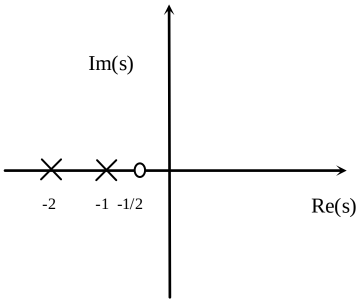
ANALIZA SISTEMOV V ČASOVNEM PODROČJU

1. Vpliv polov in ničel na časovni odziv

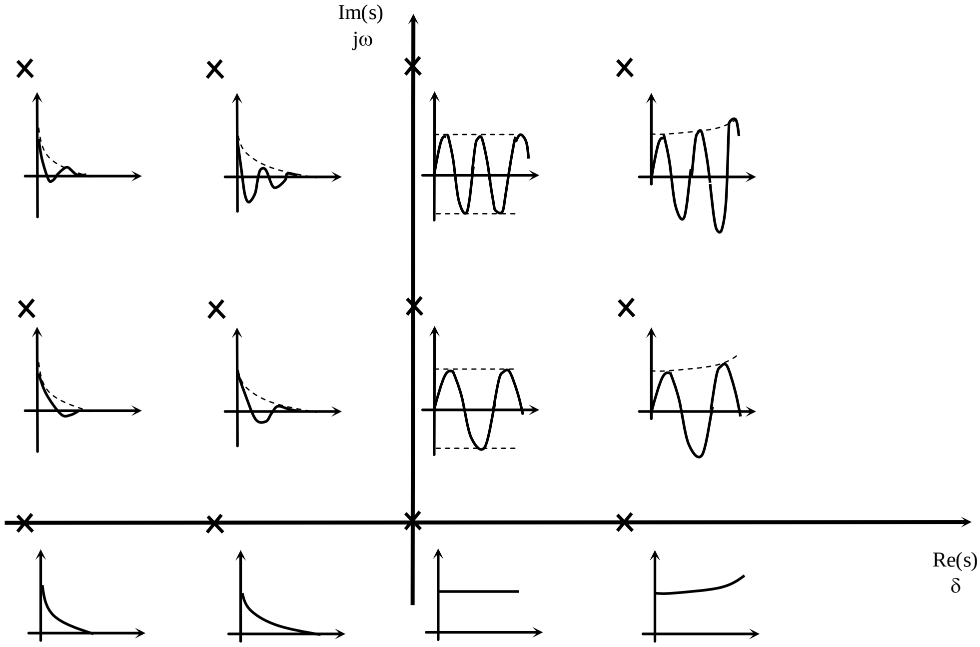
$$\ddot{c} + 3\dot{c} + 2c = 2\dot{r} + r \qquad G(s) = \frac{C(s)}{R(s)} = \frac{\check{S}(s)}{I(s)} = \frac{2s+1}{s^2+3s+2} = \frac{2(s+1/2)}{(s+1)(s+2)}$$



Prenosno funkcijo G(s) razvijemo v parcialne ulomke:

$$G(s) = \frac{2s+1}{(s+1)(s+2)} = \frac{-1}{s+1} + \frac{3}{s+2} \quad \Rightarrow \quad g(t) = -e^{-t} + 3e^{-2t}$$

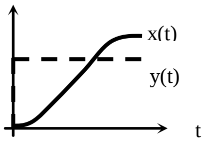
Vpliv lege polov na impulzni odziv sistema:



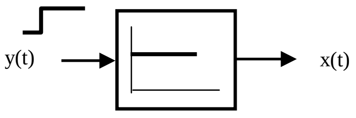
2. Tipi sistemov

Proporcionalni sistemi

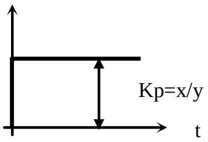
$$G(s)=\frac{\check{S}(s)}{I(s)}=\frac{X(s)}{Y(s)}=\frac{a_n s^n+a_{n-1}s^{n-1}+\ldots+a_1 s+a_0}{b_m s^m+b_{m-1}s^{m-1}+\ldots+b_1 s+b_0};\quad m\le n;\quad \check{S}(0)\neq 0;\quad I(0)\neq 0$$



