov, in $s = r - t - 1$, kadar je predhodno potrebno oceniti t parametrov.

Pogoj: vsi $np_k \geq 5$!

2. Test (neodvisnosti) s kontingenčno tabelo

Glede na vrednosti X in Y vzorec (in populacija) razpade na r razredov oz. s razredov. Celične frekvence $n_{j,k}$, robne $n_{j.}$ za X oz. $n_{.k}$ za Y . Testna statistika

$$\chi^2 = n \left(\sum \frac{n_{j,k}^2}{n_{j.} n_{.k}} - 1 \right)$$

je porazdeljena po $\chi^2((r-1)(s-1))$. Posebnost pri $r = s = 2$ in vsaj eni $n_{jk} < 50$ (Yatesova korektura):

$$\chi_Y^2 = \frac{n(|ad - bc| - n/2)^2}{(a+b)(c+d)(a+c)(b+d)}$$

3. Test z znaki (enaka porazdelitev na isti populaciji)

Vzorec parov $(x_1, y_1), \dots, (x_n, y_n)$, frekvenca pozitivnih razlik $x - y$ je testna statistika K^+ , porazdeljena po $b(n, 1/2)$, če sta X in Y porazdeljeni zvezno in enako. Za velike vzorce smiselno $Z = (2K^+ - n)/\sqrt{n}$ (po $N(0, 1)$).

4. Inverzijski (Mann, Whitney) test (enaka porazdelitev na različnih p.)

Vzorca (x_1, \dots, x_m) za X in (y_1, \dots, y_n) za Y . Vsota rangov x v združenem nizu vseh $m + n$ vrednostije R , število inverzij $U = R - m(m+1)/2$ je za $m + n \geq 20$ približno normalno porazdeljeno. Testna statistika

$$Z = \frac{2U - mn}{\sqrt{mn(m+n+1)}} \sqrt{3} \quad \text{tedaj po } N(0, 1)$$

Tabela A

V tabeli so prikazane aproksimativne vrednosti funkcije

$$\Phi(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx.$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.5000	0.5000	0.5000

Tabela B

Vrednosti funkcije $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-t^2/2} dt$

x	0	1	2	3	4	5	6	7	8	9
0-0	0-0000	0040	0080	0120	0160	0199	0239	0279	0319	0359
1	0398	0438	0478	0517	0557	0596	0636	0675	0714	0753
2	0793	0832	0871	0910	0948	0987	1026	1064	1103	1141
3	1179	1217	1255	1293	1331	1368	1406	1443	1480	1517
4	1554	1591	1628	1664	1700	1736	1772	1808	1844	1879
5	1915	1950	1985	2019	2054	2088	2123	2157	2190	2224
6	2257	2291	2324	2357	2389	2422	2454	2486	2518	2549
7	2580	2611	2642	2673	2704	2734	2764	2794	2823	2852
8	2881	2910	2939	2967	2995	3023	3051	3079	3106	3133
9	3159	3186	3212	3238	3264	3289	3315	3340	3365	3389
1-0	3413	3437	3461	3485	3508	3531	3554	3577	3599	3621
1	3643	3665	3686	3708	3729	3749	3770	3790	3810	3830
2	3849	3869	3888	3907	3925	3944	3962	3980	3997	4015
3	4032	4049	4066	4082	4099	4115	4131	4147	4162	4177
4	4192	4207	4222	4236	4251	4265	4279	4292	4306	4319
5	4332	4345	4357	4370	4382	4394	4406	4418	4429	4441
6	4452	4463	4474	4485	4495	4505	4515	4525	4535	4545
7	4554	4564	4573	4582	4591	4599	4608	4616	4625	4633
8	4641	4649	4656	4664	4671	4678	4686	4693	4699	4706
9	4713	4719	4726	4732	4738	4744	4750	4756	4762	4767
2-0	4773	4778	4783	4788	4793	4798	4803	4808	4812	4817
1	4821	4826	4830	4834	4838	4842	4846	4850	4854	4857
2	4861	4864	4868	4871	4875	4878	4881	4884	4887	4890
3	4893	4895	4898	4901	4904	4906	4909	4911	4913	4916
4	4918	4920	4922	4925	4927	4929	4931	4932	4934	4936
5	4938	4940	4941	4943	4945	4946	4948	4949	4951	4952
6	4953	4955	4956	4957	4959	4960	4961	4962	4963	4964
7	4965	4966	4967	4968	4969	4970	4971	4972	4973	4974
8	4974	4975	4976	4977	4977	4978	4979	4980	4980	4981
9	4981	4982	4983	4983	4984	4984	4985	4985	4986	4986
3-0	4987	4987	4987	4988	4988	4989	4989	4989	4990	4990
1	4990	4991	4991	4991	4992	4992	4992	4993	4993	4993
2	4993	4993	4994	4994	4994	4994	4995	4995	4995	4995
3	4995	4995	4996	4996	4996	4996	4996	4997	4997	4997
4	4997	4997	4997	4997	4997	4997	4997	4998	4998	4998
5	4998	4998	4998	4998	4998	4998	4998	4998	4998	4998
6	4998	4998	4999	4999	4999	4999	4999	4999	4999	4999
7	4999	4999	4999	4999	4999	4999	4999	4999	4999	4999
8	4999	4999	4999	4999	4999	4999	4999	5000	5000	5000

