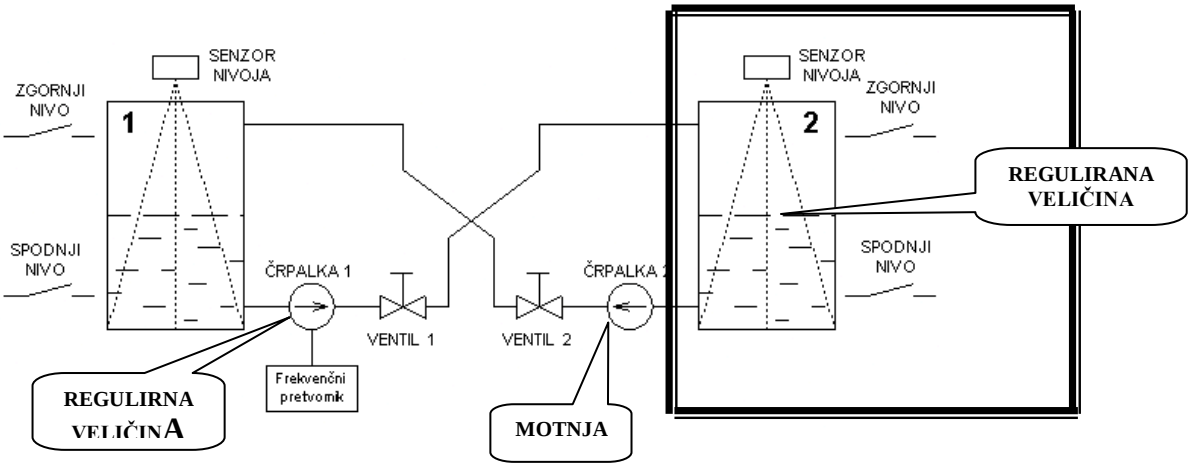
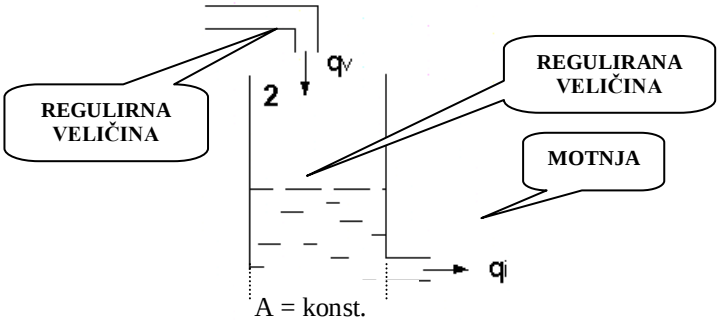
PRIMER REGULACIJE NIVOJA

1. Tehnološka shema modela



2. Modeliranje



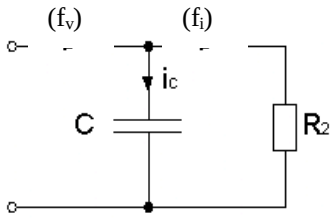
A = konst.

$$q_v(t) - q_i(t) = A\dot{h}$$

$A = \pi r^2 = 5026\ cm^2$

q_{vmax}=5 l/min $\qquad$ q_i=10,91 l/min

Električni ekvivalent posode:

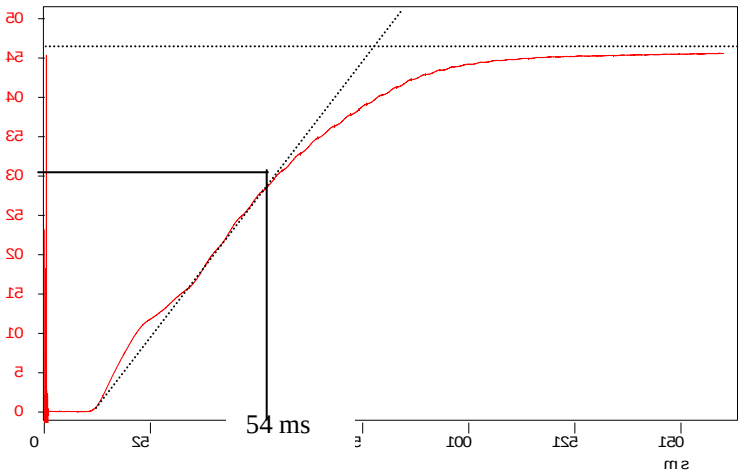


$$e = \frac{1}{C}\int_0^t (i_v(\tau) - i_i(\tau))d\tau$$

$$\dot{e}C = i_v(t) - i_i(t)$$

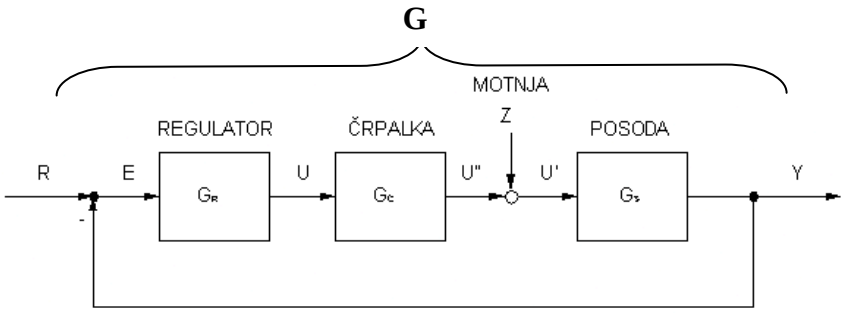
napetost tahogeneratorja (V)