Proporcionalni P člen:



P člen

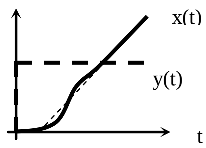


$$G(s)=\frac{X(s)}{Y(s)}=K_p$$

$$x(t)=K\cdot y(t)$$

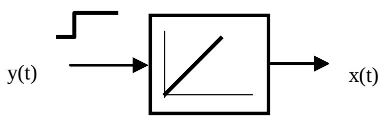
Integrirni sistemi

$$G(s)=\frac{\check{S}(s)}{I(s)}=\frac{X(s)}{Y(s)}=\frac{1}{s^j}\frac{\check{S}(s)}{I'(s)};\quad \check{S}(0)\neq 0;\quad I(0)\neq 0$$

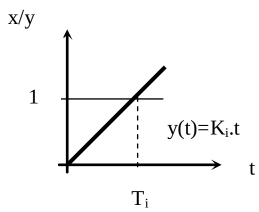


- j = 0… sistem ničelne vrste (reda) ali stopnje je proporcionalni sistem
- j = 1… sistem prve vrste
- j = 2… sistem druge vrste itd.

Integrirni člen:



I člen



$$x(t)=K_i\int_0^t y(t)dt$$

$$G(s)=\frac{X(s)}{Y(s)}=\frac{K_i}{s}=\frac{1}{sT_i}$$

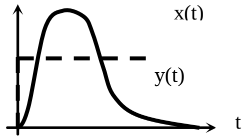
integracijska konstanta

časovna $\frac{dx}{dt}=\frac{1}{T_i}\cdot y;\quad K_i=\frac{1}{T_i};$

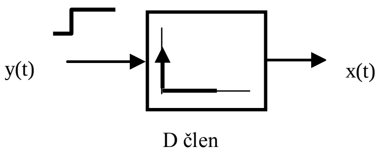
Diferencirni sistem

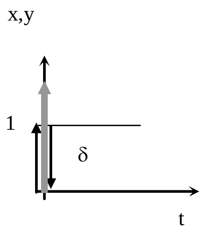
Števec prenosne funkcije vsebuje enega ali več korenov (ničel) v koordinatnem izhodišču.

$$G(s) = \frac{Š(s)}{I(s)} = \frac{X(s)}{Y(s)} = \frac{s^j Š(s)}{I^{'}(s)}; \quad Š(0) \neq 0; \quad I(0) \neq 0$$



Diferencirni D člen:

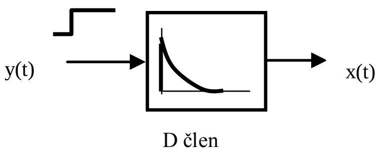


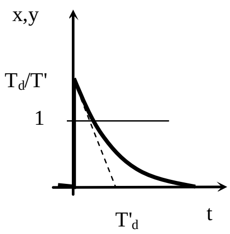


$$x(t) = T_d \frac{dy}{dt}; \quad G(s) = \frac{X(s)}{Y(s)} = T_d \cdot s$$

$$x(t) = \frac{dy}{dt} \cdot T_d = \frac{dy}{dt} \cdot K_d = \delta(t) \cdot T_d$$

T_d… diferencialna časovna konstanta
δ … odziv na stopničasto vzbujanje je Dirac-ov impulz



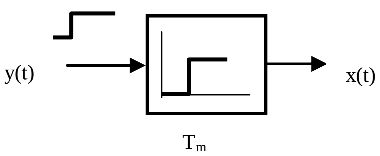


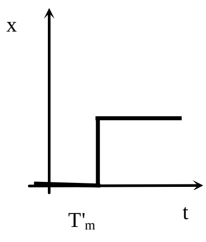
$$DE: \quad T'_d \, \dot{x}(t) + x(t) = T_d \, \dot{y}(t)$$