Tabela

Tabela C

Rešitve χ^2_α enačbe $P(\chi^2(n) > \chi^2_\alpha) = \alpha$

$n \setminus \alpha$	0-99	0-95	0-05	0-02	0-01	0-001
1	0-00	0-00	3-84	5-41	6-64	10-83
2	0-02	0-10	5-99	7-82	9-21	13-82
3	0-12	0-35	7-82	9-84	11-34	16-27
4	0-30	0-71	9-49	11-67	13-28	18-46
5	0-55	1-14	11-07	13-39	15-09	20-52
6	0-87	1-64	12-59	15-03	16-81	22-46
7	1-24	2-17	14-07	16-62	18-48	24-32
8	1-65	2-73	15-51	18-17	20-09	26-12
9	2-09	3-32	16-92	19-68	21-67	27-88
10	2-56	3-94	18-31	21-16	23-21	29-59
11	3-05	4-58	19-68	22-62	24-72	31-26
12	3-57	5-23	21-03	24-05	26-22	32-91
13	4-11	5-89	22-36	25-47	27-69	34-53
14	4-66	6-57	23-68	26-87	29-14	36-12
15	5-23	7-26	25-00	28-26	30-58	37-70
16	5-81	7-96	26-30	29-63	32-00	39-25
17	6-41	8-67	27-59	31-00	33-41	40-79
18	7-02	9-39	28-87	32-35	34-80	42-31
19	7-63	10-12	30-14	33-69	36-19	43-82
20	8-26	10-85	31-41	35-02	37-57	45-32
21	8-90	11-59	32-67	36-34	38-93	46-80
22	9-54	12-34	33-92	37-66	40-29	48-27
23	10-20	13-09	35-17	38-97	41-64	49-73
24	10-86	13-85	36-42	40-27	42-98	51-18
25	11-52	14-61	37-65	41-57	44-31	52-62
26	12-20	15-38	38-88	42-86	45-86	54-05
27	12-88	16-15	40-11	44-14	46-96	55-48
28	13-56	16-93	41-34	45-42	48-28	56-89
29	14-26	17-71	42-56	46-69	49-59	58-30
30	14-95	18-49	43-77	47-96	50-89	59-70

Pri $n > 30$ lahko vzamemo $\chi^2_\alpha = \frac{1}{2}(z_\alpha + \sqrt{2n-1})^2$. Pri tem je z_α rešitev enačbe $P(Z > z_\alpha) = \alpha$ in Z porazdeljena standardizirano normalno.