63,2 %



čas

3. Matematični model in regulacijska proga modela:



Izpeljava prenosne funkcije modela v primeru, ko je motnja enaka 0: $Z=0$

$$Y = G_S U = G_S G_Č G_R E = G_S G_Č G_R (R - Y)$$

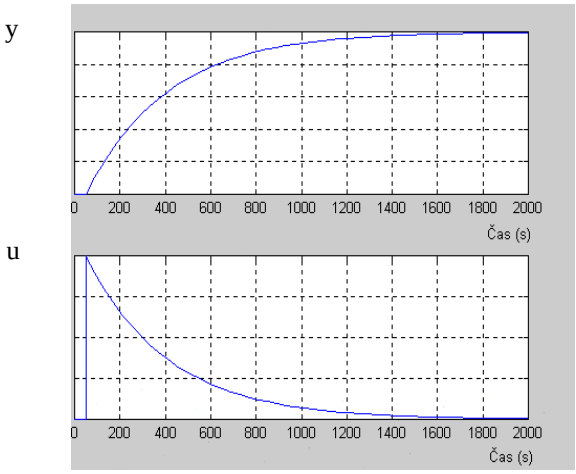
$$Y = \frac{G_S G_Č G_R}{1 + G_S G_Č G_R} R \qquad G_S = \frac{1}{T_S s} \qquad G_Č = \frac{1}{1 + sT_Č}$$

a.) P-REGULATOR $\underline{\underline{G_R = K_P}}$

$$Y = \frac{\frac{1}{sT_S}\frac{1}{1+sT_Č}K_P}{1+\frac{1}{sT_S}\frac{1}{1+sT_Č}K_P} R \qquad \underline{\underline{Y = \frac{K_P}{sT_S(1+sT_Č)+Kp} R}}$$

$$G = \frac{Y}{R} = \frac{K_P}{sT_S(1+sT_Č)+Kp} = \underline{\underline{\frac{K_P}{s^2 T_S T_Č + (T_S + T_Č)s + K_P}}}$$

$$t \to \infty \longrightarrow s \to 0 \longrightarrow \underline{\underline{G = 1}}$$



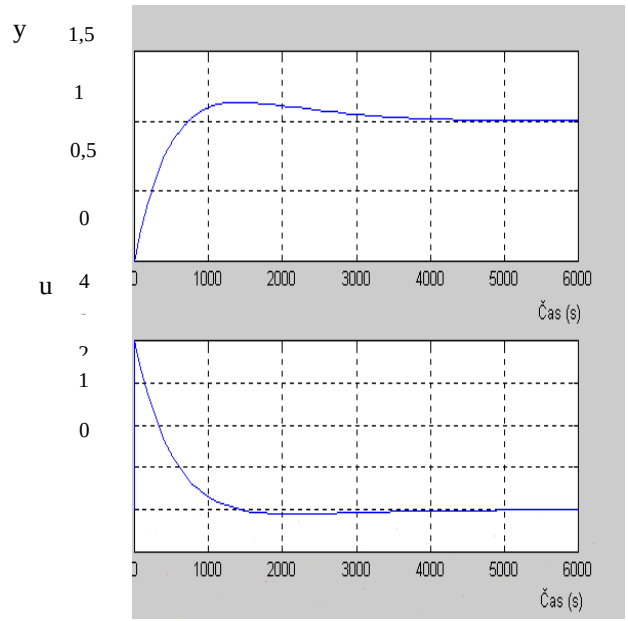
b.) PI-REGULATOR $\underline{\underline{G_R = K_P + \frac{1}{sT_i}}}$

$$Y = \frac{\frac{1}{sT_S}\frac{1}{1+sT_Č}\left(K_P + \frac{1}{sT_i}\right)}{1+\frac{1}{sT_S}\frac{1}{1+sT_Č}\left(K_P + \frac{1}{sT_i}\right)} R$$