$$PF: \quad G(s) = \frac{X(s)}{Y(s)} = \frac{sT_d}{1 + T'_d \, s}$$

stopnični odziv: $x(t) = \frac{T_d}{T_{d'}} \cdot e^{-\frac{t}{T'_d}}$

Zakasnilni člen – člen z mrtvim časom





$$PF: \quad G(s) = \frac{X(s)}{Y(s)} = e^{-sT_m}$$

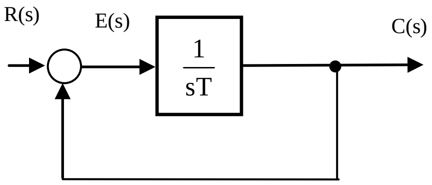
stopnični odziv:

$$x(t) = y(t - T_m) = 0; \quad t \langle T_m$$

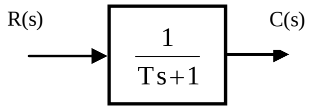
$$x(t) = y(t - T_m) = 1; \quad t \geq T_m$$

Proporcionalni sistem 1. reda PT1 (P1)

Povratno-zančna blok shema:



Poenostavljena shema:



$$\text{DE}: \quad T\dot{c}(t) + c(t) = K_p r(t)$$

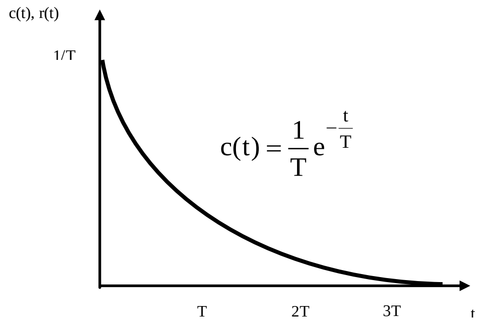
$$\text{PF}: \quad G(s) = \frac{C(s)}{R(s)} = \frac{K_p}{1+Ts}$$

K_p… ojačanje sistema
T … časovna konstanta sistema

Odziv sistema PT1 na impulz δ (naravni odziv)

$$C(s) = \frac{K_p}{1+Ts} \cdot 1$$

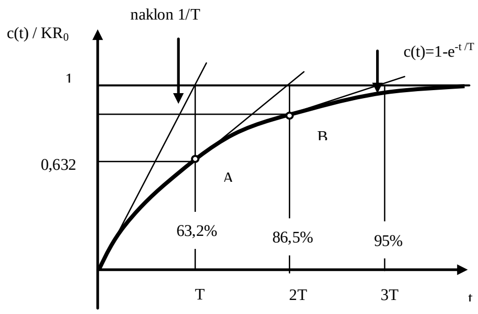
$$c(t) = \frac{K_p}{T} \cdot e^{-\frac{t}{T}}; \quad t \geq 0$$



Odziv sistema PT1 na stopničasto vzbujanje

$$C(s) = \frac{K_p}{1+Ts} \cdot \frac{R_0}{s} = K_p R_0 \left(\frac{1}{s} - \frac{T}{1+Ts} \right)$$

$$c(t) = K_p R_0 (1 - e^{-\frac{t}{T}}); \quad t \geq 0$$



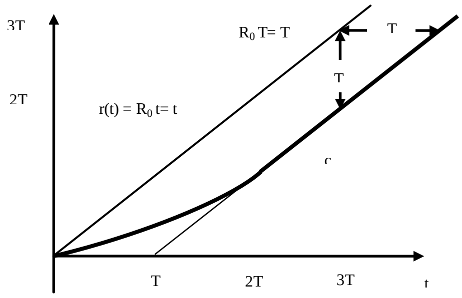
Regulacijski sistem v primeru Kp = 1 v ustaljenem stanju nima pogreška:

$$c(t) = R_0 (1 - e^{-\frac{t}{T}}) \quad \Rightarrow \quad e(t) = R_0 - c(t) = R_0 e^{-\frac{t}{T}} \quad \Rightarrow \quad e(\infty) = 0$$

Odziv sistema PT1 na linearno naraščajoči vhodni signal

$$C(s) = \frac{K_p}{1+Ts} \cdot \frac{R_0}{s^2} = K_p R_0 \left(\frac{1}{s^2} - \frac{T}{s} + \frac{T^2}{1+Ts} \right)$$

$$c(t) = K_p R_0 (t - T1 + Te^{-\frac{t}{T}}); \quad t \geq 0$$



Regulacijski sistem ima v primeru Kp = 1 v ustaljenem stanju konstantni pogrešek:

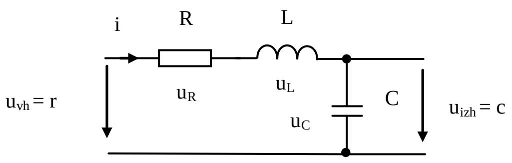
$$c(t) = R_0 (t - T + Te^{-\frac{t}{T}}) \quad \Rightarrow \quad e(t) = R_0 t - c(t) = R_0 T(1 - e^{-\frac{t}{T}}) \quad \Rightarrow \quad e(\infty) = R_0 T$$

Primerjava odzivov

- impulz δ: $c(t) = \frac{K_p}{T} \cdot e^{-\frac{t}{T}}; \quad t \geq 0$
- stopnica: $c(t) = K_p (1 - e^{-\frac{t}{T}}); \quad t \geq 0$
- linearno naraščajoči signal: $c(t) = K_p (t - T1 + Te^{-\frac{t}{T}}); \quad t \geq 0$

Sistemi drugega reda PT2 (P2)

Splošna oblika modela



$$u_{vh} = u_R + u_L + u_c = i \cdot R + L\frac{di}{dt} + \frac{1}{C}\int i \cdot dt; \quad i = C\frac{du_C}{dt};$$

$$RC\frac{du_C}{dt} + LC\frac{d^2u_C}{dt^2} + u_C = u_{vh}$$

$$\frac{d^2u_C}{dt^2} + \frac{R}{L}\frac{du_C}{dt} + \frac{1}{LC}u_C = \frac{1}{LC}u_{vh}$$

$$\ddot{c}(t) + 2\zeta\omega_n\dot{c}(t) + \omega_n{}^2 c(t) = \omega_n{}^2 r(t); \quad \zeta = \frac{R}{2}\sqrt{\frac{C}{L}}; \quad \omega_n = \frac{1}{\sqrt{LC}}$$

Dobljeni izraz preko Laplace-ove transformacije transformiramo:

$$C(s)\cdot(s^2 + 2\zeta\omega_n s + \omega_n^2) = \omega_n^2 \cdot R(s) \quad \Rightarrow \quad G(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s-s_1)(s-s_2)}$$

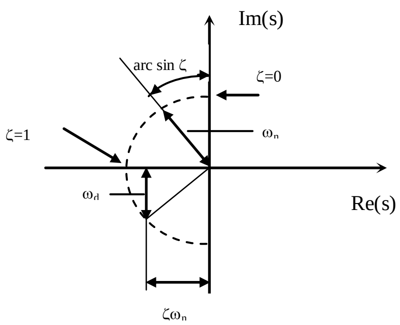
$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0; \quad \Rightarrow \quad s_{1,2} = -\frac{2\zeta\omega_n}{2} \pm \sqrt{\frac{4\zeta^2\omega_n^2 - 4\omega_n^2}{4}} = \omega_n(-\zeta \pm j\omega_n\sqrt{1-\zeta^2}) = \delta \pm j\omega_d;$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

- ω_d faktor dušenega nihanja.

Glede na vrednost dušenja ζ ima stopničasti odziv štiri značilne oblike.

a) dušeno nihanje – podkritično dušenje: $0 < \zeta < 1$



Stopnični odziv R(s) = 1/s

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}\; t - \varphi\phi\right)$$