Tabela D

Student

Rešitve t_α enačbe $P(|T(n)| > t_\alpha) = \alpha$

α n	0.05	0.02	0.01	0.001
1	12.71	31.82	63.66	636.6
2	4.30	6.96	9.92	31.60
3	3.18	4.54	5.84	12.92
4	2.78	3.75	4.60	8.61
5	2.57	3.36	4.03	6.87
6	2.45	3.14	3.71	5.96
7	2.36	3.00	3.50	5.41
8	2.31	2.90	3.36	5.04
9	2.26	2.82	3.25	4.78
10	2.23	2.76	3.17	4.59
11	2.20	2.72	3.11	4.44
12	2.18	2.68	3.06	4.32
13	2.16	2.65	3.01	4.22
14	2.14	2.62	2.98	4.14
15	2.13	2.60	2.95	4.07
16	2.12	2.58	2.92	4.02
17	2.11	2.57	2.90	3.96
18	2.10	2.55	2.88	3.92
19	2.09	2.54	2.86	3.88
20	2.09	2.53	2.84	3.85
21	2.08	2.52	2.83	3.82
22	2.07	2.51	2.82	3.79
23	2.07	2.50	2.81	3.77
24	2.06	2.49	2.80	3.74
25	2.06	2.48	2.79	3.72
26	2.06	2.48	2.78	3.71
27	2.05	2.47	2.77	3.69
28	2.05	2.47	2.76	3.67
29	2.04	2.46	2.76	3.66
30	2.04	2.46	2.75	3.65
40	2.02	2.42	2.70	3.55
50	2.01	2.40	2.68	3.50
60	2.00	2.39	2.66	3.46
80	1.99	2.37	2.64	3.42
100	1.98	2.36	2.63	3.39
200	1.97	2.35	2.60	3.34
300	1.97	2.34	2.59	3.32
500	1.96	2.33	2.59	3.31
1000	1.96	2.33	2.58	3.30
∞	1.96	2.33	2.58	3.29

Rešitve F enačbe $P(F(m, n) > F) = 0.05$

Tabela E

m n	1	2	3	4	5	6	7	8
1	161	200	216	225	230	234	237	239
2	18.51	19.00	19.16	19.25	19.30	19.33	19.36	19.37
3	10.13	9.55	9.28	9.12	9.01	8.94	8.88	8.84
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95
12	4.75	3.88	3.49	3.26	3.11	3.00	2.92	2.85
14	4.60	3.74	3.34	3.11	2.96	2.85	2.77	2.70
17	4.45	3.59	3.20	2.96	2.81	2.70	2.62	2.55
20	4.35	3.49	3.10	2.87	2.71	2.60	2.52	2.45
24	4.26	3.40	3.01	2.78	2.62	2.51	2.43	2.36
30	4.17	3.32	2.92	2.69	2.53	2.42	2.34	2.27
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07
100	3.94	3.09	2.70	2.46	2.30	2.19	2.10	2.03
150	3.91	3.06	2.67	2.43	2.27	2.16	2.07	2.00
200	3.89	3.04	2.65	2.41	2.26	2.14	2.05	1.98
400	3.86	3.02	2.62	2.39	2.23	2.12	2.03	1.96
1000	3.85	3.00	2.61	2.38	2.22	2.10	2.02	1.95
∞	3.84	2.99	2.60	2.37	2.21	2.09	2.01	1.94

Tabela E (nadaljevanje)