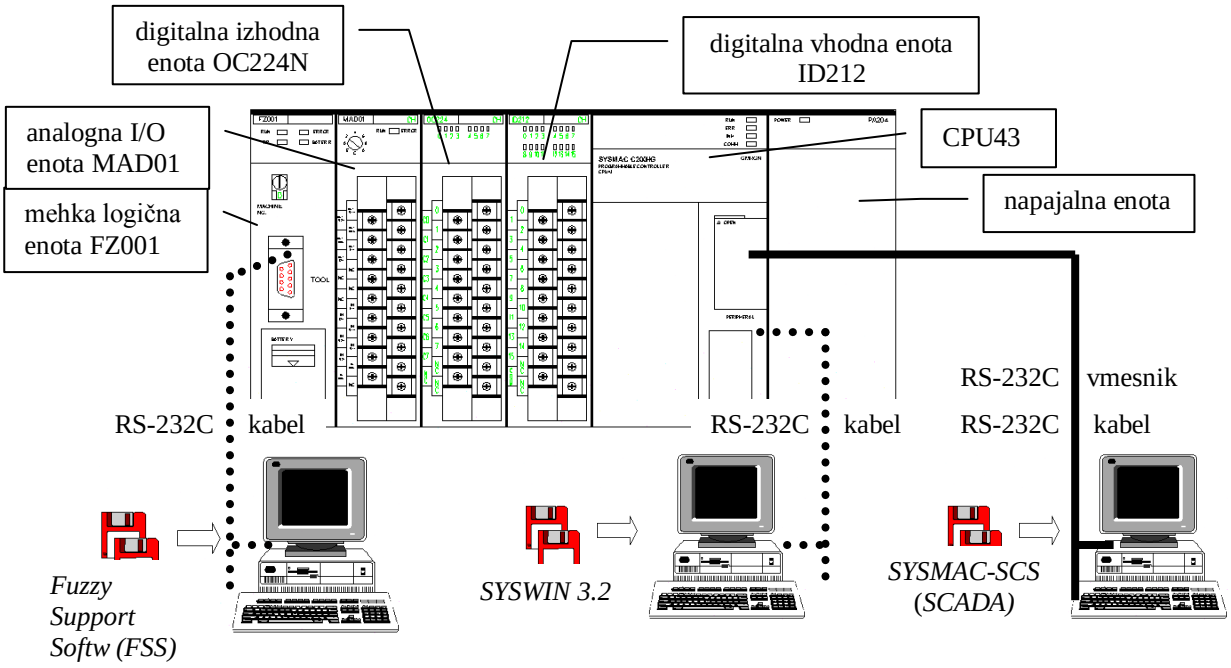
$$\underline{\underline{Y = \frac{sT_i K_P + 1}{sT_S(1+sT_Č)sT_i + K_P sT_i + 1} R}}$$

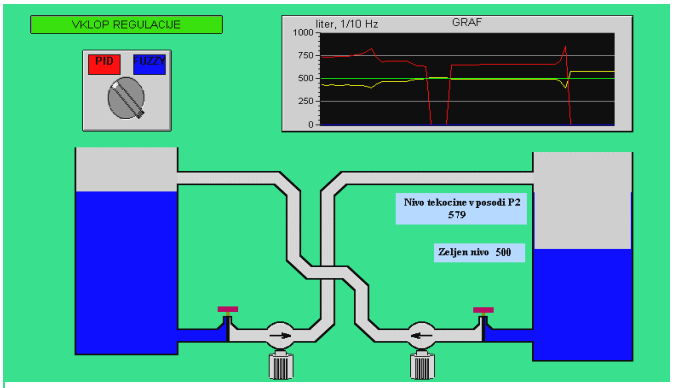
$$\underline{\underline{G = \frac{Y}{R} = \frac{sT_i K_P + 1}{s^3 T_S T_Č T_i + s^2 T_S T_i + K_P sT_i + 1}}}$$

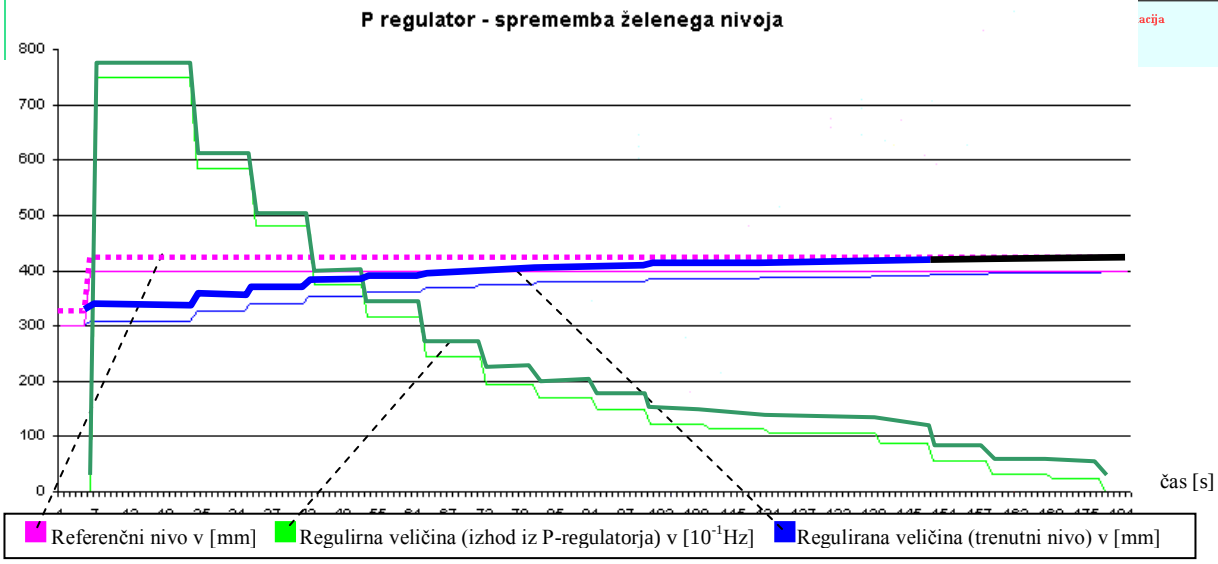
$$t \to \infty \longrightarrow s \to 0 \longrightarrow \underline{\underline{G = 1}}$$