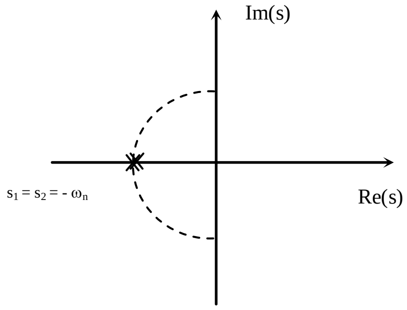
$$\phi = arc\, tg\frac{\sqrt{1-\zeta^2}}{\zeta}$$

b) nedušeno nihanje: $\zeta = 0$

Ko je dušenje $\zeta = 0$, ležita oba pola na imaginarni osi in velja: $s_{1,2} = \pm j\omega_n$. Odziv na stopničasti vhod R(s) = 1/s je nedušeno nihanje:

$$c(t) = 1 - \cos \omega_n t$$

c) meja aperiodičnosti – kritično dušenje: $\zeta = 1$

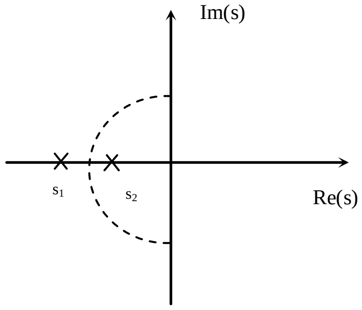


Stopnični odziv R(s) = 1/s

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s+\omega_n)^2}$$

$$c(t) = 1 - e^{-\omega_n t}(1 + \omega_n t)$$

d) aperiodični odziv – nadkritično dušenje: $\zeta > 1$



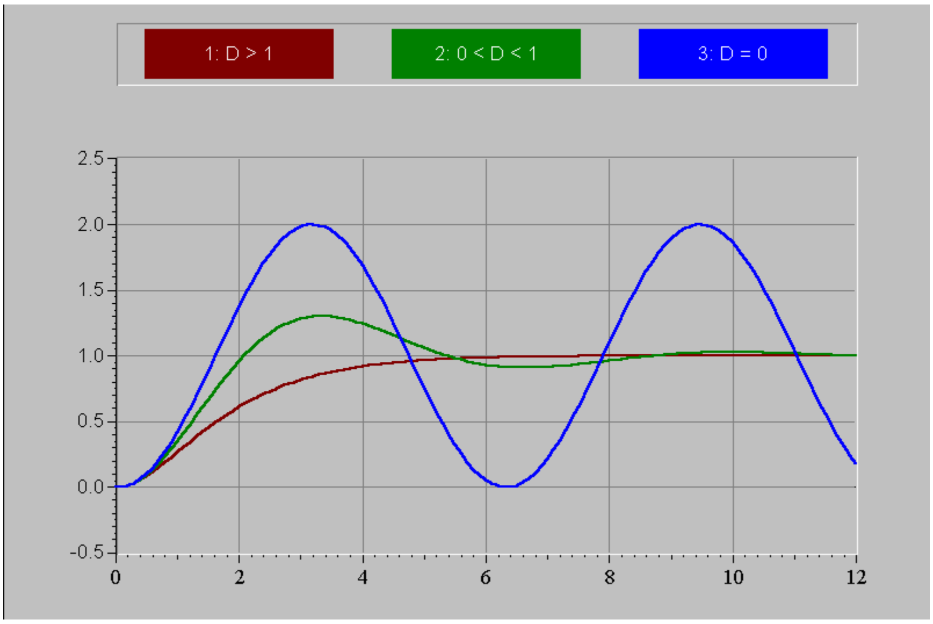
$$s_{1,2} = \omega_n(-\zeta \pm \sqrt{\zeta^2 - 1}); \quad s_1 = \cdot \quad s_2 = -\frac{1}{T_2}$$

Stopnični odziv R(s) = 1/s:

$$c(t) = 1 - \frac{\omega_n}{2\sqrt{\zeta^2 - 1}}\left(T_1 e^{-\frac{t}{T_1}} - T_2 e^{-\frac{t}{T2}}\right)$$

Sistemi z dušilnim faktorjem $0{,}5 < \zeta < 0{,}8$ najhitreje dosežejo bližino referenčne vrednosti, med aperiodičnimi odzivi pa je najhitrejši odziv pri kritičnem dušenju. Aperiodični odzivi relativno počasi reagirajo na vhodne signale.

Značilni odziv sistema PT2 na stopnico



$\zeta \geq 1$
kritično in nadkritičo dušenje

$0 < \zeta < 1$
dušeno nihanje-podkritično dušenje

$\zeta = 0$
nedušeno nihanje

Tipični parametri odziva sistema