Fischer

$n \backslash m$	9	10	11	12	14	16	20	24
1	241	242	243	244	245	246	248	249
2	19-38	19-39	19-40	19-41	19-42	19-43	19-44	19-45
3	8-81	8-78	8-76	8-74	8-71	8-69	8-66	8-64
4	6-00	5-96	5-93	5-91	5-87	5-84	5-80	5-77
5	4-78	4-74	4-70	4-68	4-64	4-60	4-56	4-53
6	4-10	4-06	4-03	4-00	3-96	3-92	3-87	3-84
7	3-68	3-63	3-60	3-57	3-52	3-49	3-44	3-41
8	3-39	3-34	3-31	3-28	3-23	3-20	3-15	3-12
9	3-18	3-13	3-10	3-70	3-20	2-98	2-93	2-90
10	3-02	2-97	2-94	2-91	2-86	2-82	2-77	2-74
11	2-90	2-86	2-82	2-79	2-74	2-70	2-65	2-61
12	2-80	2-76	2-72	2-69	2-64	2-60	2-54	2-50
14	2-65	2-60	2-56	2-53	2-48	2-44	2-39	2-35
17	2-50	2-45	2-41	2-38	2-33	2-29	2-23	2-19
20	2-40	2-35	2-31	2-28	2-23	2-18	2-12	2-08
24	2-30	2-26	2-22	2-18	2-13	2-09	2-02	1-98
30	2-21	2-16	2-12	2-09	2-04	1-99	1-93	1-89
40	2-12	2-07	2-04	2-00	1-95	1-90	1-84	1-79
50	2-07	2-02	1-98	1-95	1-90	1-85	1-78	1-74
70	2-01	1-97	1-93	1-89	1-84	1-79	1-72	1-67
100	1-97	1-92	1-88	1-85	1-79	1-75	1-68	1-63
150	1-94	1-89	1-85	1-82	1-76	1-71	1-64	1-59
200	1-92	1-87	1-83	1-80	1-74	1-69	1-62	1-57
400	1-90	1-85	1-81	1-78	1-72	1-67	1-60	1-54
1000	1-89	1-84	1-80	1-76	1-70	1-65	1-58	1-53
∞	1-88	1-83	1-79	1-75	1-69	1-64	1-57	1-52

Tabela E (nadaljevanje)

Fischer

$n \backslash m$	30	40	50	75	100	200	500	∞
1	250	251	252	253	253	254	254	254
2	19-46	19-47	19-47	19-48	19-49	19-49	19-50	19-50
3	8-62	8-60	8-58	8-57	8-56	8-54	8-54	8-53
4	5-74	5-71	5-70	5-68	5-66	5-65	5-64	5-63
5	4-50	4-46	4-44	4-42	4-40	4-38	4-37	4-36
6	3-81	3-77	3-75	3-72	3-71	3-69	3-68	3-67
7	3-38	3-34	3-32	3-29	3-28	3-25	3-24	3-23
8	3-08	3-05	3-03	3-00	2-98	2-96	2-94	2-93
9	2-86	2-82	2-80	2-77	2-76	2-73	2-72	2-71
10	2-70	2-67	2-64	2-61	2-59	2-56	2-55	2-54
11	2-57	2-53	2-50	2-47	2-45	2-42	2-41	2-40
12	2-46	2-42	2-40	2-36	2-35	2-32	2-31	2-30
14	2-31	2-27	2-24	2-21	2-19	2-16	2-14	2-13
17	2-15	2-11	2-08	2-04	2-02	1-99	1-97	1-96
20	2-04	1-99	1-96	1-92	1-90	1-87	1-85	1-84
24	1-94	1-89	1-86	1-82	1-80	1-76	1-74	1-73
30	1-84	1-79	1-76	1-72	1-69	1-66	1-64	1-62
40	1-74	1-69	1-66	1-61	1-59	1-55	1-53	1-51
50	1-69	1-63	1-60	1-55	1-52	1-48	1-46	1-44
70	1-62	1-56	1-53	1-47	1-45	1-40	1-37	1-35
100	1-57	1-51	1-48	1-42	1-39	1-34	1-30	1-28
150	1-54	1-47	1-44	1-37	1-34	1-29	1-25	1-22
200	1-52	1-45	1-42	1-35	1-32	1-26	1-22	1-19
400	1-49	1-42	1-38	1-32	1-28	1-22	1-16	1-13
1000	1-47	1-41	1-36	1-30	1-26	1-19	1-13	1-08
∞	1-46	1-40	1-35	1-28	1-24	1-17	1-11	1-00