3. Krmilno – regulacijska oprema







Sistemi analize in načrtovanje vodenja procesov

1. Laplace-ova transformacija $\mathcal{L}$

$$L(f(t)) = F(s) = \int_0^\infty f(t)\cdot e^{-st}dt$$

$$L^{-1}(F(s)) = f(t) = \frac{1}{2\pi j}\int_{e-j\omega}^{e+j\omega} F(s)\cdot e^{st}ds$$

Teoremi Laplace-ove transformacije

$$L[k\cdot f(t)] = k\cdot F(s)$$

$$L[f_1(t)\pm f_2(t)] = F_1(s)\pm F_2(s)$$

$$L\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0^+); \quad f(0^+) = \lim_{t\to 0^+} f(t)$$

$$L\left[\frac{d^n}{dt^n}f(t)\right] = s^nF(s) - \lim_{t\to 0^+}\left[s^{n-1}f(t) + s^{n-2}\frac{d\,f(t)}{dt} + \cdots + \frac{d^{n-1}f(t)}{dt^{n-1}}\right] = s^nF(s) - s^{n-1}f(0^+) - s^{n-2}f^{(1)}(0^+) - \cdots - f^{(n-1)}(0^+)$$

$$L\left[\int_0^t f(\tau)d\tau\right] = \frac{F(s)}{s}$$

$$L\left[\int_0^{t_1}\int_0^{t_2}\cdots\int_0^{t_n} f(\tau)d\tau dt_1\cdots dt_{n-1}\right] = \frac{F(s)}{s^n}$$

$$L[f(t-\tau)] = e^{-\tau s}F(s)$$

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

$$L[e^{-\alpha t}f(t)] = F(s+\alpha)$$

$$L\left[f\left(\frac{t}{\alpha}\right)\right] = \alpha\cdot F(\alpha s)$$

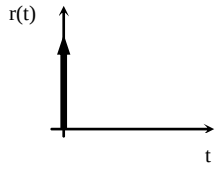
$$L[t^n f(t)] = (-1)^n\frac{d^n}{ds^n}F(s) \quad n = 1,2,3,\ldots$$

Laplace-ovi transformi

1	1	$\delta(t)$
2	$\frac{1}{s}$	$1(t)$
3	$\frac{1}{s^2}$	$\frac{1}{s}$
4	$\frac{2!}{s^3}$	t^2
5	$\frac{3!}{s^4}$	t^3
6	$\frac{m!}{s^{m+1}}$	t^m
7	$\frac{1}{s+a}$	e^{-at}
8	$\frac{1}{(s+a)^2}$	te^{-at}
9	$\frac{1}{(s+a)^3}$	$\frac{1}{2!}t^2e^{-at}$
10	$\frac{1}{(s+a)^m}$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{a}{s(s+a)}$	$1-e^{-at}$
12	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}\left(at-1+e^{-at}\right)$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{\omega^2}{s^2+2\zeta\omega s+\omega^2}$	$\frac{\omega}{\sqrt{1-\zeta^2}}e^{-\zeta\omega t}\sin\omega\sqrt{1-\zeta^2 t}$
16	$\frac{a^2}{s(s+a)^2}$	$1-e^{-at}(1+at)$
17	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-bt}-ae^{-at}$
18	$\frac{\omega}{s^2+\omega^2}$	$\sin\omega t$
19	$\frac{s}{s^2+\omega^2}$	$\cos\omega t$
20	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$
21	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin\omega t$
22	$\frac{a^2+\omega^2}{s\left[(s+a)^2+\omega^2\right]}$	$1-e^{-at}\left(\cos\omega t+\frac{a}{\omega}\sin\omega t\right)$

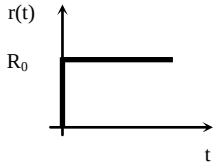
Signali v Laplace-ovem prostoru

Impulzna funkcija:



$$r(t) = \delta(t) \quad R(s) = 1$$

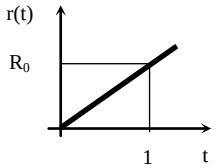
Stopničasta funkcija:



$$r(t) = R_0; t \geq 0 \quad R(s) = \frac{R_0}{s}$$

$$r(t) = 0; t \langle 0$$

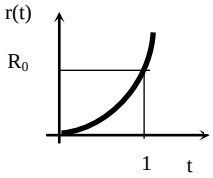
Linearno naraščajoča funkcija (rampa):



$$r(t) = R_0 t; t \geq 0 \quad R(s) = \frac{R_0}{s^2}$$

$$r(t) = 0; t \langle 0$$

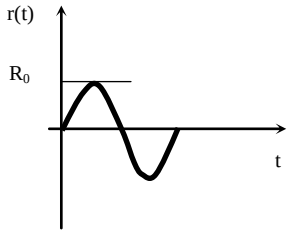
Parabolična funkcija:



$$r(t) = R_0 t^2; t \geq 0 \quad R(s) = \frac{R_0}{s^3}$$

$$r(t) = 0; t \langle 0$$

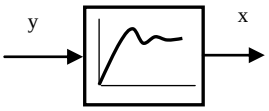
Sinusna funkcija:



$$r(t) = R_0 \sin \omega t; t \geq 0 \quad R(s) = \frac{R_0 \omega}{s^2 + \omega^2}$$

$$r(t) = 0; t \langle 0$$

2. Diferencialne enačbe



$$a_n \cdot \frac{d^n y}{dt^n} + a_{n-1} \cdot \frac{d^{n-1} y}{dt^{n-1}} \cdots + a_1 \cdot \frac{dy}{dt} + a_0 \cdot y \quad = \quad b_m \cdot \frac{d^m x}{dt^m} + b_{m-1} \cdot \frac{d^{m-1} x}{dt^{m-1}} \cdots + b_1 \cdot \frac{dx}{dt} + b_0 \cdot x$$

$$a_1, a_0, b_0 \neq 0$$

$$a_1 \cdot \frac{dy}{dt} + a_0 \cdot y \quad = \quad b_0 \cdot x$$

Primer:

$$\ddot{x} + 3\dot{x} + 2x = 0 \qquad x(0) = a,\ \dot{x}(0) = b$$

$$L[x(t)] = X(s); \quad L[\dot{x}(t)] = s \cdot X(s) - x(0); \quad L[\ddot{x}(t)] = s^2 \cdot X(s) - s \cdot x(0) - \dot{x}(0)$$

$$(s^2 \cdot X(s) - s \cdot x(0) - \dot{x}(0)) + 3(s \cdot X(s) - x(0)) + 2X(s) = 0$$

$$(s^2 \cdot X(s) - as - b) + 3(s \cdot X(s) - a) + 2X(s) = 0 \quad ali \quad X(s) \cdot (s^2 + 3s + 2) = as + b + 3a$$

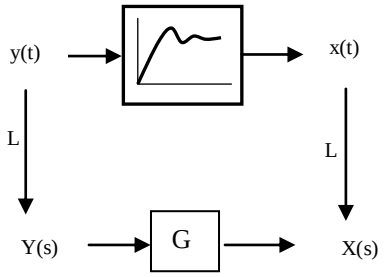
$$X(s) = \frac{as + b + 3a}{s^2 + 3s + 2} = \frac{as + b + 3a}{(s+1)(s+2)} = \frac{2a + b}{s+1} - \frac{a+b}{s+2}$$

Inverzna Laplace-ova transformacija:

(pravilo Laplace-ove transformacije št.7)

$$x(t) = L^{-1}[X(s)] = L^{-1}\left[\frac{2a+b}{s+1}\right] - L^{-1}\left[\frac{a+b}{s+2}\right] = (2a+b)e^{-t} - (a+b)e^{-2t} \qquad \Big| t \geq 0$$

3. Prenosna funkcija G(s)



$$G(s) = \frac{L(izhod)}{L(vhod)} = \frac{X(s)}{Y(s)} = \frac{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}$$

$$X(s) = G(s) \cdot Y(s)$$

$$G(s) = \frac{k(s+z_1)(s+z_2)\cdots(s+z_n)}{(s+p_1)(s+p_2)\cdots(s+p_m)}$$

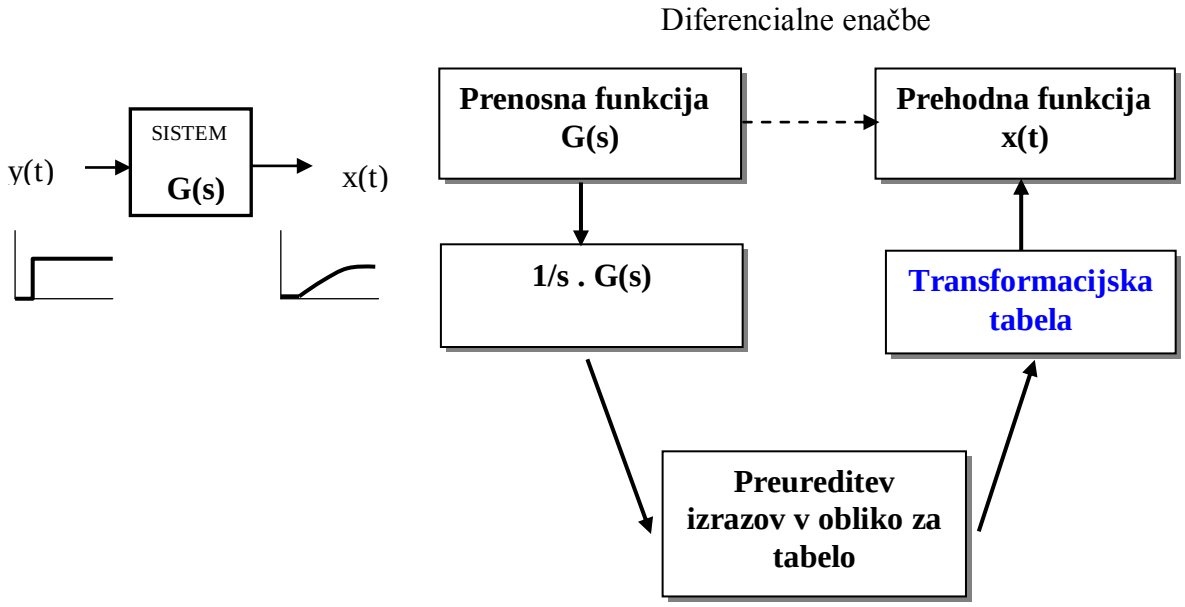
$$G(s) = \frac{K(T_{b1}s+1)(T_{b2}s+1)\cdots(T_{bn}s+1)}{(T_{a1}s+1)(T_{a2}s+1)\cdots(T_{am}s+1)}$$

Osnovna pravila za transformacijo:

$$L(kx(t)) = kX(s), \qquad L(x_1(t) \pm x_2(t)) = X_1(s) \pm X_2(s)$$

$$L\left(\frac{d}{dt}x(t)\right) = sX(s), \quad L\left(\frac{d^2}{dt^2}x(t)\right) = s^2 X(s), \quad L(\int x(t)dt) = \frac{1}{s}X(s)$$

4. Prehodna funkcija



Transformacijska tabela najpogostejših funkcij:

$\frac{1}{s}\cdot G(s)$	$x(t)$
$\frac{1}{s^2T}$	$y(t)\cdot\frac{t}{T}$
$\frac{sT}{s(1+sT)}$	$y(t)\cdot e^{-\frac{t}{T}}$
$\frac{1}{s(1+sT)}$	$y(t)\cdot(1-e^{-\frac{t}{T}})$
$\frac{1}{(1+sT_1)(1+sT_2)}$	$y(t)\cdot\frac{1}{T_1-T_2}(e^{-\frac{t}{T_1}}-e^{-\frac{t}{T_2}})$
$\frac{1}{s(1+sT_1)(1+sT_2)}$	$y(t)\cdot(1-\frac{T_1e^{-\frac{t}{T_1}}-T_2e^{-\frac{t}{T_2}}}{T_1-T_2}$
$\frac{1}{s}$	$y(t)\cdot 1$

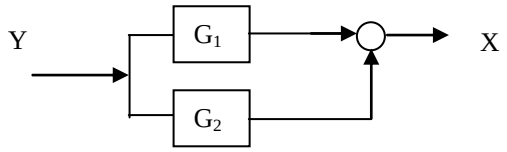
Primer:

$$G(s)=\frac{1}{1+sT}$$

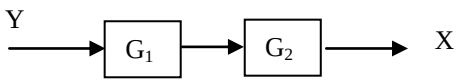
$$\frac{1}{s}G(s)=\frac{1}{s(1+sT)} \quad\Rightarrow\quad x(t)=y(t)\cdot(1-e^{-\frac{t}{T}})$$

5. Blok diagram

$X(s) = (G_1(s) \pm G_2(s)).Y(s)$

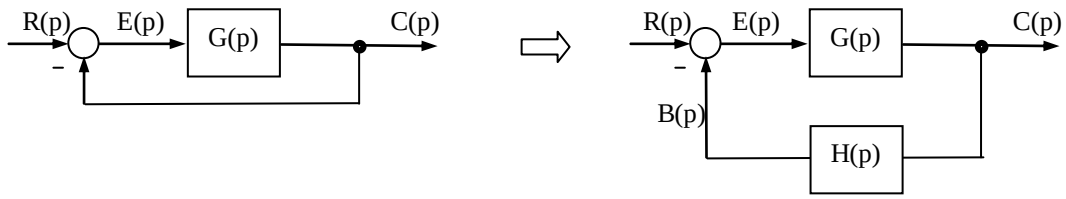


$X(s) = G_1(s) .G_2(s).Y(s)$



6. Modeliranje

Blok diagram zaprtozančnega sistema:

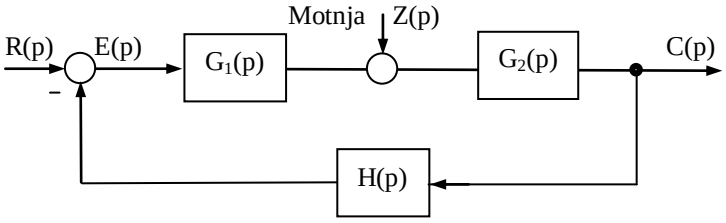


$$G(p)=\frac{C(p)}{E(p)};\quad \textit{prenosna funkcija direktne veje}$$

$$G(p)\cdot H(p)=\frac{B(p)}{E(p)};\quad \textit{prenosna funkcija odprte zanke}$$

$$\frac{G(p)}{1+G(p)H(p)}=\frac{C(p)}{R(p)};\quad \textit{prenosna funkcija zaprte zanke}$$

Zaprtozančni sistemi pri motilnem in referenčnem signalu:

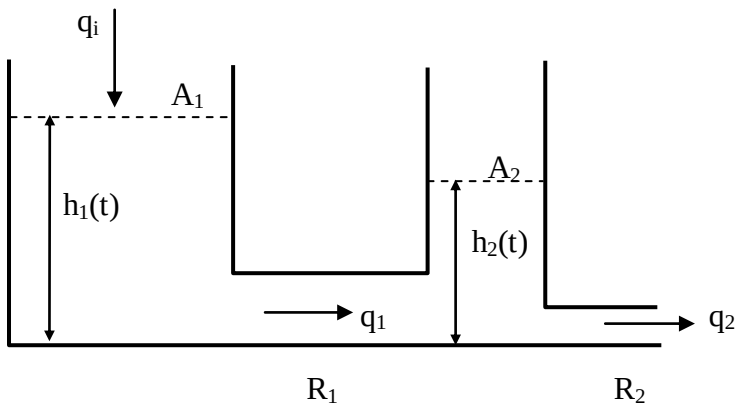


$$\frac{G_2(p)}{1+G_1(p)G_2(p)H(p)} = \frac{C_Z(p)}{Z(p)}; \qquad |G_1(p)G_2(p)H(p)| \gg 1; \; |G_1(p)H(p)| \gg 1; \quad \Rightarrow \quad \left|\frac{C_Z(p)}{Z(p)}\right| \approx 0$$

$$\frac{G_1(p)G_2(p)}{1+G_1(p)G_2(p)H(p)} = \frac{C_R(p)}{R(p)}; \qquad |G_1(p)G_2(p)H(p)| \gg 1; \quad \Rightarrow \quad \left|\frac{C_R(p)}{R(p)}\right| \approx \frac{1}{|H(p)|}$$

$$C(p) = C_R(p) + C_Z(p) = \frac{G_2(p)}{1+G_1(p)G_2(p)H(p)}\left[G_1(p)R(p) + Z(p)\right]$$

Primer – hidravlični sistem z dvema nivojskima posodama:



$q_i(t)$	vhodni pretok		
$h_1(t)$	nivo v prvi posodi	$h_2(t)$	nivo v drugi posodi
A_1	površina gladine v prvi posodi	A_2	površina gladine v drugi posodi
$q_1(t)$	pretok med obema posodama	$q_2(t)$	iztok iz druge posode

Ravnotežna enačba prve posode:

$$A_1 \frac{dh_1(t)}{dt} = q_i(t) - q_1(t)$$

$$q_1(t) = \frac{h_1(t) - h_2(t)}{R_1}$$

Ravnotežna enačba druge posode:

$$A_2 \frac{dh_2(t)}{dt} = q_1(t) - q_2(t)$$

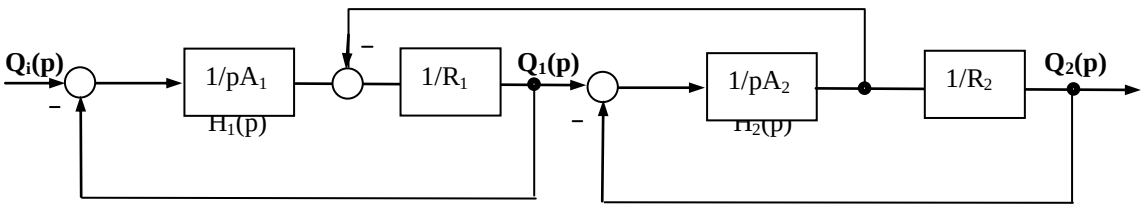
$$q_2(t) = \frac{h_2(t)}{R_2}$$

Laplace-ova transformacija enačb:

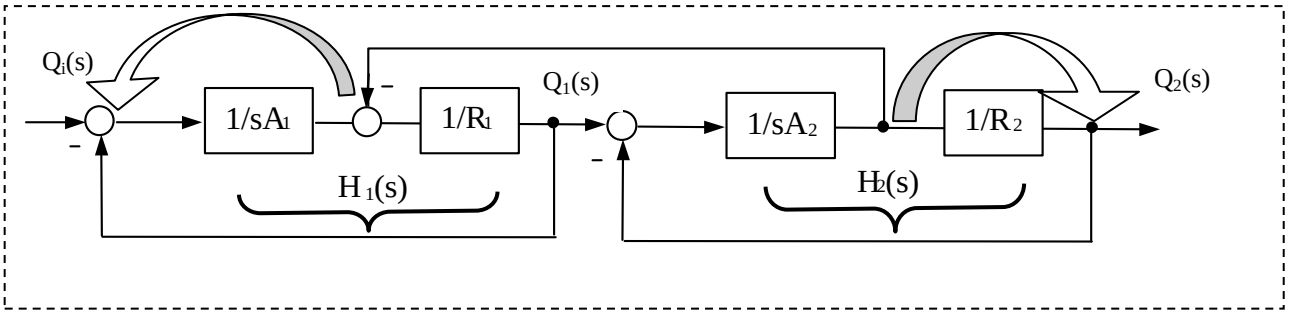
$$A_1 p H_1(p) = Q_i(p) - Q_1(p) \quad \Rightarrow \quad H_1(p) = \frac{1}{A_1 p}\left[Q_i(p) - Q_1(p)\right]; \quad Q_1(p) = \frac{H_1(p) - H_2(p)}{R_1}$$

$$A_2 p H_2(p) = Q_1(p) - Q_2(p) \quad \Rightarrow \quad H_2(p) = \frac{1}{A_2 p}\left[Q_1(p) - Q_2(p)\right]; \quad Q_2(p) = \frac{H_2(p)}{R_2}$$

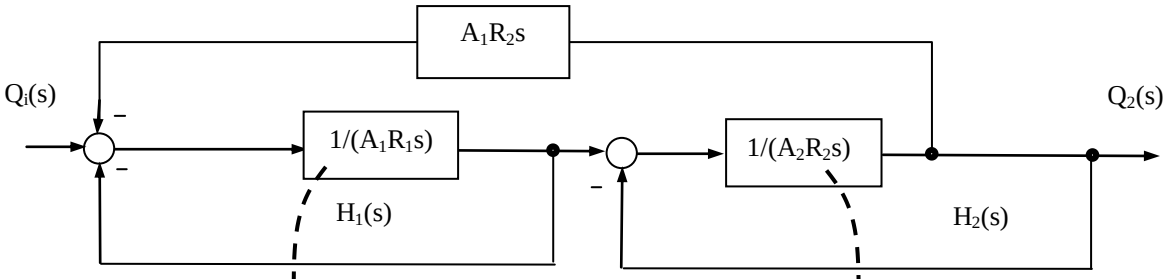
Blok diagram hidravličnega sistema:



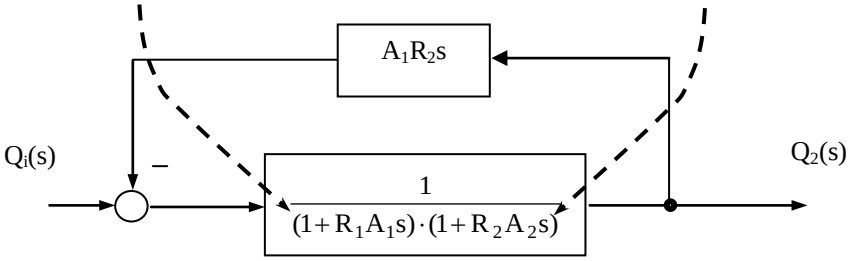
Poenostavitev blok diagrama hidravličnega sistema



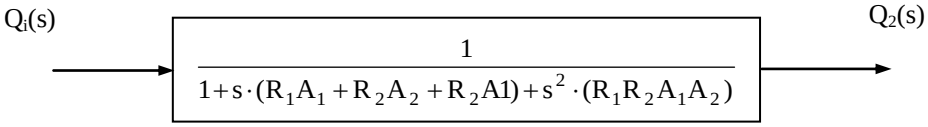
PRAVILO 5, PRAVILO 4, PRAVILO 1



PRAVILO 6, PRAVILO 1



PRAVILO 6



Primer določitve odvisnosti H1 od Qi